PROGRAMA DE DOCTORADO EN CIENCIAS ECONÓMICAS, EMPRESARIALES Y JURÍDICAS

DEPARTAMENTO DE ECONOMÍA Y EMPRESA

UNIVERSIDAD DE ALMERÍA

TESIS DOCTORAL

ANOMALÍAS EN LA ELECCIÓN INTERTEMPORAL: LOS EFECTOS PLAZO E INTERVALO

ANOMALIES IN INTERTEMPORAL CHOICE: THE DELAY AND INTERVAL EFFECTS

TESIS PRESENTADA POR:

Dª Piedad Ortiz Fernández

DIRIGIDA POR:

Dr. D. Salvador Cruz Rambaud

Almería, mayo de 2021

La perseverancia no es una carrera larga;

es una serie de carreras, una tras otra.

Walter Elliot

AGRADECIMIENTOS

En primer lugar, quería agradecer a mi director Salvador Cruz Rambaud todo el apoyo que me ha dado, así como toda la paciencia mostrada a lo largo de los últimos cinco años. Él fue quien me brindó la posibilidad de conocer este mundo, el mundo de la investigación, y que, a pesar de ser un arduo camino y de las dificultades encontradas, siempre tuvo una palabra de aliento que me ayudó a continuar. También quería darle las gracias por todo el tiempo dedicado, pues sin él no habría sido posible llegar hasta aquí.

Del mismo modo, quería agradecer a mi familia por toda la fe depositada en mí, así como, el apoyo incondicional que me han brindado.

Tampoco puedo olvidar a mi marido por todo el tiempo que me ha regalado. Gracias por comprender la situación, por tu paciencia y por no perder la esperanza en mí.

Para terminar, quería agradecer también a todos mis compañeros por cada una de sus palabras de apoyo, ánimo y cariño, pero, sobre todo, quería agradecer todos los consejos y toda la ayuda que me han ofrecido, fundamentalmente, a Jorge, mi querido compañero y amigo.

RESUMEN

La presente Tesis Doctoral tiene como objetivo analizar dos de las principales anomalías en la elección intertemporal: el efecto plazo y el efecto intervalo. La primera ha sido una de las anomalías más estudiadas por los economistas y psicólogos, mientras que la segunda apenas ha sido analizada. Uno de los principales motivos de esta circunstancia ha sido la tradicional identificación de ambas anomalías en un único efecto: el efecto plazo. Es por ello que el principal objetivo de esta tesis ha sido analizar ambas anomalías desde un punto de vista teórico. Esto posibilitará la diferenciación entre ambos efectos y, por tanto, la aportación de una definición matemática del efecto intervalo. Consideramos que esta base teórica contribuirá a que este efecto se analice con mayor detalle, como lo está haciendo el efecto plazo y el resto de las anomalías del Modelo de Utilidad Descontada. Para conseguir este objetivo, presentamos, en primer lugar, una revisión sistemática de la literatura de las principales anomalías en la elección intertemporal. En ella, pueden observarse las diferentes áreas de estudio de estos efectos (Economía, Psicología, Medicina y Neurología) y la evolución de los mismos a lo largo de los últimos 20 años. Esta revisión nos permite constatar que el efecto intervalo apenas es conocido, pues solo encontramos tres artículos que tratan sobre el mismo. Una vez justificada la importancia de promover la investigación acerca del efecto intervalo, en el siguiente capítulo analizamos el efecto plazo pues, para delimitar el efecto intervalo, se hace necesario el análisis previo del efecto plazo. En este capítulo, presentamos una definición matemática del efecto plazo desde un punto de vista estacionario y desde un punto de vista dinámico, y lo relacionamos con la subaditividad (inicialmente, el efecto intervalo también fue confundido con este concepto). Una vez analizados el efecto plazo y la subaditividad en ambos contextos (estacionario y dinámico), proponemos una nueva función dinámica que explican ambos conceptos simultáneamente, denominada "función de descuento exponencial asimétrica".

Por último, en el siguiente capítulo, introducimos una definición matemática del efecto intervalo y lo comparamos con el efecto plazo para mostrar las diferencias entre ambos. Posteriormente, analizamos los posibles casos en los que puede aparecer este efecto y los estudiamos, desde un punto de vista matemático, junto con el efecto plazo y la subaditividad. Esto nos permitirá redefinir el concepto de efecto intervalo y dividirlo en dos nuevos subefectos: el efecto intervalo creciente y el efecto intervalo decreciente. Finalmente, planteamos el efecto intervalo desde un punto de vista dinámico y concluimos que esta anomalía no tiene sentido en este contexto. Como consecuencia, podemos afirmar que ambos efectos son independientes desde un punto de vista dinámico.

ABSTRACT

This Doctoral Dissertation aims to analyze two of the main anomalies in intertemporal choice: the delay and the interval effects. The delay effect has been one of the anomalies most studied by economists and psychologists but, on the contrary, the interval effect has hardly been analyzed. One of the main reasons for this fact has been the traditional confusion of both anomalies as a single effect: the delay effect. Therefore, the main objective of this Thesis has been to analyze both anomalies from a theoretical point of view. This will allow us to distinguish both effects and therefore, to provide a mathematical definition of the interval effect. We consider that this theoretical basis will help to expand this effect, the same as the delay effect and the rest of the anomalies of the Discounted Utility Model. First, we present a systematic review of the existing literature on the main effects in the field of intertemporal choice. In this chapter, the areas of study of these effects (Economics, Psychology, Medicine, and Neuroscience) and their evolution over the last 20 years can be observed. This review shows that the interval effect is hardly known, as we have only found three articles dealing with it. Once justified the importance of promoting the research on the interval effect, in the next chapter we have analyzed the delay effect because, in order to define the interval effect, it is necessary a previous analysis of the delay effect. In this chapter, we have presented a mathematical definition of the delay effect from a stationary point of view and from a dynamic point of view, and we have related it to the subadditivity (initially, the interval effect was also confused with this concept). Once analyzed the delay effect and the subadditivity in both contexts (stationary and dynamic), we have proposed a new dynamic function which explains both concepts simultaneously, called the “asymmetric exponential discount function”. Finally, in a next chapter, we have provided a mathematical definition of the interval effect and compared it with the delay effect to show the

differences between both concepts. Subsequently, we have analyzed the possible cases in which this effect is present and then we have mathematically studied it together with the delay effect and subadditivity. This has allowed us to redefine the concept of interval effect and to divide it into two new sub-effects: the so-called increasing interval effect and the decreasing interval effect. Finally, we have analyzed the interval effect from a dynamic point of view by observing that this anomaly does not make sense it in this context. Consequently, we could state that both effects are independent from a dynamic point of view.

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CAPÍTULO I.

INTRODUCCIÓN

1. Introducción

La mayoría de las decisiones que tomamos los seres humanos tienen consecuencias para nuestro futuro. Un ejemplo de ello es cuando decidimos qué carrera vamos a estudiar, pues indudablemente esto condicionará nuestra forma de ganarnos la vida en el futuro. O cuando decidimos que vamos a ahorrar para comprar un coche o una vivienda lo que supone que, en el momento actual, vamos a renunciar a la satisfacción de otras necesidades. Sin embargo, no todas las decisiones que tienen consecuencias sobre nuestro futuro implican recompensas monetarias, sino que también hay decisiones que conllevan recompensas no monetarias. Un ejemplo de ello es cuando decidimos llevar una vida sana para mantener una buena salud a lo largo de nuestra vida. Todas estas decisiones, que son consideradas cotidianas para las personas, también pueden aplicarse en otros ámbitos. Un ejemplo podría ser el mundo empresarial. El éxito de las organizaciones está íntimamente ligado a una buena elección de las decisiones empresariales en todos sus niveles jerárquicos, e incluso a nivel político, cuando se decide entre el progreso y el desarrollo sostenible. Pues bien, este tipo de decisiones, que implican el intercambio de recompensas a lo largo del tiempo, se inscriben en lo que se denomina proceso de elección intertemporal.

La elección intertemporal fue tratada por primera vez por Rae (1834), quien argumentó que cualquier desarrollo de la riqueza debía tener en cuenta factores psicológicos (deseo de acumulación y capacidad de ejercer autocontrol), así como factores ambientales (que disminuyan o aumenten la incertidumbre acerca del futuro). Más tarde, Fisher (1930), un economista neoclásico, modeló las decisiones de consumo a lo largo del tiempo, partiendo de una curva de indiferencia. Este trabajo fue la base del denominado modelo de Utilidad Descontada (DU), propuesto por Samuelson (1937). Este modelo se basa en que los decisores descuentan los resultados futuros mediante una tasa de descuento constante a partir de la siguiente expresión:

$$U_0 = \sum_{t=0}^{T} \delta^t u_t ,$$

donde U_0 es el valor actual de la corriente de dichos resultados futuros, u_t representa la utilidad obtenida del resultado esperado en el momento t y δ es el factor de descuento, cuyo valor se encuentra normalmente entre 0 y 1 al tratarse de una preferencia positiva por el tiempo.

Este modelo, al igual que el de Utilidad Esperada (EU), fue rápidamente aceptado por su simplicidad. Ambos modelos están basados en la suma ponderada de las utilidades; sin embargo, las ponderaciones en el modelo EU son probabilidades y no aplazamientos temporales como ocurre con el modelo DU.

Por su parte, el modelo DU se basa en el descuento exponencial, según el cual las decisiones por parte de los agentes son racionales, ya que el aplazamiento de todas las recompensas mediante los mismos intervalos de tiempo no introduce incongruencias dinámicas en las preferencias. En definitiva, las preferencias temporales no cambian por el mero trascurso del tiempo en el vencimiento de las recompensas analizadas (Strotz, 1956). Pero ¿son realmente racionales todas las decisiones? Tradicionalmente, se ha considerado que la mente humana era perfecta y que, por tanto, la toma de decisiones era racional. Sin embargo, numerosos estudios empíricos han demostrado que esta racionalidad es realmente limitada. Un ejemplo de esto es la teoría de la racionalidad limitada (Simon, 1957) según la cual los agentes económicos muestran limitaciones en la memorización, percepción y capacidad cognitiva, lo que hace que no puedan comparar todas las posibles alternativas y, como consecuencia, no puedan tomar decisiones completamente racionales. Es por ello que las personas que trabajan en una empresa, independientemente del nivel jerárquico que ocupen, no pueden ser completamente racionales en sus decisiones y actuaciones. Los estudios de Simon (1957) y Kahneman y Tvesky (1979) fueron la base de nuevas investigaciones tanto en el campo de la Psicología como en el de la Economía (estrategia de comportamiento, neuroestrategia, finanzas comportamentales etc.). Fue a partir de los años 80, con el trabajo de Thaler (1981), cuando comienzan a surgir estudios empíricos que contradicen el modelo normativo propuesto por Samuelson (modelo DU). En efecto, estos trabajos llegaron a conclusiones similares en cuanto a las tasas de descuento obtenidas, pues demostraron que no eran constantes como afirmaba el modelo de Samuelson, surgiendo lo que se denominan anomalías en el proceso de elección intertemporal. Así, las principales anomalías son las siguientes:

- <u>Efecto plazo</u>: Esta anomalía ha sido denominada con numerosos nombres. Los principales han sido "efecto diferencia común" (*common difference effect*), "efecto plazo" (*delay effect*) o "impaciencia decreciente" (*decreasing impatience*). Este efecto consiste en que las tasas de descuento de un individuo varían inversamente a la longitud del tiempo de espera, es decir, a medida que el plazo aumenta, las tasas de descuento se reducen (Kirby y Maraković, 1995; Myerson y Green, 1995; Chapman y Elstein; 1995; Chapman, 1996; Kirby, 1997; Green, Myerson y McFadden, 1997; Kirby, Petry y Bickel, 1999; Green, Myerson y Macaux, 2005; Scholten y Read, 2013). Este efecto se ha manifestado tanto con decisiones monetarias (Benzion et al., 1989; Thaler, 1981), como no monetarias (Christensen-Szalanski, 1984; Chapman, 2001; Thaler, 1981). Según este efecto, una persona puede ser indiferente entre 10 € hoy y 20 € dentro de un mes, pero puede preferir los 20 € dentro de 11 mes a los 10 € en 10 meses. Como puede observarse, el plazo ha sido incrementado en 11 meses, habiéndose producido una reversión de las preferencias. Este efecto puede ser planteado matemáticamente de la siguiente forma (Scholten y Read, 2006):

$$(x,s) \sim (y,\ t) \text{ implica } (x,s\ +\varepsilon) \prec (y,\ t\ +\varepsilon),$$

donde x e y ($x < y$) representan las recompensas equivalentes en los instantes s y t, respectivamente, y $\varepsilon > 0$ hace referencia al incremento del plazo aplicado a cada recompensa.

El efecto plazo puede ser explicado tanto por modelos hiperbólicos como exponenciales (Green et al., 1981; Bocquého, Jacquet y Reynaud, 2013; entre otros).

- Efecto magnitud: Esta anomalía ha sido una de las más estudiadas en la elección intertemporal y ha recibido diversas denominaciones como efecto magnitud (*magnitude effect*) o efecto magnitud absoluta (*absolute magnitude effect*). Este efecto supone un mayor descuento para las cuantías pequeñas que para las más grandes (Green, Myerson y McFadden, 1997; Chapman y Winquist, 1998; Kirby, Petry y Bickel, 1999; Schoenfelder y Hantula, 2003; Estle et al., 2006; Benhabib, Bisin y Schotter, 2010; Andersen et al., 2013; Meyer, 2015). Por ejemplo, un decisor podría preferir 10 € hoy a 15 € dentro de un año, pero también podría preferir 1.500 € en un año a 1.000 € ahora. En ambas elecciones, se ofrecen ganancias de un 50% por esperar durante un año pero, debido a la magnitud de la recompensa, hay una reversión en las preferencias. Su expresión matemática es (Prelec y Loewenstein, 1991):

$$(x,s) \sim (y,t) \text{ implica } (\alpha x, s) \prec (\alpha y, t),$$

donde x e y ($x < y$) representan las recompensas equivalentes en los instantes s y t, respectivamente, y $\alpha > 0$ hace referencia al incremento de la recompensa (en %).

- Efecto signo: Esta anomalía es denominada así (*sign effect*) o asimetría pérdidas-ganancias (*gain-loss asymmetry*). Este efecto implica ratios de descuento más bajos para las pérdidas que para las ganancias (Benzion, Rapoport y Yagil, 1989; Chapman y Winquist, 1998; Estle et al., 2006; Scholten y Read, 2013). Un ejemplo que permite detectar este efecto es el siguiente. Consideremos un decisor que es indiferente entre una ganancia de 50 €, en el momento actual, y una ganancia de 100 €, dentro de un año (como se puede observar el ratio de descuento es del 100%). Sin embargo, este mismo decisor, en el caso de las pérdidas, podría ser indiferente entre una pérdida de 50 €, en el momento actual, y una pérdida de 100 €, dentro de un año (ratio de descuento del 50%). Como podemos observar, el ratio de descuento para las ganancias (100%) es mayor que para las pérdidas (50%), es decir, las personas están dispuestas a pagar menos por aplazar una pérdida, pero exigirán más al posponer una ganancia. Su expresión matemática sería la siguiente (Prelec y Loewenstein, 1991):

$$(x,s) \sim (y,t) \text{ implica } (-x,s) > (-y,t),$$

donde x e y ($x < y$) representan las recompensas equivalentes en los instantes s y t, respectivamente, y $-x$ y $-y$ representan las pérdidas equivalentes en los instantes s y t.

- <u>Efecto secuencia</u>: Esta anomalía se denomina efecto secuencia (*sequence effect*) o efecto secuencia creciente (*improving sequence effect*) y consiste en la preferencia por las secuencias de pagos crecientes frente a las de pagos decrecientes, siempre que sumen el mismo montante total (Loewenstein y Sicherman, 1991). Más concretamente, estos autores demostraron que la mayoría de los encuestados preferían una secuencia de salarios crecientes, a pesar de no tener el mayor valor actualizado en su conjunto, condición que sí cumplían las secuencias de salarios decrecientes. Por ello, pudieron afirmar que las preferencias por secuencias de resultados difieren de las elecciones de resultados individuales. Para las primeras, la preferencia es negativa en el tiempo, mientras que para los segundos la preferencia es positiva. Esta anomalía ha sido detectada en trabajos empíricos, tanto en unidades monetarias como no monetarias (Loewenstein, 1987; Loewenstein y Sicherman, 1991; Loewenstein y Prelec, 1991; Chapman, 1996; Chapman, 2000; Cruz Rambaud et al., 2018; Garcia et al., 2020). Para comprender mejor este efecto, pondremos un ejemplo basado en el trabajo de Loewenstein y Prelec (1991). Estos autores pusieron de manifiesto que los encuestados preferían cenar en un restaurante francés en un mes mejor que en dos meses, es decir, eligieron recibir la recompensa lo antes posible. Sin embargo, cuando implementaron el experimento con secuencias, propusieron una cena en un restaurante griego, en un mes, y una cena en un restaurante francés en dos meses (la mejor recompensa es la última). En este caso, observaron que los encuestados prefirieron el resultado mostrado en secuencia a pesar de tener la mejor recompensa más tardía. Este experimento mostró que, en los resultados individuales, existe una preferencia positiva por el tiempo, mientras que en las elecciones por secuencias existe una preferencia negativa por el tiempo, es decir, una preferencia por secuencias crecientes. Su expresión matemática fue introducida por Muñoz Torrecillas y Cruz Rambaud (2004):

Para todo C_1 y C_2, con $0 < C_1 < C_2$ y t_1 y t_2, con $t_1 < t_2$, se verifica que existe un C_0 suficientemente grande tal que, para todo $C_2 > C_0$ se cumple que:

$$\left\{\left(C_1, t_1\right), \left(C_{2,} t_2\right)\right\} \succ_p \left\{\left(C_2, t_1\right), \left(C_{1,} t_2\right)\right\}$$

- <u>Efecto asimetría respecto al aplazamiento-anticipación</u>: Este efecto, también denominado *delay-speedup asymmetry*, implica ratios de descuento mayores para las elecciones que implican retrasar recompensas que para aquellas decisiones que implican adelantarlas (Loewenstein, 1988;

Benzion, Rapoport y Yagil, 1989; Shelley, 1993; Malkoc y Zauberman, 2006). Un ejemplo de este efecto lo encontramos en el trabajo de Loewenstein (1988) en el que los encuestados estaban dispuestos a pagar 54 dólares por recibir inmediatamente un reproductor de vídeo, cuya recepción estaba prevista para dentro de un año; sin embargo, aquéllos que iban a recibirlo de inmediato, exigían 126 dólares por posponer la recepción del mismo en un año.

- Efecto fecha-aplazamiento: Esta anomalía, también denominada *date-delay effect*, implica que los pagos futuros se descuentan con tasas mayores cuando el tiempo se expresa como plazos (por ejemplo, 3 meses) que cuando se expresa en una fecha concreta del calendario (por ejemplo, el 4 de septiembre). Esta anomalía fue detectada por Read et al. (2005) y, posteriormente, ha sido demostrada en otros trabajos (Scherbaum et al., 2012; Klapproth, 2012; Dshemuchadse et al., 2013; Wang et al., 2015, Breuer y Soypak, 2015; Schoemann et al., 2019).

- Efecto intervalo: Este efecto también es denominado efecto intervalo (*interval effect*) o efecto duración del intervalo (*interval length effect*) y, tradicionalmente, ha sido uno de los menos analizados de la elección intertemporal, posiblemente por haber sido identificado con el efecto plazo. La distinción entre ambos fue realizada, por primera vez, por Read (2001). Esta anomalía implica que las tasas de descuento suelen ser más altas cuanto más cerca están las recompensas, es decir, que las tasas de descuento se reducen a medida que la longitud del intervalo se incrementa. Un ejemplo para comprender este efecto podría ser el siguiente: Un decisor preferirá 100 € en 6 meses a 150 € en 12 meses, o 150 € en 12 meses a 200 € en 18 meses. Observemos que el intervalo de tiempo entre ambos pares de recompensas es de 6 meses y que la opción elegida ha sido la primera. Si ahora incrementamos la longitud del intervalo, el decisor podría cambiar su preferencia, decidiendo esperar, es decir, ahora el decisor podría decidir esperar 18 meses para obtener 200 € a obtener 100 € en 6 meses. En este ejemplo, se puede observar que hay una reversión de la preferencia del decisor con la amplitud del intervalo (que ha pasado de 6 meses a 12 meses). Como ya se ha comentado anteriormente, este efecto apenas ha sido estudiado y ésta es la razón principal de que esta anomalía haya sido objeto de la presente Tesis Doctoral.

Las anomalías enumeradas anteriormente han sido objeto de estudio por parte de muchos investigadores aunque, como ya hemos señalado, no todas han sido igualmente tratadas. En cuanto al ámbito de investigación, no solo han sido investigadas por economistas y psicólogos, sino que sus aplicaciones se han extendido a otras áreas del conocimiento como la Medicina, la Neurociencia o las Ciencias Políticas. Así, la mayor parte de sus investigaciones son empíricas y se han llevado a cabo con recompensan monetarias. Sin embargo, existen algunos estudios que han utilizado otro tipo de recompensas como la salud (Chapman, 2000; Chapman y Weber, 2006; Kang e Ikeda, 2016; Ikeda et al., 2010), las drogas (Giordano et al., 2002; Johnson et al., 2015), el tabaco (Baker et al., 2003; Khwaja et al., 2007; Kang e Ikeda, 2014), la comida (Estle et al., 2007) o incluso las propinas (Chapman y Winquist, 1998; Hesketh (2000).

Uno de los principales objetivos de la investigación en el ámbito de la elección intertemporal es encontrar una función que consiga explicar todas las anomalías que presenta el modelo propuesto por Samuelson. Sin embargo, hasta ahora, este objetivo no ha sido aún conseguido, aunque sí se han aportado una gran cantidad de modelos que explican una o varias anomalías. Los modelos de la elección intertemporal pueden clasificarse en tres grandes categorías:

- <u>Modelos basados en alternativas (Alternative Based Models)</u>: Estos modelos son los denominados modelos de descuento, entre los que podemos encontrar el modelo exponencial, el cuasi-hiperbólico, el de sensibilidad constante y el doble exponencial (Samuelson, 1937; Herrnstein, 1981; Mazur, 1987; Loewenstein y Prelec, 1992; Laibson, 1997; Elbert and Prelec, 2007; McClure et al., 2007; Scholten et al., 2014).

- Modelos basados en atributos (Attibute Based Models): Estos modelos plantean una visión completamente diferente al enfoque del descuento, ya que proponen que los individuos comparen los atributos con opciones. Cada uno de los modelos utiliza una técnica, pero la idea general es comparar los valores dentro de cada atributo; por ejemplo, una pequeña cantidad comparada con una gran cantidad o un corto plazo comparado con un largo plazo. De esta forma, se puede evaluar si algún atributo impulsa la elección. Entre ellos, podemos encontrar el modelo PD, DRIFT, ITCH, BTM y TM (Cheng y González-Vallejo, 2016; Ericson et al., 2015; Read et al., 2003; Dai and Busemeyer, 2014; Scholten y Read, 2013; Sholten et al., 2014).
- Modelos híbridos (Hybrid Models): Estos modelos son una mezcla de los dos anteriores, entre los que podemos destacar el modelo propuesto por Schoten et al. (2014).

La mayor parte de las funciones aportadas explican el efecto plazo y el efecto magnitud, siendo la contribución al resto de efectos bastante menor. Esto muestra la importancia de seguir trabajando e investigando en el resto de las anomalías menos estudiadas, en particular, el efecto intervalo.

2. Justificación

El efecto intervalo es uno de los efectos menos conocidos de la elección intertemporal y consideramos que esto se debe a que siempre ha sido identificado con el efecto plazo. El efecto plazo implica tasas decrecientes a medida que el plazo aumenta; sin embargo, el efecto intervalo supone tasas decrecientes a medida que aumenta la longitud del intervalo. Los conceptos plazo e intervalo no son sinónimos, sino que el plazo es la distancia existente entre el momento 0 y el instante *t*, mientras que el intervalo es la diferencia entre dos instantes t_1 y t_2, es decir, es la diferencia entre dos plazos, de modo que, cuando el primer plazo es igual a 0, entonces los términos plazo e intervalo coinciden. Esta distinción fue introducida, por primera vez, por Read (2001); sin embargo, en un estudio posterior, el efecto intervalo se identificó con la subaditividad (Read et al., 2003). Como podemos observar, la investigación de este efecto ha venido marcada por la identificación con otros conceptos, lo que ha llevado a que su conceptualización sea bastante confusa, y, por tanto, que su investigación teórica y empírica sea extremadamente escasa. Precisamente, es por ello que surge la necesidad de clarificar qué es el efecto intervalo, llevando a cabo una clara distinción entre los efectos plazo e intervalo, y relacionar ambas anomalías entre sí y con el concepto de subaditividad. Esto permitiría establecer una sólida fundamentación teórica de esta anomalía y, por tanto, crear la base necesaria para que este efecto se siga investigando al igual que otras anomalías como el efecto magnitud o el efecto plazo, entre otras.

3. Objetivos

El **objetivo principal** de esta Tesis Doctoral es clarificar el concepto del efecto intervalo, así como llevar a cabo un análisis lo más completo posible del mismo, desde un punto de vista teórico. Esto permitirá establecer las bases necesarias para que este efecto se siga investigando desde un punto de vista teórico y empírico.

Entre los **objetivos secundarios** de este trabajo, tenemos los siguientes:

- Analizar el estado actual de investigación sobre los principales efectos en la elección intertemporal (efecto plazo, efecto magnitud, efecto signo, efecto secuencia, efecto asimetría respecto a la anticipación y al aplazamiento, efecto fecha-aplazamiento y efecto intervalo), que permita conocer cuál es el grado general de conocimiento sobre cada uno de ellos. Esto nos ayudará a conocer cuáles son los posibles campos de investigación que todavía están sin explorar en cada uno de los efectos, en concreto del efecto intervalo, cuya investigación es, hasta la fecha, prácticamente nula.

- Analizar del efecto plazo. Antes de estudiar el efecto intervalo, primero es necesario conocer a fondo el efecto plazo. Por ello, se analizará este efecto, desde un punto de vista matemático-financiero, y se relacionará con la subaditividad y la impaciencia decreciente. Todos ellos se analizarán tanto desde un punto de vista estacionario como desde un punto de vista dinámico.
- Proponer un modelo matemático de descuento que explique el efecto plazo y la subaditividad.
- Definir el efecto intervalo. Una vez analizado y conceptualizado matemáticamente el efecto plazo y la subaditividad, es necesario analizar las diferentes definiciones de este efecto en la literatura. Además, se estudiarán los posibles casos en los que pueda darse este efecto. De esta forma, se propondrá una definición, lo más general posible, del efecto intervalo.
- Analizar la relación del efecto intervalo con el efecto plazo y la subaditividad, de acuerdo con la definición propuesta. Esto se llevará a cabo tanto en un contexto estacionario como dinámico.

4. Metodología

La metodología seguida en esta Tesis ha sido la siguiente:

- Revisión sistemática de la literatura, que ha sido la metodología seguida en la recopilación de los trabajos sobre el objeto de la Tesis. Este método se caracteriza por ser preciso y estructurado, lo que nos ha permitido localizar, seleccionar y evaluar las contribuciones existentes en la literatura existente sobre las anomalías en la elección intertemporal. Posteriormente, esta metodología nos ha permitido analizar y sintetizar la información recabada para extraer conclusiones y proponer futuras líneas de investigación (Tranfield, Denyer y Smart, 2003; Denyer y Tranfield, 2009).
- Utilización del razonamiento matemático para caracterizar los efectos plazo e intervalo, así como para relacionar estos efectos con la subaditividad.
- Análisis del efecto plazo, desde un punto de vista dinámico, a partir de tres metodologías distintas:
    - Por comparación de los ratios de descuento correspondientes a dos intervalos diferidos de la misma duración.
    - Por comparación del valor de la función de descuento con el ratio de descuento correspondiente a un intervalo aplazado con la misma amplitud.
    - Por comparación de los valores de la función de descuento en dos intervalos con la misma amplitud.
- Método deductivo, para la establecer las consecuencias lógicas derivadas del efecto plazo y del efecto intervalo.

5. Estructura de la Tesis Doctoral

Esta Tesis se ha estructurado en cinco capítulos (véase la Figura 1). A continuación, se va a explicar detalladamente la estructura de cada uno de estos capítulos:

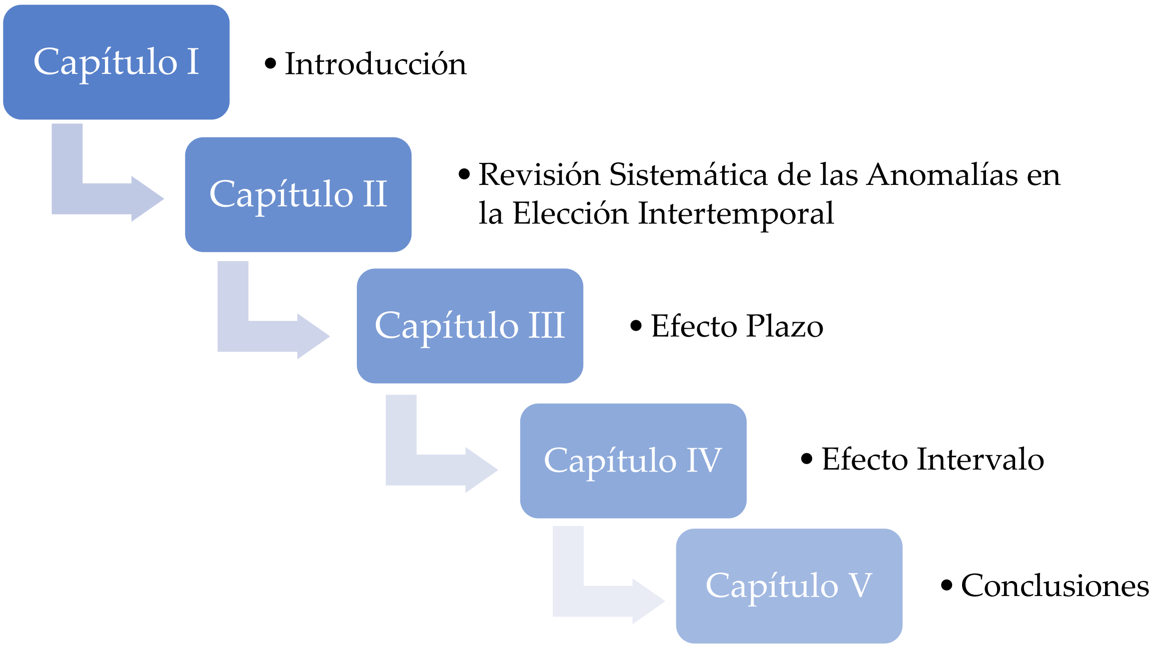

Figura 1. Estructura de la Tesis Doctoral. **Fuente.** Elaboración propia.

- Capítulo I. Introducción: El objetivo de este capítulo introductorio es aportar una visión general de la Tesis y justificar los objetivos planteados. Para ello, se exponen el origen y evolución de las anomalías en la elección intertemporal y, posteriormente, se lleva a cabo una justificación de las razones que han llevado a estudiar el efecto intervalo. En el siguiente epígrafe, se exponen el objetivo principal de esta tesis, así como los objetivos secundarios de la misma. En el cuarto epígrafe, se detalla la metodología utilizada en todo el trabajo y, finalmente, se pormenoriza la estructura de la tesis y se enumera la bibliografía utilizada en este capítulo.

- Capítulo II. Revisión Sistemática de las Anomalías en la Elección Intertemporal: En este capítulo, se lleva a cabo una revisión sistemática de las principales anomalías en la elección intertemporal, de tal manera que queda justificado uno de los objetivos secundarios de esta tesis, pues nos va a permitir conocer cuál es el estado actual de las anomalías y su relación con el efecto intervalo. Asimismo, este capítulo nos permite justificar claramente la necesidad de analizar el efecto intervalo. Este artículo fue enviado a la revista *Systematic Reviews* indexada en el *Journal Citation Reports* (JCR) con un factor de impacto de 2,479 (2019), situada en el cuartil Q2, y en el *Scimago Journal Reports* (SJR), con un factor de impacto de 1,249 y posicionada en el cuartil Q2.

- Capítulo III. El Efecto Plazo: Análogamente al anterior, este capítulo nos permite alcanzar algunos de los objetivos secundarios enumerados en la Sección 4 de este capítulo. Para conseguir el objetivo principal de analizar el efecto intervalo, es necesario determinar y definir correctamente el efecto plazo. En este capítulo, analizaremos este efecto desde un punto de vista matemático-financiero y lo relacionaremos con el concepto de subaditividad en los ámbitos estacionario y dinámico. Además, propondremos una función de descuento exponencial que explique tanto el efecto plazo como la subaditividad. Este trabajo fue publicado en la revista *Mathematics* en marzo de 2020, bajo el título "Delay effect and Subadditivity. Proposal of a New Discount Function: The Asymmetric Exponencial Discounting". Esta revista está indexada en el *Journal Citation Reports* (JCR) con un factor de impacto de 1,747 (2019), situada en el decil D1 y en el cuartil Q1, y en el *Scimago Journal Reports* (SJR), con un factor de impacto de 1,4 y posicionada en el cuartil Q2.

- Capítulo IV. El Efecto Intervalo: Este capítulo es el que nos va a permitir alcanzar el objetivo principal de esta Tesis, así como también algunos de los objetivos secundarios, ya que se da una definición matemática del efecto intervalo y se analizan cada uno de los casos en lo que puede presentarse este efecto. Esto nos va a permitir aportar una definición de este efecto y, posteriormente, relacionarlo con el efecto plazo y la subaditividad. Además, se efectuará un análisis de este efecto en dos contextos: el estacionario y el dinámico. Este capítulo fue publicado bajo el nombre de "Are Delay and Interval Effects the Same Anomaly in the Context of Intertemporal Choice in Finance?", en la revista *Symmetry* en enero de 2021. Esta revista está indexada en el *Journal Citation Reports* (JCR) con un factor de impacto de 2,645 (2019), situada en el cuartil Q2 y en el *Scimago Journal Reports* (SJR), con un factor de impacto de 2,5 y posicionada en el cuartil Q2.

- Capítulo V. Conclusiones: En este último capítulo se expondrán los principales resultados obtenidos en esta Tesis, así como las limitaciones encontradas y las futuras líneas de investigación.

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CAPÍTULO II.

REVISIÓN SISTEMÁTICA DE LAS ANOMALÍAS EN LA ELECCIÓN INTERTEMPORAL

ARTÍCULO 1. A Systematic Review of the Anomalies in Intertemporal Choice

Artículo envíado en 2021 a la revista Systematic Reviews

Factor de impacto (2019) 2,479 (Journal Citation Reports-Thomson-Reuters)

A Systematic Review of the Anomalies in Intertemporal Choice

Abstract: In this paper, a systematic review of the existing literature about the main anomalies in intertemporal choice has been carried out (delay effect, magnitude effect, sign effect, sequence effect, delay/speed up asymmetry, d effect date-delay effect and interval effect). In this analysis, we have differentiated the works by area of knowledge (economics, medicine, neuroscience, psychology and political science) in order to know the causes and consequences of the aforementioned anomalies within each field. Moreover, some proposals for future lines of research in each of these areas have been made.

Keywords: Intertemporal choice, Magnitude effect, Delay effect, Sign effect, Delay/Speed Up Asymmetry, Date-delay effect, Sequence effect, Interval effect.

1. Introduction

Intertemporal choice is the financial process whereby decision makers must choose between two or more rewards available at different moments of time. There are many examples of intertemporal choice such as, for example, to decide between spending certain money amount now or in the future, or stop smoking now for better health later. This concept was first presented by Rae (1834) and later refined by Fisher (1930), leading to the so-called Discounted Utility Model, introduced by Samuelson (1937). This exponential model became the main paradigm for the evaluation of intertemporal decision making. However, from the 1980s onwards, a series of criticisms of this model have arisen (Thaler, 1981) due to the inconsistency or anomalies observed by decision makers in numerous empirical works. The main anomalies are delay effect, magnitude effect, sign effect, sequence effect, delay/speed up asymmetry, sequence effect and Interval effect.

Over the past 30 years, there has been a wide research on these anomalies as well as on the models explaining them, which has derived in an increase of publications, possibly stimulated by the 2017 Nobel Prize in economics received by Richard Thaler because of extensive background in behavioral economics.

The objective of this paper is to conduct a systematic review of the existing literature about the anomalies in intertemporal choice which will allow researchers in different field to have a background of the main works and potential lines of research. To the best of our knowledge, the latest review of all effects can be found in Frederick et al. (2002). However, in the last 20 years there has been an intense research in this field being necessary to delimit the advances obtained.

The structure of this paper is the following. Firstly, Section 2 describes the main effects and delimitates the choice scenarios, viz intertemporal choice or delay discounting (for choices under certainty) and expected discounting (for choices under uncertainty). However, in this paper, we will mainly focus on the first scenario. In Section 3, we will describe the methodology followed for the systematic literature review. In Section 4, we will analyze the results obtained by providing the descriptive analysis of the papers (Subsection 4.1), the main contributions of each of the analyzed effects (Subsection 4.2) and the proposal of several lines of future research in each of the analyzed areas (Subsection 4.3). Finally, Section 5 summarizes and concludes.

2. Background

Intertemporal choice analyzes the preferences in trade-offs involving costs and benefits which occur at different times (Loewenstein et al., 2003). For example, consumption now or saving for the future, smoking now or having better health in the future, or choosing between receiving €100 now or €150 in 1 year. As indicated, in this paper we will deal with decisions which are made in an environment of certainty. Research in this field has been growing in recent decades and its contribution is increasingly recognized in various disciplines such as economics, psychology, neuroscience and medicine.

In order to explain the choice between a smaller-sooner and a larger-later amount, economists and psychologists have introduced the concept of delay discounting which states that the earlier availability of a future reward is a decreasing function of the waiting time (Frederick et al., 2002; Green et al., 2004. Alternatively, in choices under uncertainty, an analogous concept, called the probability discounting, was introduced (Rachlin et al., 1991). The models used in both scenarios are the Discounted Utility (DU) model (for choices under certainty) and the Expected Utility (EU) model (for choices under uncertainty). Despite the criticisms received in recent decades, both models continue to be widely used in several contexts as normative and descriptive models of choice rewards. These theories have been characterized mainly by their simplicity and similarity to the financial present value and the actuarial models. However, there are several empirical studies which show how people contradict the axioms of the DU and EU models in some specific choices. These anomalies have received greater attention in the case of the EU model than in the DU model, giving rise to several alternative models. In the case of the DU model, the interest in these anomalies has increased in recent years but the new models have not had sufficient impact. On the other hand, the anomalies of the EU model are of greater complexity than those of the DU model and, consequently, have been more studied.

Traditionally, the analysis of intertemporal choice (DU model) and choices under risk (EU model) has been carried out separately, focusing on the decision biases or deviations from the normative theory. However, recent papers have analyzed the parallelism between these two models. In effect, Loewenstein and Prelec (1992) and Prelec and Loewenstein (1991) were among the first scholars in observe that some effects in intertemporal choice are similar to those presented in choices under uncertainty, but other studies have shown that their behavior is different. For example, the magnitude of the involved reward affects the degree of discounting in different ways depending on whether such reward is delayed or probabilistic. Thus, in the case of delay discounting, larger rewards are discounted less abruptly than smaller ones (Green et al., 1997; Kirby, 1997; Thaler, 1981) whilst, in the case of probability discounting, the opposite occurs, that is to say, larger rewards are discounted more abruptly than smaller ones (Green et al., 1999). Table 1 shows a comparison between the effects in each of the scenarios formerly described:

Table 1. Comparing the main anomalies. **Source:** Own elaboration.

Delayed Discounting	**Expected Discounting**
Delay effect/Common difference effect	Common ratio effect
Magnitude effect	Peanuts effect
Sign Effect	Reflection effect
Delay/Speed up Asymmetry	---
Sequence effect	---
Date-Delay effect	---
Interval effect	---

These effects may be defined as follows:

- *Anomalies of delay discounting or intertemporal choice:*
  - **Magnitude effect:** The magnitude effect is a bias present in intertemporal choice which means that, multiplying the magnitude of the outcome by a constant factor, greater than 1, may reverse the preference from the smaller, earlier option to the larger, later one. For example, someone may prefer $10 now to $20 in 1 year, but also $200 in 1 year to $100 now. So, the magnitude effect is characterized by a higher discount rate for small rewards than for large ones.

- **Delay effect:** This is an anomaly of intertemporal choice in which, as the deferral of both options is increased by a constant, there is a reversal of preferences from the smaller, sooner outcome to the larger, later one, whereby it is also known as preference reversal. For example, someone might prefer $10 today to $20 in one year, but if the time horizon of both preferences is increased by 2 years then someone might prefer $20 in 3 years to $10 in 2 years. Therefore, the temporal discount rate decreases as the time until receipt of the reward increases. A specific case of the delay effect is the immediacy effect, in which more immediate options are given a greater weight. Thus, the highest discount rates appear for small delays.
- **Sign effect:** This effect consists in that decisions involving gains have higher discount rates than those involving losses. For example, a gain of $100 at the present time may be indifferent to a gain of $200 in a year, but a loss of $100 at the present time would also be seen as the same as a loss of $150 in a year. In this example, we can see that gains are discounted more than losses.
- **Delay/Speed up Asymmetry:** This effect implies discount rates for decisions involving delayed rewards higher than for decisions involving anticipated rewards. Loewenstein (1988) demonstrated this effect through an experiment in which respondents who did not expect immediate consumption of a video player would pay an average of $54 to receive it immediately rather than in a year; even those who expected to receive it at that time asked an average of $126 for delaying its receipt by a year.
- **Sequence effect:** This effect consists of a preference for sequences of increasing outcomes. Thus, whilst for individual outcomes there is a positive time preference, for sequences there is a negative time preference. Chapman (1996) showed that, in the short run, decision makers prefer increasing sequences of money and health because they expect to improve

their position in the long run. However, for very long-term sequences, in monetary decisions they still prefer increasing sequences but, in the health area, they prefer decreasing sequences as they expect that, with the passage of time, they will start to have health problems.

- **Date-Delay effect:** This effect implies that the discount rates imputed when time is described using calendar dates (e.g., October 17) are markedly lower than those revealed when future outcomes are described in terms of the corresponding delay (e.g., six months). This anomaly was discover by Read et al. (2005).
- **Interval effect:** This effect consists in the fact that the discount rate will tend to be higher the closer the rewards are to each other. For example, a decision-maker may be indifferent between receiving $100 in 6 months or $150 in 12 months (the interval is 6 months) but would wait for receiving of $200 in 18 months rather than $100 in 6 months (the interval is now 12 months).

➢ *Anomalies of expected discounting*

- **"Peanuts" Effect:** Outcome magnitude also influences choices under uncertainty, although very few studies have examined this effect. The peanuts effect occurs when increasing the magnitude of the outcome by a constant factor shifts preferences from the larger, less likely reward to the smaller but more likely reward. In this case, someone might prefer to receive $2 with a 50% probability to $1 for sure but might also prefer $100 for sure to $200 with a 50% probability. In other words, decision-makers are more risk averse as the magnitude increases, so they are more willing to take risks for small rewards. The "peanuts" effect is the reversal of the magnitude effect for probabilistic rewards, since individuals have higher discount rates for large amounts than for small amounts because risk seeking corresponds to a lower time discount.

- **Common ratio effect:** The effect parallel to the delay effect in choices under uncertainty is the common ratio effect. In this case, the reduction of the probabilities for both options by a common ratio results in a shift in preferences from smaller and more likely to larger and less likely outcomes. That is to say, a person may prefer $100 with a 50% probability to $200 with a 25% probability of winning, but if we reduce the probabilities by a ratio of 10, we obtain that $200 with a 2.5% probability is preferred to $100 with a 5% probability. Therefore, the lower the probabilities of obtaining a reward, the higher the risk-taking tendency of decision-makers. The common ratio effect presents a specific case, viz the certainty effect, where the smallest and most likely option is always the preferred one. For example, $30 with 100% probability will be preferred to $45 with 80% probability, but $45 with 20% probability will also be preferred to $30 with 25% probability.

- **Reflex Effect:** Analogously to the sign effect, in choices under uncertainty, the reflex effect shows risk aversion in case of gains whilst, in case of losses, there is a shift towards risk seeking. Prelec and Loewenstein (1991) offered a joint explanation of these two effects, based on the importance of each attribute (money, time and probability), viz decreasing absolute sensitivity, incremental proportional sensitivity and loss amplification. This means that, adding a constant to the values of an attribute, then it loses importance; moreover, by proportionally increasing the values of an attribute or changing the sign of an attribute from positive to negative, then the attribute becomes more important.

In this paper, we will primarily focus on intertemporal choices in situations of certainty, although on numerous occasions we will consider decisions in situations of uncertainty.

3. Methodology

In this paper, the so-called "Systematic Literature Review" has been applied to the analysis of the anomalies in intertemporal choice. This technique determines the current state of the knowledge in a field (Tarifa-Fernández and De Burgos-Jiménez, 2017; Tranfield et al., 2003), which allows to identify the research areas, main findings, research directions and gaps.

The search for the most relevant articles was carried out through two of the main bibliographic databases, Web of Science (WoS) and Scopus, because of the high impact of their publications, being the two most important international academic databases covering interdisciplinary publications. This means a significant strength of our analysis and allows comparing different scientific fields (Archambault et al., 2006).

The keywords chosen for the search were grouped into three categories: the first delimits the field of study with the concept of "intertemporal choice" or "delay discounting"; the second category restricts the previous search to articles dealing with exclusively the anomalies or effects in intertemporal choice ("anomalies" and "effect"); and the last category limits the search to the effects which we want to analyze by using the possible names of each of them ("loss-gain asymmetry", "delay-speed up asymmetry", "sign effect", "sequence effect", "time consistency", "magnitude effect", "framing effect", "interval effect", "delay effect", "present-bias effect", "common difference effect" and "interval length effect"). These keywords were chosen to achieve the greatest possible coverage on this topic. The articles included in the analysis were from the aforementioned databases until December 2020. Likewise, given their relevance in the analysis, two articles of 2021 were added.

Table 2 shows the criteria followed in searching articles, being conducted at the beginning of January 2021.

Table 2. The process of systematic review. **Source:** Own elaboration.

KEYWORDS	*WOS*	*SCOPUS*
"Intertemporal Choice" OR "Delay Discount"*	1925	1700
"Anomal" OR "Effect*"*	767	660
"Loss-Gain Assymmetry" OR "Delay-Speed Up Asymmetry" OR "Sign Effect" OR "Sequence Effect" OR "Time Consistency" OR "Magnitude Effect" OR "Framing Effect" OR "Interval Effect" OR "Delay Effect" OR "Present-Bias Effect" OR "Common Difference Effect" OR"Interval Length Effect"	82	91
Article	80	77
English language	74	74
Total articles	148	
Duplicates	–53	
Not considered in the analysis	–17	
Articles 2021	+2	
Total articles analyzed	80	

The first search resulted in 3,625 articles: 1,925 were from WoS and 1,700 were from Scopus. This search was limited to 1,427 with the words "Anomal*" or "effect*", and to 173 articles with the inclusion of the anomalies described in Table 2. Additionally, the search was limited to articles written in English, which meant a total of 148 articles. 53 duplicate articles were found in the two databases, and 17 articles were removed for not meeting the objectives of this work. Finally, two works from 2021, which were available in the 2020 databases, were included. Finally, a total of 80 articles were analyzed.

4. Results

4.1. Descriptive Results

As indicated in Section 3, the total number of analyzed articles was 80, of which 44 were published in the last 5 years (55%). Table 3 shows the articles published in periods of 5 years and the effects investigated within them. Observe that the most studied anomalies have been the magnitude effect, which has been analyzed in 47 works; the delay effect, which appears in 41 papers; and the sign effect, in 30 works. On the other hand, the less investigated effects are the sequence effect, researched in 10 articles, the delay/speed up asymmetry and date-delay effect in 7, and the interval effect in only 3 articles, all in the last 5 years.

Table 3. Number of articles per year and effect. **Source:** Own elaboration.

Period	Articles	ME	IE	DE	SE	DSUA	DDE	SQE
1997-2004	5	2	0	2	2	1	0	1
2005-2009	10	7	0	7	6	2	1	4
2010-2014	21	12	0	11	8	2	3	0
2015-2020	44	26	3	22	13	2	3	5
Total	**80**	**47**	**3**	**42**	**29**	**7**	**7**	**10**

On the other hand, Table 4 shows the countries and areas in which the anomalies in intertemporal choice have been most investigated. Observe that the countries with a higher number of works are USA, with 32 jobs, and Japan and Spain with 9 studies each. Whilst USA and Japan stand out for their empirical contribution, in Spain most of the studies are theoretical. Regarding the areas of study, Economics and Psychology stand out as the most involved in the research of these effects. Specifically, USA is prominent in both areas, Spain in Economics and Japan in Psychology. The other areas, in which these effects have been dealt with, but to a lesser extent, are Medicine, Neuroscience and Political Science.

Table 4. Number of articles per country, type of study and area. **Source:** Own elaboration.

Country	No. Articles	Type of Study		Area of Study				
		Theo.	Emp.	Econ.	Med.	Psycho.	Neuro.	Polit. Sci.
Australia	6	1	6	3		3		
Austria	1	1		1				
Canada	2	2		1		1		
China	6		6	3		2	1	
USA	32	11	24	9	6	13	3	1
England	7	5	5	4	1	2		
France	1	1		1				
Germany	5	2	5	1		3	1	
Italy	5		5	2		3		
Japan	9	1	9	3		5	1	
Luxembourg	1		1			1		
Netherlands	2		2	1		1		
New Zealand	2		2			2		
Norway	1	1	1	1				
Portugal	1	1	1	1				
Spain	9	7	4	7	1	1		
Total	**90**	**33**	**71**	**38**	**8**	**37**	**6**	**1**

Table 5 shows the articles per effect, framing (delayed or expected discounting) and type of study (theoretical, empirical or both). Observe that all effects have been studied in both framings, that is to say, under delayed and expected discounting, except for the date-delay effect. However, the most relevant scenario is delayed discounting on which our study will be focuses. Although all effects have been theoretically and empirically analyzed, it is worth noting the need for further studies which investigate in greater depth the date-delay effect, the delay/speed up asymmetry, the sequence effect, and mainly the interval effect.

Table 5. Number of articles per effect, framing and area. **Source:** Own elaboration.

Effects	Articles	Framing		Type of Study	
Date-Delay Effect	**7**	Delay discounting	7	Theoretical/Empirical	3
				Empirical	4
Delay Effect	**42**	Delay discounting	33	Theoretical/Empirical	1
				Empirical	20
				Theoretical	12
		Probability	9	Theoretical/Empirical	1
				Theoretical	4
				Empirical	4
Delay/Speed Up Asymmetry	**7**	Delay discounting	6	Theoretical	3
				Empirical	3
		Probability	1	Theoretical	1
Interval Effect	**3**	Delay discounting	2	Theoretical/Empirical	1
				Empirical	1
		Both	1	Theoretical	1
Magnitude Effect	**47**	Delay discounting	37	Theoretical/Empirical	4
				Theoretical	11
				Empirical	22
		Probability	10	Theoretical/Empirical	1
				Theoretical	5
				Empirical	4
Sequence Effect	**10**	Delay discounting	9	Theoretical/Empirical	2
				Theoretical	4
				Empirical	3
		Probability	1	Empirical	1
Sign Effect	**29**	Delay discounting	25	Theoretical	5
				Empirical	20
		Probability	4	Theoretical	1
				Empirical	3

Table 6 shows the different names given to the anomalies in their corresponding works. Only the date-delay effect and the delay/speed up asymmetry have kept the same name in all the analyzed papers. However, the other anomalies have received different and, in some cases, numerous names, which can make research quite puzzling. This justifies the need to unify the nomenclature of all effects in all the areas of study.

Table 6. The different names of the effects. **Source:** Own elaboration.

Effect	Articles	Names	
Date-Delay Effect	**7**	Date-delay effect	7
Delay Effect	**42**	Common difference effect	7
		Declining impatience	2
		Delay discounting	1
		Delay effect	14
		Dynamic inconsistency Effect	2
		Effect of self-control	1
		Hyperbolic discounting	6
		Impatience	1
		Impulsivity	2
		Present bias	1
		Short/long-term asymmetry	1
		Time delay	1
		Time effect	1
		Time inconsistency	1
		Preference reversals	1
Delay/Speed Up Asymmetry	**7**	Delay/speed up asymmetry	7
Interval Effect	**3**	Interval effect	2
		Interval length effect	1
Magnitude Effect	**47**	Magnitude effect	43
		Absolute magnitude effect	3
		Size effect	1
Sequence Effect	**10**	Sequence effect	8
		Negative time preference	1
		Preference for improving sequences	1
Sign Effect	**30**	Sign effect	23
		Gain-loss asymmetry	4
		Instant endowment	1
		Gain-loss	2

4.2. Discussion

In this section, we are going to classify all the selected articles, by differentiating among effects, areas of study (Economics, Medicine, Neuroscience, Psychology and Political Sciences), choice scenarios (delay and probability discounting) and types of work (theoretical and empirical).

4.2.1. The magnitude effect

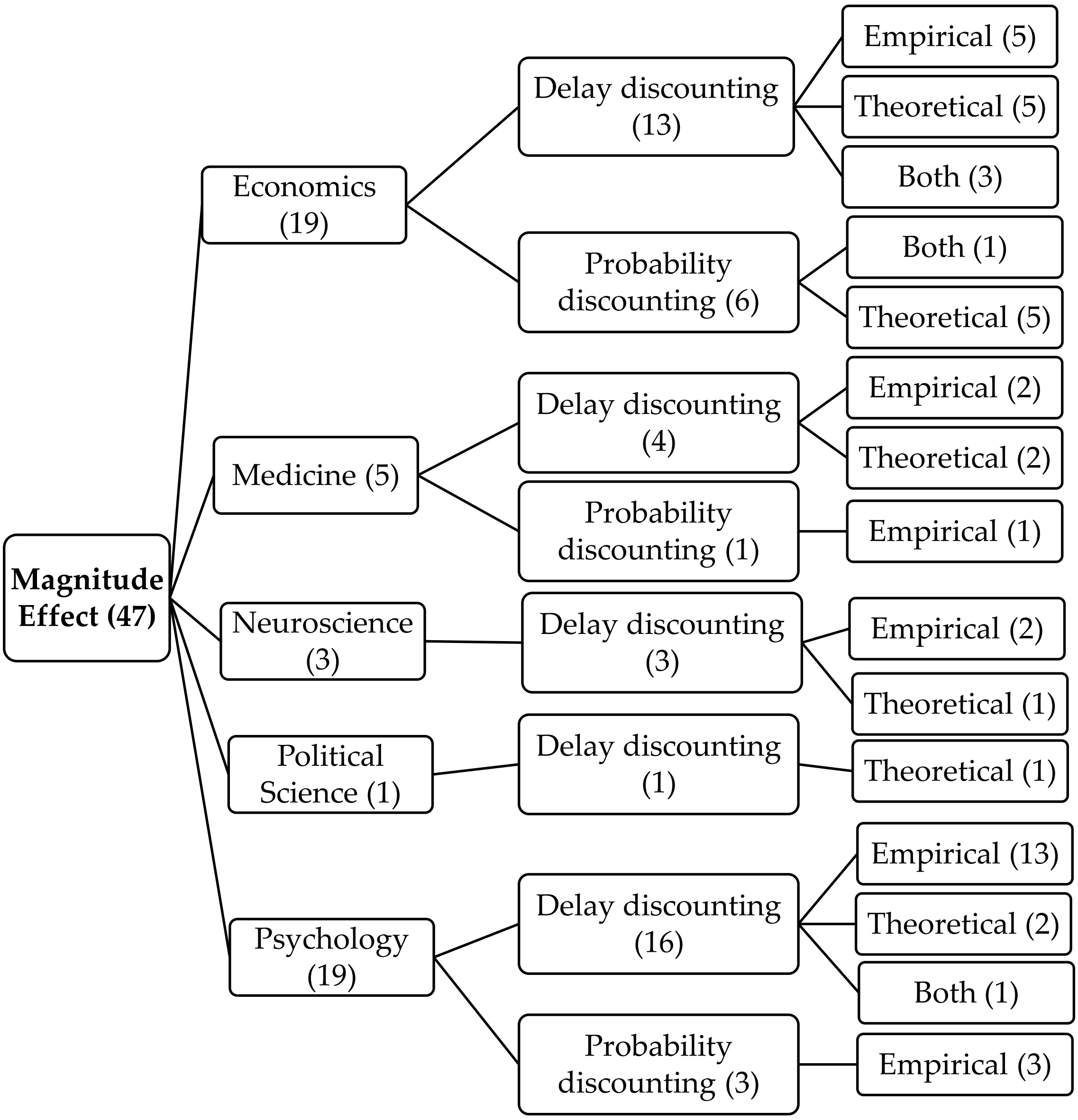

Chart 1. The magnitude effect. **Source:** Own elaboration.

The magnitude effect has been analyzed in a total of 47 articles, whose characteristics have been described in Chart 1. Now, we are going to analyze these works classified by their area of study.

In the economic area and focused on the delay discounting scenario, we have analyzed 13 articles, of which 5 are theoretical, 3 theoretical-empirical and 5 empirical. As for theoretical and theoretical-empirical works, it should be noted that most of them provide a proposal of mathematical model which explains the magnitude effect. Firstly, in 2011, Noor proposed the magnitude effect model (MED), which generalizes the separable discounting model, making the discount factor dependent on the reward amount (Noor, 2011). Subsequently, Read et al. (2013) provided an empirical-theoretical study in which they developed the DRIFT model, a heuristic description of how framing influences intertemporal choice; and empirically analyzed the delay and magnitude effects. These authors showed that, if the experimental interest rate is explicit, the magnitude effect is drastically reduced. In this way, the interest frames increase patience when the rewards are small, and reduce patience in case of large amounts. Later, Baucells and Bellezza (2017) proposed a descriptive model, called the anticipation-event-recall (AER) model, which explains both the magnitude effect and the delay-speed up asymmetry. Afterwards, Cruz Rambaud et al. (2018) proposed an alternative model, called the q-exponential discount function deformed by amount, which is able to describe the magnitude and delay effects jointly. Finally, Drouhi (2020) defined an additive and non-stationary discounted utility function, which can explain the delay and magnitude effects, what differs from previous literature.

Other studies contribute to the literature with mathematical developments related to the magnitude effect. Among them, Al-Nowaihi and Dhami (2009) started from the explanation provided by Loewenstein and Prelec (1992) for the magnitude effect, based on the incremental elasticity property of discount functions, to build a theoretical framework from which it is possible to obtain functions (simple increasing elasticity utility functions, SIE) exhibiting this property. Likewise, Cruz Rambaud et al. (2019) proposed an index inspired in the hyperbolic factor of Rohde (2010), the so-called ME-index, to determine whether a discount function is able to explain the magnitude effect. Finally, Kinari et al. (2009) presented a study in which the delay effect was conceptually distinguished from the interval effect. Additionally, they conducted an experiment demonstrating the occurrence of these effects together with the magnitude effect.

Regarding empirical works, Guyse and Simon (2011) demonstrated the simultaneous presence of both the sign and the magnitude effects. Subsequently, Wang et al. (2015) conducted an empirical study of the magnitude effect along with other anomalies, in which the subjective perception of time was taken into account They observed that, if time was objective, the anomalies appeared, while if time was subjective, the anomalies were not present. Likewise, Meyer (2015) empirically demonstrated the magnitude effect starting from two different elicitation mechanisms (a matching task and a choice task) without finding major differences in both cases. Faralla et al. (2017) confirmed this effect in their experiments. Finally, Lu et al. (2020) conducted several experiments to demonstrate the existence of the so-called sequence effect in a loan, along with the magnitude effect, confirming that the magnitude effect does not influence the preference pattern.

In probability discounting, we have analyzed 6 articles of which 5 are theoretical and only 1 is theoretical-empirical. As in the delay discounting scenario, there are several works which provide mathematical models in order to explain the different anomalies in intertemporal choice. This is the case of Xia (2011) and Baucells and Heukamp (2012). The first one introduces the expected utility model, with uncertainty, risk aversion, and preference for precautionary savings, thus explaining the three main anomalies in intertemporal choice (magnitude effect, delay effect and sign effect). The second paper provided a general model able to reconcile the DU and EU models, and explain the anomalies arising in both intertemporal choice and in choices under uncertainty. Analogously, Holden and Quiggin (2017) presented the Zooming model to explain the magnitude and delay effects, which was confirmed by an empirical study. On the other hand, Walther (2010) demonstrated that the delay effect, the sign effect, the delay-speed up asymmetry and the magnitude effect (only in losses) can be explained in the framework of an expected state-dependent intertemporal utility, that is to say, by considering the uncertainty as an aspect of intertemporal decisions. Later, Shoji and Kanehiro (2012) carried out a numerical analysis of the magnitude and sign effects under different risk tolerances. Finally, Adriani and Sonderegger (2020) presented a simple cost-benefit analysis to derive optimal similarity judgments, demonstrating the magnitude, delay and interval effects in the delay and probability discounting scenarios.

In the field of Medicine, within the delay discounting scenario, we have analyzed 4 articles: 2 theoretical and 2 empirical. The theoretical works are reviews of the existing literature. The first one analyzes several effects, including the magnitude effect, related to the field of health (Ortendahl and Fries, 2005). However, the second one analyzes the same effects in the context of health education (Ortendahl, 2006). Regarding the empirical works, Lazaro et al. (2002) analyzed intertemporal money and health decisions in several effects, revealing the magnitude effect in both types of decision and higher discount rates in life-saving decisions compared to economic ones. On the other hand, Johnson et al. (2015) demonstrated that opportunity costs may replace the effect of magnitude for consumable commodities.

Considering the probability discounting scenario, we can find a work by Chapman and Weber (2006) which analyzes the delay and magnitude effects in delay and probability scenarios for decisions on money and health. They conclude that the magnitude and the peanut effects are correlated in both scenarios, but not in monetary and health domains.

In the field of neuroscience, we have analyzed 2 empirical articles and 1 theoretical-empirical. Ballard et al. (2018) carried out a repetitive transcranial magnetic stimulation (rTMS) to transiently disrupt dlPFC neural activity. This manipulation dramatically reduced the magnitude effect, providing causal evidence that the magnitude effect depends on dlPFC. Moreover, Wagner et al. (2020) found that the magnitude effect was attenuated under haloperidol, and Gershman and Bhui (2020) demonstrated that the optimal allocation of mental effort can give rise to the magnitude effect in intertemporal choice.

In the field of Psychology, we can find 2 theoretical articles, 1 theoretical-empirical and 13 empirical in the scenario of delay discounting. With respect to theoretical papers, the work by Killeen (2009) provided a novel discount function which was generated by making the marginal discount rate time-sensitive and by assuming that it is utility, not monetary value, which is discounted. The additive utility model is unique in that it posits a disutility to waiting which is added to the utility of the good. Moreover, it predicts a number of standardized anomalies, among them the magnitude effect. Finally, Vanderveldt et al. (2016) carried out a literature review of the delay and magnitude effects in nonhuman animals.

From an empirical point of view, it is necessary to distinguish between studies with animals and humans. Regarding the former ones, we can find the work by Grace et al. (2012) who showed the magnitude effect in pigeons which confirms that this effect is not unique in humans. However, De Petrillo et al. (2015) analyzed the magnitude effect in capuchin monkeys coming to the opposite conclusion, that is to say, nonhuman animals showed the reverse magnitude effect in intertemporal choices. Regarding the experiments in humans, various aspects have been analyzed: the empirical validation of the different proposed models, the behavior of people with some type of addition, the influence of life habits in the decision-making, and the decisions in different choice domains. Starting with the validation of some theoretical models, Stevens (2016) tested discounting models against attribute-based models, which use similarity judgments to make choices. His results showed that similarity judgments permit to account for behaviors which contradict many discounting models, such as the magnitude and the sign effects. Therefore, the attribute-based models, such as the similarity models, provide some alternatives to discounting, what may offer several insights into the process of decision-making in intertemporal choices. Analogously, Cheng and González-Vallejo (2016) analyzed two attribute-wise models: the trade-off model (Scholten et al., 2014) and the proportional difference model (González-Vallejo, 2002); and an alternative hyperbolic model based on Rachlin (2006). They noted that the attribute-wise models are better suited to describe intertemporal elections.

Other works in humans focused on examining the behavior of people with problems alcohol and drug consumption or with ADHD. For example, Paloyelis et al. (2010) analyzed the delay and magnitude effects in people with ADHD (attention deficit/hyperactivity disorder) by distinguishing between hypothetical and real rewards. Their results showed no magnitude effect in people with ADHD in case of hypothetical rewards. Additionally, greater impulsivity was observed in the group with ADHD when carrying out hypothetical tasks. For his part, Klapproth (2012) conducted an empirical study with drug addicts, in which the delay discounting task described the time with dates and delay. This allowed him to observe that future rewards were discounted to a lesser extent when the choice implied a date. Likewise, drug addicts were affected by this effect more than people without addictions. On the other hand, they found that the magnitude effect was present in both groups. Finally, Oberlin et al. (2015)

conducted an experiment with people exhibiting alcohol problems, whose results suggested that the magnitude effect may be more sensitive to alcohol-induced problems than single discounting measures.

Other empirical works focused on analyzing the process of decision making in different domains. Regarding the decisions on tips, Chapman and Winquist (1998) analyzed the magnitude and sign effects in both monetary and restaurant tips decisions. As for the magnitude effect, they found evidence in both domains. Green et al. (2003) also studied the tips and the magnitude effect by obtaining that, as the amount of the bill increases, the percent tip tends to decrease. On the other hand, Olsen et al. (2018) observed the magnitude effect in decisions on academic tasks.

Regarding the influence of the social habits on intertemporal decisions, Paglieri et al. (2013) analyzed whether the religion affected the decision-making by people, focusing on the delay and magnitude effects. The results showed the existence of the magnitude effect and that temporal discounting was specifically modulated by religion. Muñoz Torrecillas et al. (2018) studied whether the dietary habits affected to individuals' decisions confirming a greater presence of the magnitude effect in people with worse habits of life.

Some studies (Holt et al., 2008) focused on the analysis of the magnitude effect in losses by concluding that smaller losses were not discounted more abruptly larger losses. On the other side, Ballard et al. (2017) studied the influence of self-control on this effect. The results provided empirical evidence that the visceral (for example, being hungry) and cognitive factors which reduce self-control, also reduce the magnitude effect. Finally, a recent study analyzed the magnitude effect in children by using candy as rewards (Faralla et al., 2021).

Regarding the scenario of expected discounting, we have analyzed 3 empirical articles. Firstly, Sun and Li (2010) considered both the immediacy and magnitude effects with delayed and probability rewards, by confirming their presence in both scenarios. For their part, Dai and Busemeyer (2014) conducted several experiments to elucidate the models which better explain the delay and magnitude effects in choices under uncertainty. In this way, they concluded that the DFT model was the most appropriate. Finally, Luckman et al. (2017) compared the magnitude effect in both scenarios and found that participants overwhelmingly preferred the delayed to the risky option; that is to say, people tend to wait longer when the choice is risky.

In the field of political science, Streich and Levy (2007) analyzed the different effects in intertemporal choice, including the magnitude effect, and supported the use of the quasi-hyperbolic discount model to explain this anomaly. Based on this premise, these scholars applied this model to the problem of cooperation in iterated prisoner's dilemma games.

4.2.2. The delay effect

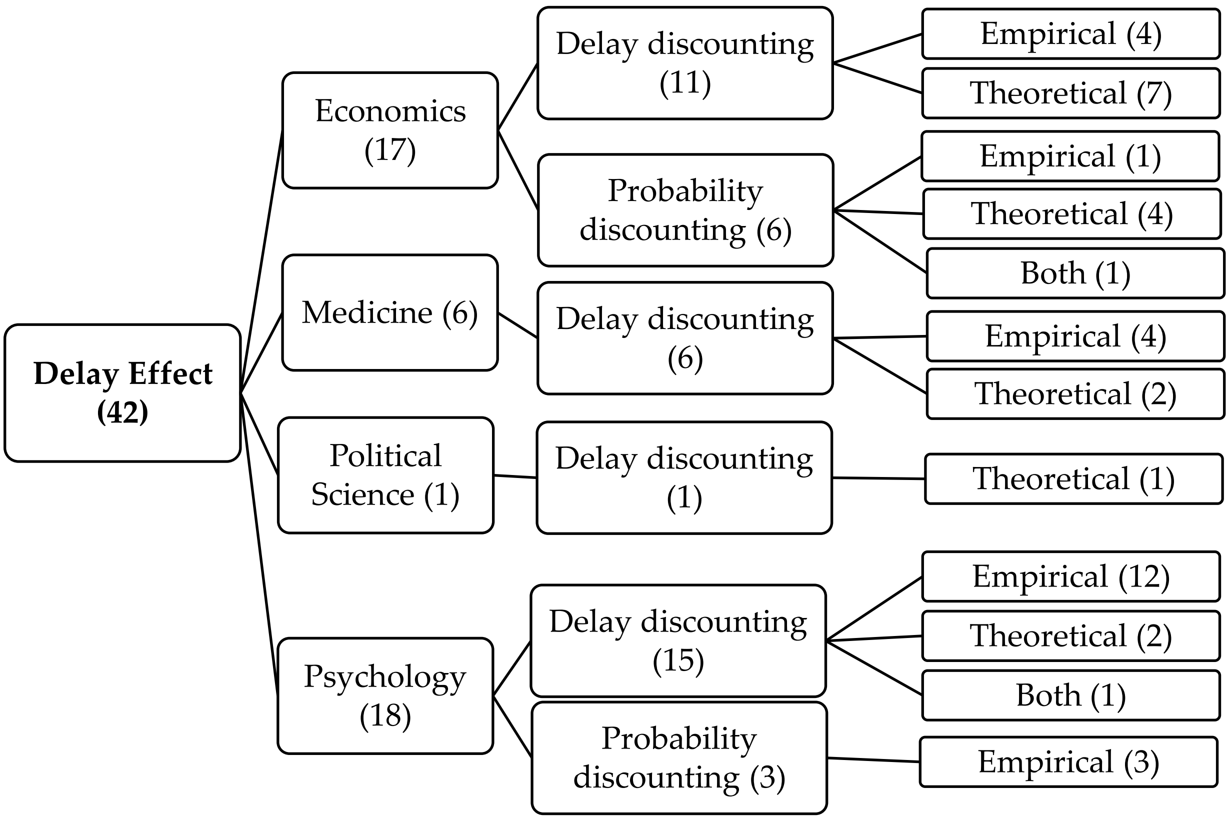

Chart 2. The delay effect. **Source:** Own elaboration.

The delay effect has received lot of attention by researchers. In this way, we have found a total of 42 articles which analyze and explain this anomaly. Chart 2 presents a classification of these works based on their area, scenario and type of study. Below, we have commented the most important contributions derived from the study of this anomaly.

In the field of Economics, we have found 7 theoretical and 4 empirical articles about delayed discounting. Focusing on the theoretical ones, Wathieu (1997) conceptualized mathematically the delay and sequence effects. Subsequently, Read et al. (2013) provided an empirical-theoretical study which develops the DRIFT model, a heuristic description of how framing influences intertemporal choice; moreover, they empirically analyze the delay and magnitude effects. With respect to the delay effect, these scholars concluded that the interest-rate frame induces somewhat greater discounting for longer time periods, thus reversing the common finding of hyperbolic discounting. For their part, Cruz Rambaud et al. (2018) proposed another model called the *q*-exponential discount function deformed by the amount which can describe the magnitude and delay effects jointly. Later, Cruz Rambaud and Ortiz Fernández (2020) proposed a dynamic discount model, called the asymmetric exponential discounting, which explains the delay effect and the subadditivity. They also carried out a mathematical development, from a stationary and dynamic point of view, in which the delay effect is considered a more general concept than subadditivity. Likewise, Drouhin (2020) defined an additive and non-stationary discounted utility function which can explain the delay and magnitude effects, against previous existing literature.

It is worth mentioning the works focused on distinguishing the delay from the interval effect, because these two effects had traditionally been considered as the same anomaly. Kinari et al. (2009) developed a study in which the delay effect was conceptually distinguished from the interval effect. In addition, they conducted an experiment demonstrating the occurrence of these effects together with the magnitude effect. On the other hand, Cruz Rambaud and Ortiz Fernández (2021) mathematically showed that the delay effect and the subadditivity are a particular case of the interval effect from a stationary point of view, and that both effects are independent from a dynamic point of view.

From an empirical point of view, in the field of economics we can find the work by Ikeda and Kang (2015) which revealed that debt holding is related to time discounting through the present bias, the sign effect and the impatience. Therefore, people presenting a steeply declining impatience (present bias or delay effect) are more likely to be debtors. For their part, Wang et al. (2015) conducted an empirical study of the delay effect along with other anomalies, in which the subjective perception of time was taken into account. They observed that, when the perception of time is either objective or subjective, such anomalies are present. Likewise, Tiezzi and Xiao (2016) empirically studied how tax information influences citizens' decisions. They concluded that, when explicit information is given about implicit intertemporal tax competition, the delay effect practically disappears, being stronger otherwise. Finally, Takeuchi and Tsubuku (2018) analyzed the intertemporal choice on goods with a limited time to enjoy them. Their results showed that the intertemporal discount rate increases over time, leading to a reverse delay effect.

In the expected utility scenario, there are 4 theoretical works, 1 theoretical-empirical and 1 empirical. Starting with the theoretical studies, Walther (2010) demonstrated that the delay effect, the sign effect, the delay-speed up asymmetry and the magnitude effect (only in losses) can be explained in the common framework of an expected state-dependent intertemporal utility, that is to say, by considering uncertainty as an aspect of intertemporal decisions. He concluded that the delay effect appears if the probabilities are weighted in a non-linear manner. In addition, he stated that the hyperbolic discount will be more pronounced if the aversion or risk rate increases. On the other hand, Xia (2011) provided an expected utility model, with uncertainty, risk aversion, and preference for precautionary saving, which explained the three main anomalies in intertemporal choice (viz magnitude effect, delay effect and sign effect). On the other hand, Baucells and Heukamp (2012) provided a general model able to reconcile the DU and the EU models, as well as to explain the anomalies arising in intertemporal choices and in choices under uncertainty. Analogously, Holden and Quiggin (2017) provided the Zooming model to explain the delay and magnitude effects, which was corroborated by an empirical study. Finally, Adriani and Sonderegger (2020) carried out a simple cost-benefit analysis in order to derive optimal similarity judgments, demonstrating the magnitude, delay and interval effects in the delayed and probabilistic discounting scenarios. With respect to the empirical works in an environment of uncertainty, we can find the work by Liu et al. (2014) who analyzed the delay effect on environmental risks, obtaining that the more distant in time the occurrence of an environmental risk, the less in intensity subjects will perceive it as a severe threat.

In the field of medicine, we have analyzed 6 articles, 2 theoretical and 4 empirical. The theoretical works are reviews of the existing literature. The first one analyzes several effects, by including the delay effect, related to the field of health (Ortendahl and Fries, 2005), whilst the second one analyzes the same effects in health education (Ortendahl, 2006). Regarding the empirical works, Lazaro et al. (2002) analyzed the delay effect in money and health decisions, finding higher discount rates in life-saving decisions compared to economic ones. Analogously, Guyse et al. (2020) also found no evidence of this effect when the participants were making decisions on human-mortality outcomes. For their part, Johnson et al. (2015), in their study over the function of opportunity cost, demonstrated the presence of the delay effect. Finally, Berry et al. (2017) studied the delay effect in monetary, respiratory health and air quality decisions. The results revealed a rapid delay discounting of air quality, what is a barrier for people engage in long-term sustainable behaviors.

In the field of psychology, we have analyzed 18 articles in the scenario of delayed discounting, of which 2 are theoretical, 1 theoretical-empirical and 12 empirical, whereas in the scenario of expected utility, only 3 empirical articles have been analyzed. Regarding the theoretical works, Killeen (2009) provided a novel discount function generated by making the marginal discount rate time-sensitive and by assuming that it is utility, not monetary value, which is discounted. The additive utility model is unique in that it proposes a disutility to waiting which was added to the utility of the good. Moreover, it predicts the most important anomalies, among them the delay effect. For their part, Scherbaum et al. (2012) provided a dynamic connectionist model of intertemporal choice focused on the delay and date-delay effects, and subsequently, this model was validated from an empirical perspective. The obtained results showed higher discount rates when time is framed in delay rather than dates. On the other hand, Vanderveldt et al. (2016) carried out a literature review of the delay and magnitude effects in nonhuman animals.

Regarding the empirical works analyzing the influence of social habits on intertemporal decisions, Ikeda et al. (2010) confirmed that time discounting is related to body weight. In their study, they analyzed the delay and sign effects, and showed that body mass index is positively associated with survey responses indicative of impatience and hyperbolic discounting (delay effect). Later, Kang and Ikeda (2016) demonstrated that the delay effect is positively associated with unhealthy behaviors, especially with naïve people (people who are not aware of their self-control problems). Additionally, Muñoz Torrecillas et al. (2018) came to similar conclusions, that is to say, there is a greater presence of the delay effect in people with worse healthy habits. Other works focused on the decision-making by people with some additions. Khwaja et al. (2007) analyzed how smokers and non-smokers make economic and health decisions. The results confirm the presence of the delay effect, and that it does not depend on the smoking status across choice domains. In a similar study, Kang and Ikeda (2014) empirically demonstrated that smoking is positively related to the discount rate and the degree of hyperbolic discounting, that is, the delay effect. They also showed that this effect is stronger for naïve people, who are not aware of their self-control problems. Other works focused on examining the behavior of people with ADHD. For example, Paloyelis et al. (2010) analyzed the delay and magnitude effects in people with ADHD (attention deficit/hyperactivity disorder) by distinguishing between hypothetical and real rewards. Moreover, greater impulsivity was observed in case of hypothetical tasks by the group with ADHD. A similar study, only focused on delay effect, was presented by Jackson and Mackillop (2016).

On the other hand, Paglieri et al. (2013) showed that intertemporal discounting was specifically modulated by religion. The work by Olsen et al. (2018) observed the delay and magnitude effects in decisions on academic tasks. On the other hand, Holt et al. (2008) conducted an experiment with students in order to demonstrate that the preference reversals also occur with losses. For their part, Han and Takahashi (2012) demonstrated that the delay and sign effects are due to psychophysical effects of time perception (i.e., nonlinearity and sign effect), by using a q-exponential temporal discounting model introduced in the ambit of Tsallis' thermostatistics. Finally, Shen et al. (2019) analyzed the change in the delay effect by including a delay common in the original choices. The results showed that people are more patient towards the receipt of later, larger outcomes by adding a common delay.

Considering the expected utility scenario, the work by Chapman and Weber (2006) analyzed the delay and magnitude effects in delay and probabilistic scenarios for decisions on money and health. They concluded that the delay and the common ratio effects are uncorrelated, but each is correlated across monetary and health domains. Likewise, Sun and Li (2010) analyzed the immediacy and magnitude effects in the delay and probabilistic discounting scenarios, by confirming their presence, in opposite direction, in both scenarios. For their part, Dai and Busemeyer (2014) conducted several experiments in order to verify which models best explain the delay and magnitude effects in choices under uncertainty. They concluded that the DFT model was the most appropriate.

In the field of political science, there is a theoretical article which analyzes the different effects in intertemporal choice, by including the magnitude effect, and supports the use of the quasi-hyperbolic discount model as an explanation for them. Based on this premise, Streich and Levy (2007) applied this model to the problem of cooperation in iterated prisoner's dilemma games.

4.2.3. The sign effect

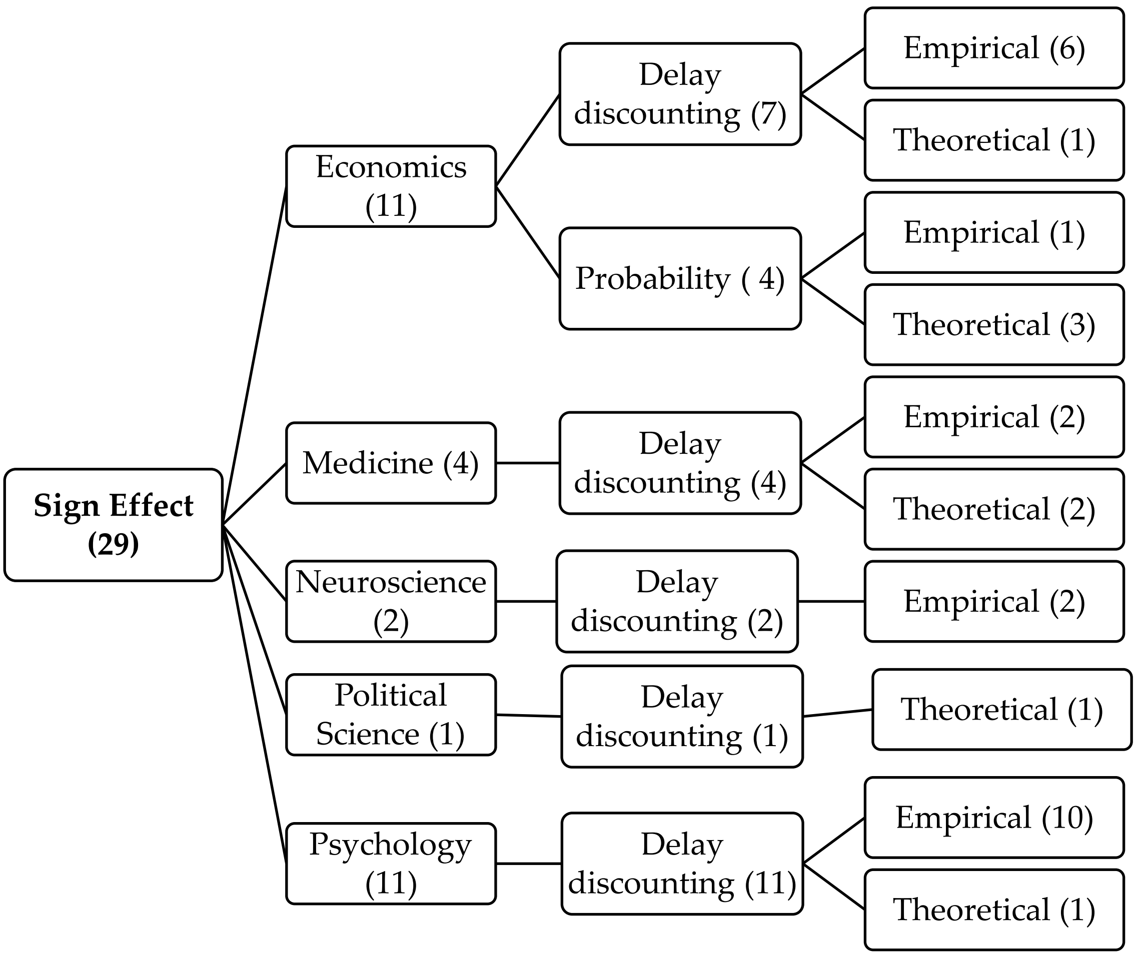

Chart 3. The sign effect. **Source:** Own elaboration.

The sign effect is one of the most robust and analyzed effects in the existing literature, along with the magnitude and delay effects. Therefore, it is logical that, starting from our search, we have obtained 29 articles, whose classification appears in Chart 3. Below, the main contributions on this anomaly will be commented.

In the field of economics, there are 1 theoretical and 6 empirical works which analyzed the sign effect under the delay discounting perspective. Theoretically, Al-Nowaihi and Dhami (2009) insisted on the explanation provided by Loewenstein and Prelec (1992) for the sign effect, based on the property of incremental elasticity of discount functions, in order to build a theoretical framework from which it is possible to obtain functions exhibiting this property, known as simple increasing elasticity (SIE) utility functions. With respect to the empirical works, most of them are demonstrations of theoretical concepts related to the sign effect. First, McAlvanah (2010) empirically demonstrated the relationship of subadditivity with the sign effect. Their results showed that the differential concavity of utility for gains and the convexity for losses imply that the discounting of losses is even more subadditive than the discounting of gains. Thus, individuals display even more relative impatience over divided time intervals for negative than for positive amounts of money. For their part, Abdellaoui et al. (2010) presented a parameter-free method in order to measure the discounted utility model as a whole. Moreover, they found some evidence for a sign effect in the time weights, which contradicts earlier conclusions that the gain-loss asymmetry is due to a framing effect. Subsequently, Wang et al. (2015) conducted an empirical study of the sign effect along with other anomalies, in which the subjective perception of time was taken into account. They observed that, when the perception of time is either objective or subjective, such anomalies are present. Guyse and Simon (2011) also demonstrated empirically the sign and the magnitude effects, jointly. Additionally, Breuer and Soypak (2015) analyzed the sign effect along with other effects, such as the delay-speed up asymmetry. They concluded that framing effects (choice and matching tasks) are stronger for negative outcomes and explain the correlation between the framing effects and the outcome sign based on the different treatment of out-of-pocket and opportunity costs. Due to a weaker loss aversion with respect to opportunity costs, delay and speedup discount rates in choice tasks are more similar for questions involving positive outcomes. Finally, Ikeda and Kang (2015) demonstrated that the sign effect was related negatively to borrowing. In effect, the survey responses indicative of high or declining impatience are associated with credit card borrowing and other overborrowing indicators.

In probabilistic discounting, we have found 4 articles, of which 3 are theoretical and one empirical. Firstly, Walther (2010) demonstrated that the delay effect, the sign effect, the delay-speed up asymmetry and the magnitude effect (only in losses) can be explained in the common framework of an expected state-dependent intertemporal utility, that is to say, by considering uncertainty as an aspect of intertemporal decisions. He concluded that the sign effect evolves if the subject is either relative risk-averse or relative disappointment averse (or both). However, this effect disappears if, and only if, the subject is risk-neutral and the probability weighting is symmetric with regard to elation and disappointment. In addition, Xia (2011) provided an expected utility model, with uncertainty, risk aversion and preference for precautionary saving, which simultaneously explained three anomalies (magnitude effect, delay effect and sign effect). On the other hand, Shoji and Kanehiro (2012) carried out a numerical analysis of the magnitude and sign effects under different risk tolerances. Finally, Molouki et al. (2019) analyzed the sign effect of past and future events and found a tendency towards discount gains more than losses. This tendency emerges more strongly and consistently for future events than for past ones.

In the field of medicine, we have analyzed two empirical articles and other two theoretical, in the scenario of delay discounting. The theoretical works are reviews of the existing literature, the first of them analyzing the sign effect and other effects (the magnitude, delay and sequence effects) in the field of health (Ortendahl and Fries, 2005). The second one analyzes the same effects in health education, based on different choice frames (Ortendahl, 2006). With respect to empirical works, Berry et al. (2017) studied the sign effect in monetary, respiratory health and air quality decisions. The results revealed that the sign effect is present in monetary decisions, but not in health and air quality decisions. Analogously, Guyse et al. (2020) also found no evidence of this effect when the participants were making decisions on human-mortality outcomes.

In the field of neuroscience, we analyzed two empirical works. The paper by Qu et al. (2013) showed some evidence on the sign effect and concluded that this effect could be encoded in the FRN (feedback-related negativity) at the initial stage of the results evaluation. On their part, Tanaka et al. (2014) analyzed the sign effect in relation to the delay and magnitude effects. They concluded that the participants with the sign effect exhibit an isolated response to the magnitude of losses greater than that of gains, and also a striatal response to the delay of losses greater than that of gains.

In the field of psychology, in the delay discounting scenario, we can find 1 theoretical and 10 empirical papers. The theoretical study by Killeen (2009) provided a novel discount function generated by making the marginal discount rate time-sensitive and by assuming that it is utility, not monetary value, which is discounted. The additive utility model was unique in that it proposes a disutility to waiting which was added to the utility of the good. Moreover, it predicts the most important anomalies, among them the sign effect. Regarding the empirical works, we can classify them in categories according to the analyzed topic. Firstly, we will analyze the articles focused on mathematical models and its empirical validation. Han and Takahashi (2012) demonstrated that the delay and sign effects are due to psychophysical effects of time perception (i.e., nonlinearity and sign effect), by the q-exponential intertemporal discounting model introduced in Tsallis' thermostatistics. They also confirmed that subjective time was perceived as shorter in the loss than in the gain domain. However, Xu et al. (2020) did not find evidence for the premise of Han and Takahashi (2012). This contradiction might be due to the large difference of timescales used by two studies. On the other hand, Stevens (2016) tested discounting models against attribute-based models, which use similarity judgments to make choices. His results showed that similarity judgments permit to account for behaviors which contradict many discounting models, such as the magnitude and the sign effects. Therefore, the attribute-based models, such as the similarity models, provide some alternatives to discounting, what may offer several insights into the process of decision-making in the context of intertemporal choices.

Regarding the empirical works on individuals with additions or health problems, we can find the work by Khwaja et al. (2007) who analyzed how smokers made economic and health decisions. The results in monetary elections showed that the sign effect is present, that is to say, gains are discounted more than losses. In the field of health, they showed that implicit discount rates decrease with the sign of the payoff (sign effect). In both cases, this effect does not depend on the smoking status. In a similar study, Kang and Ikeda (2014) empirically demonstrated that the sign effect retrained the likelihood of smoking and the number of cigarettes consumed. That is to say, smoking was negatively related to the sign effect. On the other hand, Ikeda et al. (2010) also showed that body weight was negatively associated with those parameters indicative of the sign effect. Other studies also found evidence of the sign effect in drug addicts both for monetary rewards and for cocaine (Johnson et al., 2015).

With respect to the different domains of choice in which the sign effect has been studied, Chapman and Winquist (1998) analyzed the sign effect in monetary and tips decisions. Although these scholars found no evidence of this effect in tips, it was present in monetary decisions (Hesketh, 2000). Recently, Faralla et al. (2021) analyzed the sign effect in children by using candy as rewards. Finally, in the field of political sciences, the theoretical article by Streich and Levy (2007) analyzed the different anomalies in intertemporal choice, by including the sign effect, and supported the use of the quasi-hyperbolic discount model as an explanation for them. Based on this premise, they applied this model to the problem of cooperation in iterated prisoner's dilemma games.

4.2.4. The sequence effect

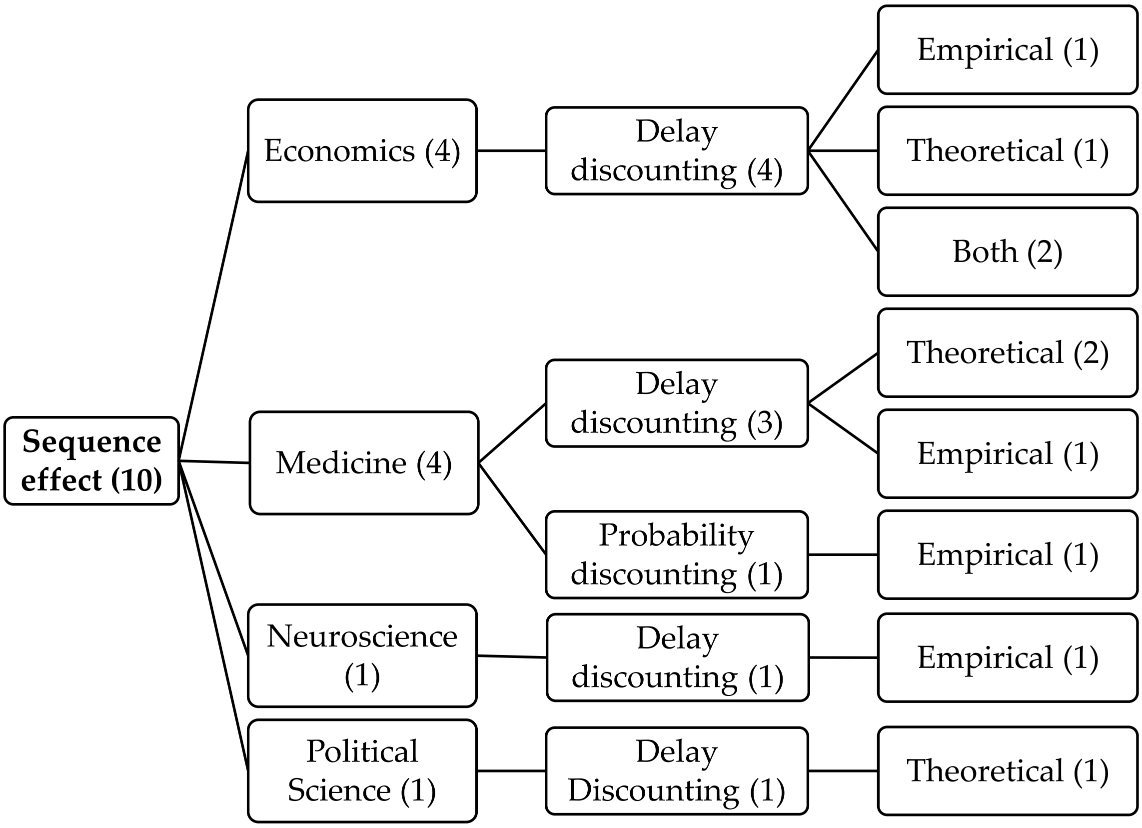

Chart 4. The sequence effect. **Source:** Own elaboration.

According to the works analyzed in this paper, the sequence effect has not received enough attention from researchers. In this way, we can find a total of 10 articles about this effect. Chart 4 shows the classification of these articles by area, scenario and type of study. The main conclusions on this anomaly have been commented below.

In the field of economics, we have analyzed four manuscripts framed in delay discounting: one theoretical, two theoretical-empirical and one empirical. The first of them is that of Wathieu (1997) who conceptualizes, in a mathematical way, the sequence and the delay effects. The following article was published 22 years later, when Cruz Rambaud et al. (2019) conducted an empirical study on this effect based on students' preferences among 3 loan proposals to buy a car. Their results confirm the sequence effect, i.e., students preferred to make higher repayments at the beginning of the loan duration, leaving the lower repayments for the end. The main characteristic of the study is that the financial sum of all loan terms is constant, contrarily to the rest of studies where the "usual sum" is constant. In addition, they propose the q-exponential discount function to explain the sequence effect. Subsequently, Lu et al. (2020) extended the works by Hoelzl et al. (2011) and Cruz Rambaud et al. (2019), by testing the robustness of the improving sequence effect with different combination of rates, amounts and repayment methods. The results suggest that the findings of previous studies are reliable and resilient to the aforementioned changes. In this same year, Garcia et al. (2020) carried out a theoretical-empirical work in which they provided a new model, which explains this effect better than the DU model. They also conducted an empirical analysis which revealed that participants, although aware of the present value maximization, preferred improving sequences of incomes in order to cover their future spending needs, to be motivated at work and to receive a signal of success and status.

In the field of medicine, in the delay discounting scenario, we have found two theoretical articles and one empirical; whilst in probability discounting we have found only one empirical paper. The theoretical works are reviews of the existing literature: the first of them analyzes the sequence effect and other effects (magnitude, delay and sign) in the field of health (Ortendahl and Fries, 2005);

whilst the second one analyzes the same effects in health education, based on different choice frames (Ortendahl, 2006). With respect to empirical works, Guyse et al. (2020) carried out an empirical study by using non-monetary choices, that is to say, the choice was between sequences of lives, lost or saved over time. The results corroborated the sequence effect. Another study related to decisions about life, but in terms of probability, was provided by Van der Pol and Ruggeri (2008). These scholars analyzed the sequence effect starting from the risk attitudes of respondents throughout their lives. They found that the sequence effect is present in the quality-of-life gamble involving the severe ill-health state. Respondents tended to be more risk seeking when the years of ill-health occurred before the years of full health. Thus, this increase is larger for individuals exhibiting negative time preferences than for those exhibiting positive time preferences.

In the field of neuroscience, we have found an empirical study in which the authors confirmed that the effects of imagination on patience do not reduce to effects of willpower. Accordingly, the sequence framing may be an especially promising means to sustain patience when the ability to exert willpower is compromised, such as under conditions of high cognitive load (Jenkins and Hsu, 2017).

Finally, in the field of political science, Streich and Levy (2007) theoretically analyzed the different anomalies in intertemporal choice (by including the sequence effect) and supported the use of the quasi-hyperbolic discount model as an explanation for them. Based on this premise, they applied this model to the problem of cooperation in iterated prisoner's dilemma games.

4.2.5. The date-delay effect

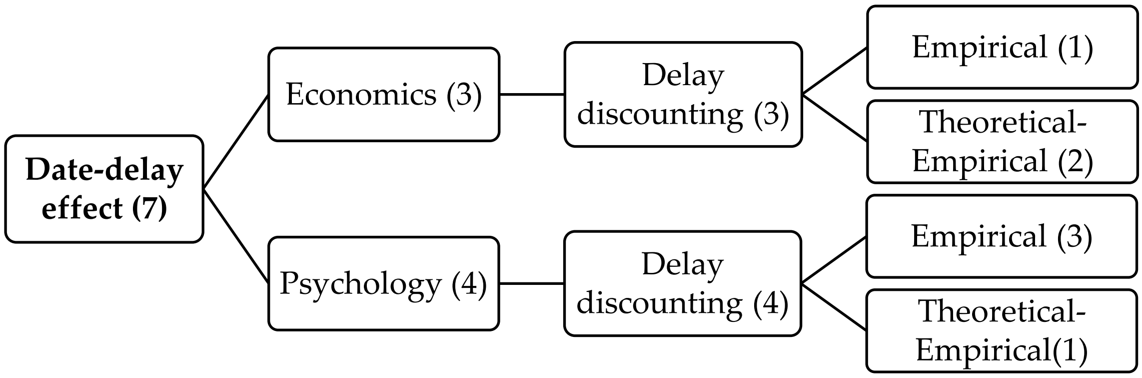

Chart 5. The date-delay effect. **Source:** Own elaboration.

Like the previous effects analyzed in this section, this effect has not been widely studied, whereby we have only found 7 articles. Their classification, according to their object of study, can be seen in Chart 5. Below, we will discuss the main contributions about this anomaly.

In the field of economics, Read et al. (2005) were the first scholars in defining the concept of date-delay effect, which was corroborated through five experiments using hypothetical and monetary rewards. Subsequently, Breuer and Soypak (2015) empirically analyzed the delay effect, the sign effect and the delay-speed up asymmetry, by distinguishing between date and delay. In the same year, Wang et al. (2015) conducted a theoretical-empirical study on this effect (including the magnitude effect, the delay effect, the sign effect and the delay-speed up asymmetry) in which they included the subjective perception of time. In this way, they observed that anomalies take place when considering the objective time was, although they disappear when introducing subjective time perception.

In the area of psychology, Klapproth (2012) conducted an empirical study with drug addicts, in which the discounting tasks were described by using dates and delays. They observed that future rewards were discounted to a lesser extent when time was presented as a date. In addition, drug addicts were affected by this effect more than people without addictions. For their part, Scherbaum et al. (2012) provided a dynamic connectionist model of intertemporal choice focused on the delay and date-delay effects and, subsequently, this model was validated from an empirical study which showed higher discount rates when the time is framed in delays rather than in dates. Likewise, Dshemuchadse et al. (2013) conducted an experiment in order to obtain some information about the mechanisms influencing intertemporal decision-making. To do this, they analyzed the choice action dynamics via a novel combination of continuously recorded mouse movements and a multiple regression approach. Specifically, they observed less direct mouse movements when the time is framed in calendar dates instead of delays, especially for later/larger options. They also found that the decision process results more strongly influenced by the differences in values of dates than of delays. As an explanation, these scholars considered that the date-delay effect is only the general consequence of more deliberative processing caused by higher cognitive demands due to the more complex format of calendar dates. Finally, Schoemann et al. (2019) replicated the former experiment but made changes to the initial procedure, thus obtaining evidence that the methodological configuration has a crucial influence on the results of the experiment.

4.2.6. The interval effect

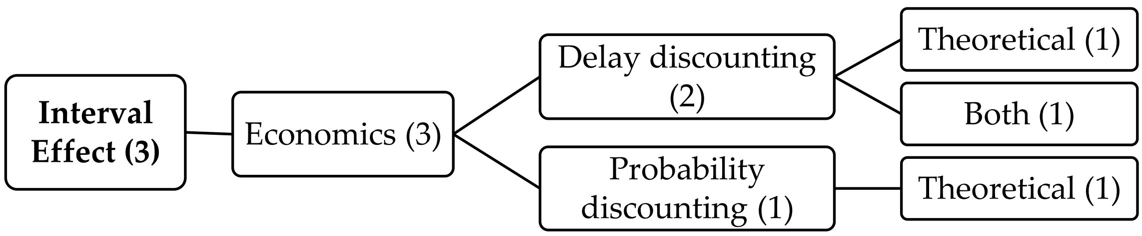

Chart 6. The interval effect. **Source:** Own elaboration.

Finally, we are going to analyze the interval effect, which is the least studied anomaly, possibly due to the fact that traditionally this effect has been confused with the delay effect. Three articles have been found, whose classification can be seen in Chart 6. The three articles belong to the field of economics: two of them (one theoretical and another theoretical-empirical) deal with delay discounting, whilst the other one is about probabilistic discounting, being of theoretical type. Considering the delay discounting scenario, Kinari et al. (2009) conducted a study in which they distinguished between the delay and the interval effects, then providing an experiment where these effects occurred together with the magnitude effect. They also confirmed that the interval effect is a sufficient condition for subadditivity, that the delay effect was a more general concept than the interval effect, and that the Weber-Fechner's law does not explain this effect in their experiment. On the other hand, and also in delay discounting, Cruz Rambaud and Ortiz Fernández (2021) mathematically defined the interval effect and related it with the delay effect and the subaditivity. Contrarily to the work by Kinari et al. (2009), they obtained, that, from a stationary point of view, the interval effect was a more general concept than the delay effect whilst, from a dynamic perspective, both effects are independent. More specifically, they showed that the interval effect only make sense from a stationary point of view. As for probability discounting, we found a theoretical article by Adriani and Sonderegger (2020) who carried out a simple cost-benefit analysis to derive optimal similarity judgments, and demonstrated the magnitude, delay and interval effects in delay and probabilistic discounting scenarios.

4.2.7. The delay/speed up asymmetry

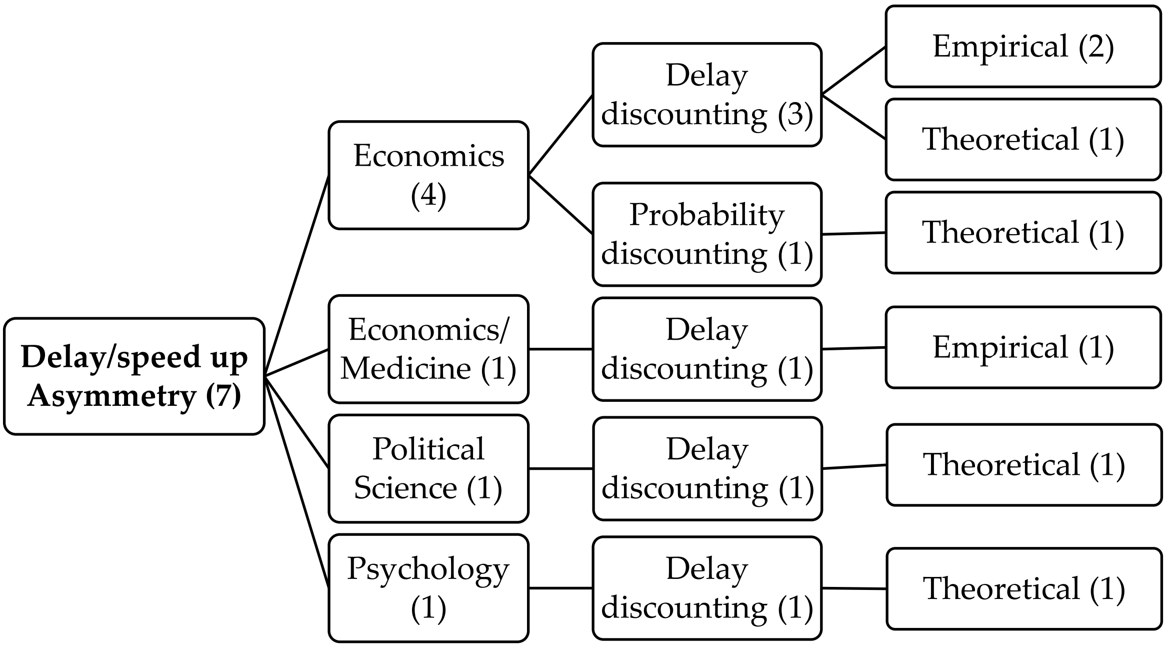

Chart 7. The delay/speed up asymmetry. **Source:** Own elaboration.

Although the intertemporal choice includes both postponement and anticipation of rewards, deferred decisions have received greater attention. Therefore, we have only found 7 articles dealing with the delay/speed up asymmetry. Chart 7 displays the classification of these works, according to the area, scenario, and type of study. We will now discuss the main findings in the study of this anomaly.

In the field of economics, we can find 4 articles, of which 3 were about delay discounting and 1 about probabilistic discounting. The first of them (McAlvanah, 2010) empirically demonstrated the relationship between the subadditivity and the delay/speed up asymmetry. It showed that, for gains, subadditivity is stronger when considering whether to delay a later rather than an earlier outcome, and weakest when anticipating an outcome from a later status quo.

However, for losses, subadditivity is weaker when delaying a loss to a later date, and strongest when anticipating a delayed loss to an earlier date. Later, Breuer and Soypak (2015) demonstrated that time-inconsistent behavior is less frequent in choice tasks compared to matching tasks. Likewise, they observed less significant differences between delay and speedup frames in intertemporal choice tasks compared to intertemporal matching tasks. Finally, Baucells and Bellezza (2017) proposed a descriptive model, called the anticipation-event-recall (AER) model, in order to explain the magnitude effect and the delay-speed up asymmetry. Regarding the probability scenario, Walther (2010) demonstrated that the delay effect, the sign effect, the delay-speed up asymmetry and the magnitude effect (only in losses) can be explained in the common framework of an expected state-dependent intertemporal utility, that is to say, by considering the uncertainty as an aspect of the intertemporal decision. He concluded that the delay-speed up asymmetry emerges if the relative risk aversion is constant and positive and disappears if the subject is risk-neutral.

In the field of medicine, Lazaro et al. (2002) conducted an empirical study in order to analyze different anomalies of the DU model in social rather than private decisions on health and money. This study corroborates the presence of the delay effect, the magnitude effect and the delay-speed up asymmetry. This indicates that the choice mechanisms are the same, regardless of whether individuals make private or social decisions about money or health.

In the field of psychology, the theoretical study by Killeen (2009) introduced a novel discount function by making the marginal discount rate time-sensitive and by assuming that it is utility (not monetary value) which is discounted. The additive utility model is unique in that it proposed a disutility to waiting which was added to the utility of the good. Moreover, it predicted the most important anomalies, among them the delay-speed up asymmetry.

In the field of political science, Streich and Levy (2007) theoretically analyzed the different anomalies in intertemporal choice, including the delay-speed up asymmetry, and supported the use of the quasi-hyperbolic discount model as an explanation for them. Based on this premise, they applied this model to the problem of cooperation in iterated prisoner's dilemma games.

4.2.8. Further research lines

In the former section, we have presented the main contributions in the existing literature on this topic. These papers were classified by areas; therefore, our aim in this section is to propose future general lines of research in each of them.

Field of economics

There have been many proposals for discounting models which attempt to explain these anomalies. However, most studies have focused on proposing some models able to explain the magnitude and the delay effects, jointly or separately. The most general model aiming to explain more anomalies (magnitude, delay and sign effects, and delay/speed up asymmetry) was that of Killeen (2009) whilst the interval effect remains without a model proposal. As indicated in other works, further work is needed in this line in order to achieve a model which explains all the possible effects, complemented with empirical work able to validate the obtained discount functions. We consider very interesting to carry out an empirical study able to compare all these new proposals (MED, DRIFT, AER, etc.) in order to continue working as accurately as possible. On the other hand, some works have analyzed the subadditivity jointly with some effects, specifically with the delay, sign and interval effects. In this way, our proposal is to mathematically analyze the subadditivity in the context of all possible effects, in particular the magnitude, the sequence effect and the delay-speed up asymmetry. Another interesting line of research is to design an index able to measure each of these effects like the one proposed by Cruz Rambaud et al. (2019) for the magnitude effect. The work by Wang et al. (2015) analyzed the subjective perception of the timing of delay, sign, delay/speed up asymmetry and date-delay effects, and observed that when timing was subjective such anomalies disappeared (except for the sequence and interval effects). Based on another work, which analyzed the delay effect in the Pigouvian taxation, we propose its generalization to other effects such as magnitude, sequence, interval and date-delay effects, by differentiating between sexes and even between age groups. Another line could be how all effects are affected when the goods are available to the decision-maker for a limited time. Specifically, in the case of the delay effect, there is a reverse effect, but what happens with the rest of effects? We also consider it necessary to carry out a review of the more specialized articles. In this way, it would be possible to revise the design of experiments (for example, a review of the empirical works of the magnitude effect) and to analyze the rewards and deadlines used in each experiment, because the differences between the conclusions could be due to changes in the units of measure. It would also be interesting to know whether the rewards are real or hypothetical, and the type of group chosen for the analysis, as well as the type of questionnaire chosen and the type of analysis applied to the data. Finally, it is important to highlight the importance of analyzing the interval effect because, in this paper, we have only considered 2 articles in the delay discounting scenario (one theoretical and the other empirical). Therefore, it is important that researchers become aware of this anomaly, and incorporate it into their experiments, since it is in its early stages of research and should be extensively considered, analogously to the magnitude and delay effects.

In this review, we have not found any work analyzing the intertemporal choice in the field of business, such as in the areas of HR, marketing, production, etc. Therefore, we consider it interesting to study these effects which allow the implementation of appropriate policies in the company able to help them to obtain added value. For example, our proposal is to analyze whether workers prefer to receive incentives monthly or all together in a single amount, whether customers or suppliers prefer to pay in several or in a single payment, whether they prefer to pay a larger amount later or prefer a prompt payment, whether these preferences are maintained when the payment amount involves larger or smaller amounts. Despite being a widely studied topic in economics, all the works have been based on the financial area, without any generalization to managerial decisions.

Field of medicine

Contrarily to the economic and psychological areas, we cannot find any paper on anomalies in the field of medicine. However, we believe that further research in this area may be of interest. For example, one of the papers compared life-saving and economic decisions; however, it only compared magnitude, delay, and delay/speed up asymmetry effects, whereby we propose to extend this study and compare life-saving/loss decisions and economic decisions, in addition to the previous effects on the sign, sequence, interval and date-delay effects.

Another paper analyzed the delay and sign effects in the context of health and air quality. Our proposal is to extend this study to the other effects. We also propose to conduct some empirical studies on health campaigns incorporating intertemporal choices, in order to analyze these effects with the aim of designing future health campaigns which positively influence people's behavior. Society is currently immersed in this type of choice with the emergence of COVID-19: have a social life now and have the virus in a few days, or stay at home and be healthy? This choice varies with age and type of person. There are many awareness campaigns, but which one is the most suitable? Perhaps, by analyzing how the effects of intertemporal choice affect our behavior (positively or negatively), campaigns could be more oriented to increase or reduce people's undesirable behaviors.

Field of neuroscience

As in the previous area, anomalies in intertemporal choice have not been studied sufficiently in the field of neuroscience. The main studies have been addressed to the magnitude and sign effects, and to a lesser extent to the delay and sequences effect. But, what about the rest of effects? Our proposal in this area is to study the rest of anomalies such as the delay/speed up asymmetry, date-delay and interval effects. Examples of future lines of research would be to know which areas of the brain are affected by the presence of these effects, to know how imagination and willpower affect them and to analyze the different intertemporal anomalies with the striatal and insular activities of the brain together with individual biological attributes (ethnicity, sex, age, obesity and genetic polymorphisms) and social (culture, income, work, social status and marital status, etc.). These works will be the basis of other future works that will emerge as this research progresses. Another possible line of research is to know the neural behavior in intertemporal choices involving animals.

Field of psychology

Undoubtedly, he fields of psychology and economics stand out for its research in empirical work. One of the investigations, and perhaps less researched because of its complexity, is to know the effects of intertemporal choice in animals. In our work, we have found two articles, one which analyzes the magnitude effect in pigeons, and another which analyzes the delay effect in capuchin monkeys. We consider that it could be interesting to increase the number of studies in other animals such as mice, but it would also be interesting to analyze, if possible, effects other than those already studied. Another possible line of research is to analyze if the attribute-based models which use similarity judgments are better than the alternative models in all effects, or if they are only better for the magnitude and the sign effects, as already demonstrated in a previous study. There are other studies which have found that people with alcohol problems have a higher magnitude effect than people without alcohol problems, but what about the other effects? We propose to investigate this duality with the other anomalies. Other studies analyzed the magnitude, delay, sign and date-delay effects in people with drug addiction problems, and the delay and sign effects in smokers, as well as the magnitude and delay effects in people with ADHD. However, there are other effects which have not been analyzed with any of these groups such as the interval effect, the sequence effect and delay/speed up asymmetry. This is why we propose to continue expanding research in this direction. On the other hand, other studies have analyzed the magnitude and sign effects in the context of tips. But, are the other effects present in the tips? and is there a difference in the effects if we differentiate by sex or age and even by culture? Other studies have analyzed whether there are some differences between religions in intertemporal choice; specifically, they analyzed the magnitude and delay effects, and indeed there were differences. So, can this statement be extended to the rest of the effects? There are also studies which have analyzed the magnitude and delay effects in academic tasks or have related these same effects to dietary habits or even to body mass as in the case of the delay and sign effects. But, in the same way, as previously indicated, these studies have not been replicated for the rest of anomalies. Another study has analyzed the influence of self-control on the magnitude effect, but does the reduction or increase of self-control affect the rest of effects? Another proposal is to extend the study of anomalies to children. So far, non-monetary rewards have been tested for the magnitude and sign effects, and we propose to extend this study to the rest of effects by discriminating by age group, sex and type of reward (monetary and jelly beans). Another work analyzed the psychophysical effects of time perception on the sign and delay effects, and then we propose to replicate this experiment to analyze the magnitude effect, the delay-speed up symmetry, and the sequence and interval effects. Finally, there are several works which analyze the date-delay effect from mouse movements, but is it possible to analyze the rest of effects with this method? We propose to study its possible application to the rest of anomalies.

Field of political science

In this area, we have only found one article which analyzes the evolution of magnitude, delay, sign, speed up asymmetry and sequence effects, without relating them to political science. We propose to carry out an experiment in which all effects can be analyzed by differentiating between sex, age group, religion and political ideology, which will allow us to know if a given political ideology influences the effects, so that, if this is confirmed, these findings can be used to design political campaigns to capture the largest possible number of voters.

5. Conclusion

In this paper, we have conducted a systematic literature review of the main intertemporal choice effects (delay effect, magnitude effect, sign effect, sequence effect, delay/speed up asymmetry, sequence effect and interval effect). We have highlighted the anomalies which require further investigation and we have proposed future lines of research in each of the analyzed areas and effects. However, we have observed that practically there is no research on intertemporal choice in the business environment, whereby we have proposed some possible lines of research on this topic. On the other hand, this analysis has allowed us to know a strong deficiency in this field, viz the high variety of names of these effects, mainly the delay effect. This prevents from a deep analysis of the effects if the variety of names which each author proposes is unknown. It is necessary that there is unanimity to refer to these effects, because only in this way a complete analysis can be made in each area of study. In this paper we propose the following denominations: delay effect, magnitude effect, sign effect, sequence effect, delay/speed up asymmetry, sequence effect and interval effect.

The main limitation of this paper was the variety of names given to all effects, so that articles that did not include our search keywords were left out of the results. Another limitation was that the first studies of these anomalies were not found in the databases chosen.

A future line of research for this article, in addition to those proposed in the previous section, is to carry out specific literature reviews of each of the above effects, by distinguishing between theoretical and empirical works, which will allow us to know in detail the current status of the research on each anomaly.

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CAPÍTULO III.

EL EFECTO PLAZO

Delay Effect and Subadditivity. Proposal of a New Discount Function: The Asymmetric Exponential Discounting

Artículo publicado en 2020 en Mathematics.

Factor de Impacto (2019) 1,747 (Journal Citation Reports-Thomson-Reuters)

Delay Effect and Subadditivity. Proposal of a New Discount Function: The Asymmetric Exponential Discounting

Abstract: The framework of this paper is intertemporal choice and, more specifically, the so-called delay effect. Traditionally, this anomaly, also known as decreasing impatience, has been revealed when individuals reverse their preferences over monetary or non-monetary rewards. In this manuscript, we will analyze the delay effect by using preference relations and discount functions. The treatment of the delay effect with discount functions exhibits several scenarios for this paradox. Thus, the objective of this paper is to deduce the different expressions of the delay effect and their mathematical characterizations by using discount functions in stationary and dynamic settings. In this context, subadditivity will be derived as a particular case of decreasing impatience. Finally, we will introduce a new discount function, the so-called asymmetric exponential discount function, able to describe decreasing impatience.

Keywords: intertemporal choice; delay effect; decreasing impatience; inconsistency; discount function; subadditivity

1. Introduction

The Discounted Utility model, originally introduced by Samuelson [1], became one of the main paradigms of asset valuation when time is involved in the decision-making. The discount function which underlies the process of intertemporal choice is an exponential function that makes preferences consistent over time. However, from the 80s onwards, Thaler [2] opened a debate on the acceptance of Samuelson's model, precisely due to the validation of the existence of time inconsistency in decision-making. This involved a large number of subsequent empirical studies which confirmed that, in effect, the discount ratios obtained in their empirical studies did not fit the theoretical basis of Samuelson's model, and that these differences were due to a series of paradoxes, labeled as "anomalies" in intertemporal choice.

In the existing literature on this topic, a large number of these anomalies have been demonstrated, of which we can highlight [3] the following effects: delay effect, magnitude effect, sign effect, sequence effect, spread or dissemination effect, and delay-anticipation asymmetry effect, among others. That is why, over the last 30 years, several attempts have been made to find mathematical models able to cover the aforementioned deficiencies of the model proposed by Samuelson. Among them, we can find the hyperbolic discounting model [4,5] in which the discount rate decreases with the passage of time, thus solving the anomaly called "delay effect" or decreasing impatience, revealed for monetary [2,6] and non-monetary decisions [7].

This anomaly implies that, when the delay increases, the discount rate decreases, that is, there is an inverse relationship between the discount rate and time, which allows us to state that as time increases, impatience diminishes. This effect gives rise to time inconsistency, due to the change in preferences of the decision-maker since, starting from two equivalent rewards available at different times, the decision-maker is willing to wait to obtain a reward greater than the smaller outcome sooner.

This concept gives rise to subadditivity, investigated by Sholten and Read [8–10] and Cruz Rambaud and Muñoz Torrecillas [11]. We can say that a discount function is subadditive if the discount rate is greater when the interval is subdivided, that is, the discount in subintervals is greater than in the whole interval; in other words, the decision-maker prefers the earliest option when deciding in a subinterval, and the latest option in the entire interval. The opposite situation is superadditivity.

However, Cruz Rambaud and Muñoz Torrecillas [12] provided a new concept, the subadditivity of the second order, which is more general than subadditivity. Moreover, in the dynamic context, apart from the two former concepts, we have to distinguish between increasing discount ratios (delay effect) and decreasing discount rates (decreasing impatience), which are equivalent situations in a stationary context.

The objective of the paper is the mathematical treatment of the delay effect by using discount functions. This will allow us to derive several expressions of this anomaly and to deduce subadditivity as a particular case of decreasing impatience.

It is well known that intertemporal choice is a topic of great relevance in the field of finance, but not exclusively because it has been studied in a lot of disciplines, including medicine and psychology. In effect, several authors have shown the importance of applying discounting processes to health [13–16], as evidence has been found that people discount their future health status, particularly in certain addictive behaviors, such as smoking [17,18], gambling addiction, excessive alcohol consumption [19], and even in obesity-related behaviors [20,21]. In all these experiments, higher discount rates have been found in the closest decisions.

As described before, in a stationary context delay effect means that the discount rate decreases as the delay increases or, what is the same, the decision-maker discounts at a higher rate when the delay is shorter. Obviously, this definition is associated with the concept of impatience (the decision-maker prefers something which happens earlier than later) [22]. Consequently, we can say that, in the context of the delay effect, the decision-maker becomes more impatient when the reward is near the current moment. In this way, impulsivity and impatience are taken as analogous concepts by some researchers, such as Takahashi et al. [23] and specifically by Cruz Rambaud and Muñoz Torrecillas [24].

On the other hand, impatience gives rise to another concept, the so-called excessive discount [25], which occurs when the applied discount rates are excessively high. This phenomenon can be mainly associated with people with problems such as schizophrenia, obese people, drug addicts, and smokers, among others. Another related concept is subadditivity, also applied to health management [26], which implies that individuals with some addiction more readily relapse into addiction when there are several short abstinent periods than when there is only one period of major abstinence. The study of this concept is very important for designing a treatment to cure an addiction.

To describe the delay effect, Mazur [27] proposed the hyperbolic function as an alternative to the exponential discounting of Samuelson [1] since it better fits the empirical works carried out by numerous researchers. Cruz Rambaud, Muñoz Torrecillas and Takahashi [28] also provided a discount function which better fits the decisions of people with addictions and, consequently, with excessive discount ratios, called the exponentiated hyperbolic discount function. This function exhibits a greater slope than the simple hyperbola presented by Mazur, reflecting the higher discount rates shown by decision-makers with problems of addiction.

In this manuscript, we are going to propose a novel dynamic discount function, the so-called asymmetric exponential discount function, which fits decreasing impatience better than the hyperbolic function of Mazur, since it exhibits the different types of delay effect presented in this work.

The structure of this paper is as follows. Section 2 presents the concept of subadditivity. In Section 3, we will define the concept of the delay effect by showing different alternatives derived from its treatment with discount functions. In particular, the concept of subadditivity will be derived from the general setting of the delay effect analyzed in this section. Section 4 provides a new discounting model that better fits the preferences of individuals, taking into account the different modalities of the delay effect. Finally, Section 5 summarizes and concludes.

2. Subadditivity

A concept closely related to the delay effect is the absence of additivity. Scholten and Read [8–10] studied subadditivity and superadditivity in intertemporal choices and, subsequently, Cruz Rambaud and Muñoz Torrecillas [11] investigated subadditivity and its relationship with the delay effect.

First, we are going to make a brief description of the additivity and subadditivity. To do this, we need the following definition.

Definition 1. *A (dynamic) discount function is a continuous real-valued function*

$$F : \Re \times \Re^{+} \to]0, 1],$$

where $\Re^{+}$ is the set of positive real numbers including zero, defined by:

$$(t, a) \mapsto F(t, a),$$

such that $F(t,0) = 1$ and $F(t,\cdot)$ is strictly decreasing, for every t.

Observe that, in Definition 1, t and a represent the variable "time", although t refers to a *delay* (a date or *calendar time*) and a refers to time as an *interval*. This distinction was already made by Scholten and Read [8–10] for whom delay refers to the time counted from 0, and interval is defined as the difference of two delays. Analogously, Cruz Rambaud and Muñoz Torrecillas [11] also made the same distinction, since they considered time as a point, that is to say, a date denoted by t, which refers to delay. On the other hand, the interval is considered as a period, for example, a week, a month, or a year, which is represented as a. Consequently, we can deduce that an interval can be considered as the difference between two delays.

Definition 2. *[29]. A discount function is said to be additive if its value in an interval is equal to the product of the values successively discounted by subintervals, that is to say:*

$$F(t,a)F(t+a,b) = F(t,a+b),$$

for every *t*, and for every *a* and *b* greater than zero.

Figure 1 illustrates this concept graphically.

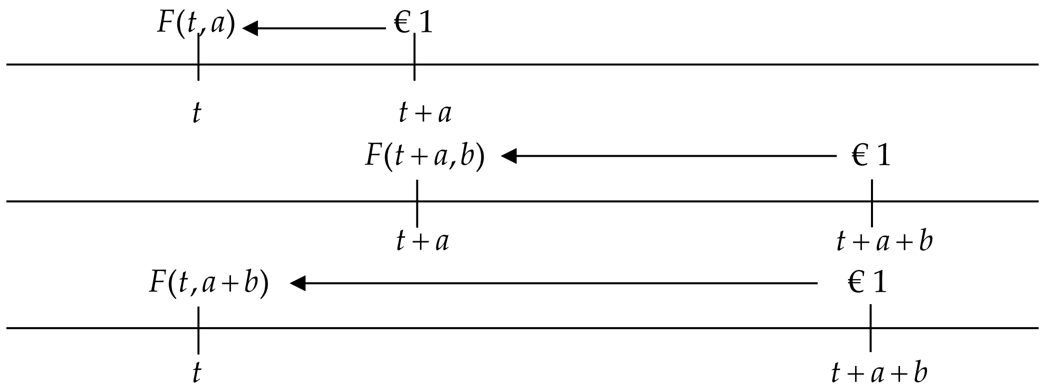

Figure 1. Additivity. **Source:** Own elaboration.

Traditional discount functions follow the premise of additivity, that is to say, the effect of the discount on several small intervals or subintervals is identical to the discount on the total or undivided interval [29]. However, many empirical studies have detected the presence of subadditivity in discounting decisions (see, for example, [29–32]). In effect, these works have shown that the discount rates are higher when the intervals are divided, that is to say, the discount is greater by using smaller than larger intervals. Mathematically, it can be written as follows:

Definition 3. *A discount function is said to be subadditive if*

$$F(t,a)F(t+a,b) < F(t,a+b),$$

for every *t*, and for every *a* and *b* greater than zero.

The following result characterizes subadditivity, but before we are going to introduce the following definition.

Definition 4. *Given two dated rewards* (x,s) *and* (y,t), *and a stationary discount function* $F(t)$, *we will say that*

$$(x,s) \sim (y,t) \text{ if } xF(s) = yF(t).$$

Obviously, ~ is an equivalence relation.

Theorem 1. *The following three conditions are equivalent:*

(i) $F(t,a)$ is subadditive.

(ii) For every t, and a and b greater than zero, if $(x,t) \sim (y,t+a)$ and $(y,t+a) \sim (z,t+a+b)$ then $(x,t) \prec (z,t+a+b)$.

(iii) For every t, a, and b greater than zero, there exists x, y, and z such that $(x,t) \succ (y,t+a)$, $(y,t+a) \succ (z,t+a+b)$ but $(x,t) \prec (z,t+a+b)$.

Proof. (i) $\Leftrightarrow$ (ii). Assume that F is subadditive. If $(x,t) \sim (y,t+a)$ then $x = yF(t,a)$. On the other hand, if $(y,t+a) \sim (z,t+a+b)$ then $y = zF(t+a,b)$. Consequently,

$$x = zF(t,a)F(t+a,b) < zF(t,a+b).$$

Therefore, $(x,t) \prec (z,t+a+b)$. Reciprocally, consider the rewards $(1,t)$, $\left(\frac{1}{F(t,a)}, t+a\right)$ and $\left(\frac{1}{F(t,a)F(t+a,b)}, t+a+b\right)$. Obviously, the first reward is equivalent to the second one, and the second is equivalent to the third. By hypothesis, the third reward is preferred to the first from which subadditivity holds.

(ii) $\Leftrightarrow$ (iii). The necessity follows by taking the rewards $(1+\varepsilon,t)$, $\left(\frac{1}{F(t,a)}+\delta,t+a\right)$ and $\left(\frac{1}{F(t,a)F(t+a,b)},t+a+b\right)$, for values of ε and δ small enough such that

$$\delta F(t,a)<\varepsilon<\frac{F(t,a+b)}{F(t,a)F(t+a,b)}-1\,.$$

The sufficiency is straightforward. □

Example 1. *Assume the choice between €100 in 6 months and €150 in 12 (which is a 6-month subinterval), another choice between €150 at 12 and €200 at 18 (which is another subinterval of 6 months) and a last choice between €100 at 6 months and €200 at 18 (the interval here is 12-months length) (see Figure 2).*

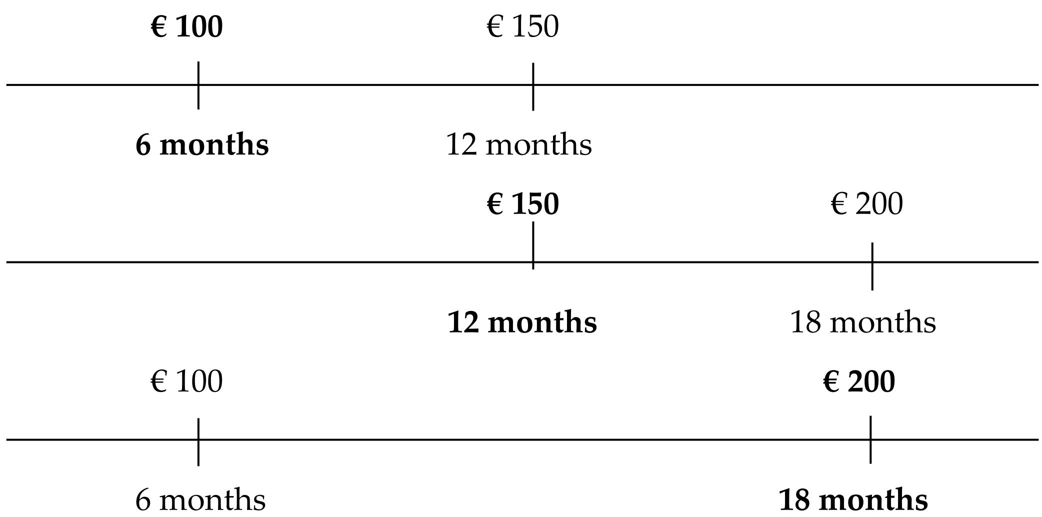

Figure 2. Subadditivity. **Source:** Own elaboration.

In the former example, the discount would be subadditive when the decision-maker prefers €100 to €150 in the first subinterval (6 months), prefers €150 to €200 in the second subinterval (another period of 6 months), but prefers €200 to €100 in the 12-month interval, corresponding to the sum of the two previous periods, that is to say, the decision-maker prefers the earliest options in the subintervals and the latest choice in the undivided interval.

3. Delay Effect

3.1. Stationary Case

The delay effect has been one of the most studied anomalies. The first authors to analyze this effect were Prelec and Loewenstein [33], who, however, labeled it as the "common difference effect".

The *delay effect* or *common difference effect* is the paradox in which the discount rate decreases as the delay increases or, said in other words, the discount rate is lower for intervals which start later.

Example 2. *A person may prefer receiving €50 in a month or receiving €75 in two months; however, this same person may prefer €75 within 13 months to €50 within 12 months. Observe that, between the two rewards, there is a difference of one month (from 1 to 2 and from 12 to 13); however, the preferences of the decision-maker have changed, resulting in a time inconsistency which is incompatible with the exponential discount function, since we have gone from preferring the €50 reward in a month to prefer the €75 reward in month 13 (in both cases, there has been an increase of the delay in 11 months).*

In Example 2, we can see that the choice of the closest rewards becomes more important than the farthest rewards. This anomaly is called the delay effect. However, this effect can be confused with the *immediacy effect*, which refers to the decision-makers giving special importance to immediate results, that is, the decision-maker prefers the reward now to waiting. Therefore, to differentiate between these two effects, we must look at the moment of choosing the reward, that is to say, the left-hand endpoint of the interval.

Example 3. *If €50 is preferred today to €75 in a month, we would talk about the immediacy effect, while, if the interval does not start at the current moment, as in the given example, €50 in a month or €75 in two months, then we talk about the delay effect. The delay effect can be graphically represented in Figure 3.*

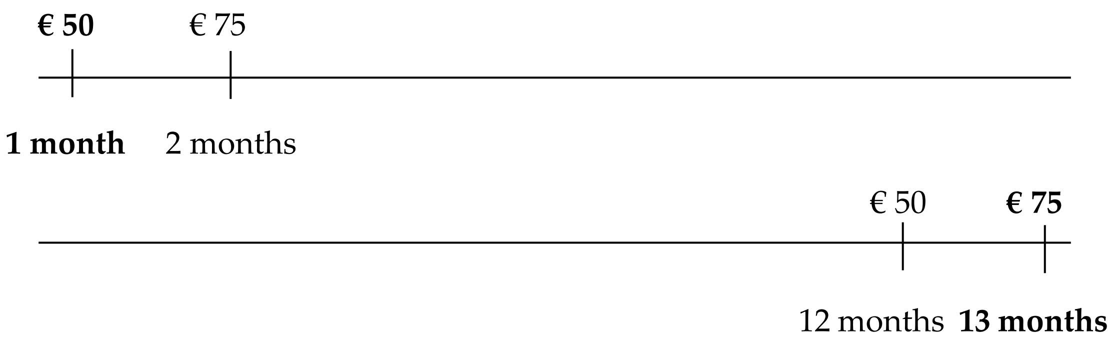

Figure 3. Delay effect (Example 2). **Source:** Own elaboration.

In Figure 3, we can observe time inconsistency because, when delaying the rewards, the individual prefers to wait longer to obtain a greater reward; however, in the first scenario, the individual prefers the small reward and not wait another month to obtain the later reward. The main problem of the situation described before is that, in the presence of time inconsistency, an individual could always prefer the first reward. Said, in other words, the former situation is not general to every couple of amounts. This could be the case of two rewards with similar amounts; for example, the individual could prefer €50 in one month and €50 in 12 months to €51 in two months and €51 in 13 months, respectively. This is why, to give a more accurate definition of the delay effect, we are going to start from an indifference relation instead of a preference. Mathematically, we can formalize this effect as follows:

$$(x,s) \sim (y,t) \text{ implies } (x,s+\varepsilon) \prec (y,t+\varepsilon),$$

where x and y ($x<y$) represent the rewards equivalent at instants s and t, respectively, and $\varepsilon > 0$ denotes the incremental delay (in Example 2, ε is 11 months) applied to each reward. On the other hand, the mathematical expression of the immediacy effect would remain in the following form [9]:

$$(x,s) \sim (y,t) \text{ implies } (x,s+\varepsilon) \prec (y,t+\varepsilon),$$

where x and y ($x<y$) represent the rewards equivalent at instants $s=0$ and t, respectively, and $\varepsilon > 0$ denotes the incremental delay.

To continue with the description of the delay effect, we are going to introduce the concept of the discount function in a stationary context.

Definition 5. *A (stationary) discount function is a continuous, strictly decreasing real-valued function*

$$F : \Re^{+} \to]0,1],$$

defined by:

$$a \mapsto F(a),$$

such that $F(0) = 1$.

The more general situation where the delay effect appears is the following: If $s < t$, the discount ratio corresponding to interval $[s, s+a]$ is less than the discount ratio of the interval $[t, t+a]$, that is to say:

$$\frac{F(s+a)}{F(s)} < \frac{F(t+a)}{F(t)} \tag{1}$$

or, equivalently,

$$F(t)F(s+a) < F(s)F(t+a). \tag{2}$$

Definition 6. *[12]. A discount function is said to be subadditive of the second order if it satisfies Equation (2).*

Specifically, if $s = 0$, then

$$F(t)F(a) < F(t+a),$$

which is subadditivity. It can be shown that, if $F(a)$ is differentiable, the following three conditions are equivalent [10]:

1. $F(a)$ is subadditive of the second order.

2. $f(a) := -\ln F(a)$ is convex.

3. The instantaneous discount rate, $\delta(a)$, is strictly decreasing.

This result could also be derived from the ambit of applied probability. In effect, in a stationary context, given a discount function $F(t)$, then $1-F(t)$ is a distribution function. It is easy to see that the instantaneous discount rate of $F(t)$ coincides with the failure (hazard) rate [34–36] of $1-F(t)$. Therefore, a decreasing instantaneous discount rate is the same as a decreasing failure rate. Chart 1 clarifies the involved implications:

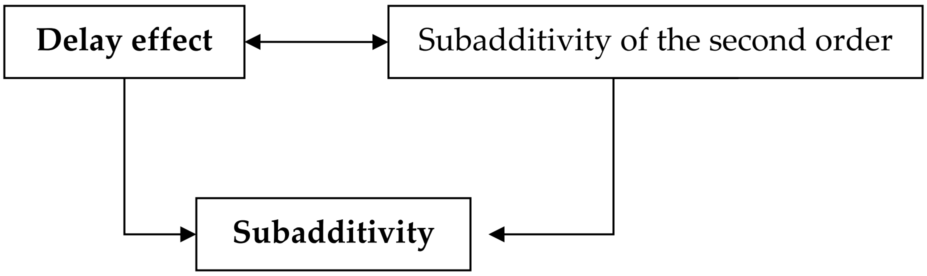

Chart 1. Delay effect and subadditivity in a stationary context. **Source:** Own elaboration.

3.2. Dynamic Case

In a dynamic setting (see Definition 1), the most general situation where the delay effect appears is the following: If $t < s$ and $s+c \leq t+a$, the discount ratio corresponding to interval $[s+c, s+c+b]$ is less than the discount ratio of the interval $[t+a, t+a+b]$, for every $b > 0$ (see Figure 4). That is to say:

$$\frac{F(s,c+b)}{F(s,c)} < \frac{F(t,a+b)}{F(t,a)} \tag{3}$$

or, equivalently,

$$F(t,a)F(s,c+b) < F(s,c)F(t,a+b)\,. \tag{4}$$

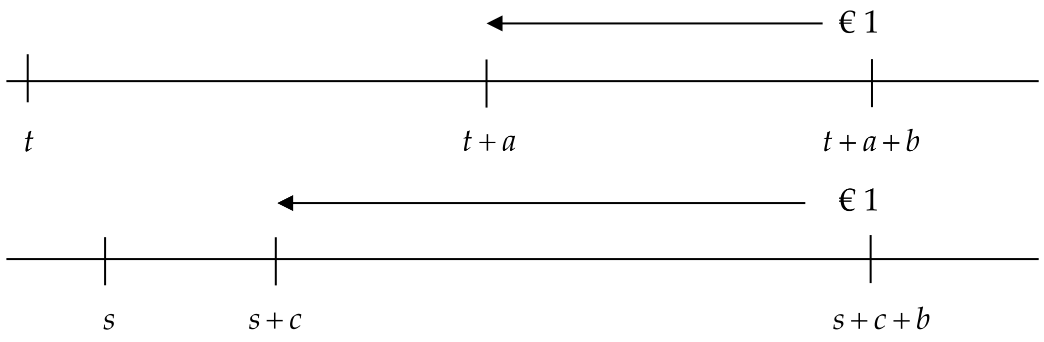

Figure 4. Delay effect in a dynamic situation. **Source:** Own elaboration.

Definition 7. *[12]. A discount function is said to be subadditive of the second order in a dynamic context if it satisfies Equation (4).*

If, in particular, $s = t + a$ in Equation (4) (which implies $c = 0$), then:

$$F(t,a)F(t+a,b) < F(t,a+b),$$

which is subadditivity (see Figure 5).

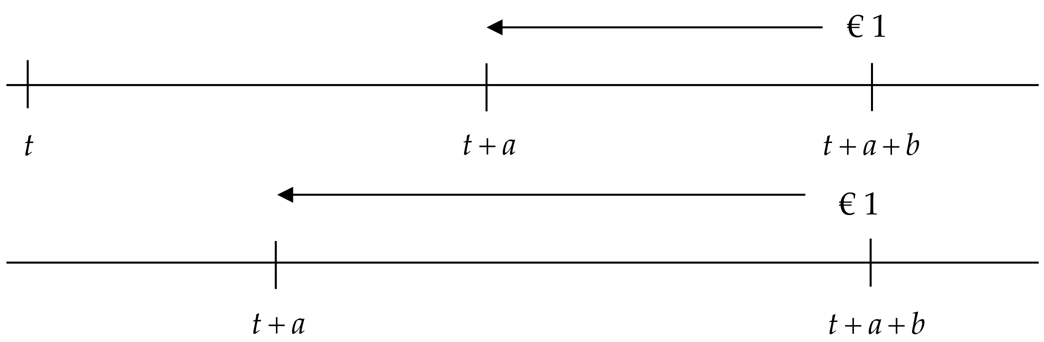

Figure 5. Subadditivity in a dynamic situation. **Source:** Own elaboration.

In the rest of this section, we will assume that the discount function is differentiable with respect to their two variables. The following two results characterize subadditive discounting of the second order.

Theorem 2. *If a discount function* $F(t,a)$ *is subadditive of the second order, then the instantaneous discount rate* $\delta(t,a)$ *is decreasing with respect to* $t+a$. *Reciprocally, if the instantaneous discount rate* $\delta(t,a)$ *is strictly decreasing with respect to* $t+a$, *there* *exists a neighborhood of 0,* $E(0)$*, such that* $F(t,a)$ *is subadditive of the second order, for every a and b in* $E(0)$.

Proof. Taking natural logarithms in Inequality (3) and dividing by b, one has:

$$\frac{\ln F(s,c+b)-\ln F(s,c)}{b} < \frac{\ln F(t,a+b)-\ln F(t,a)}{b}.$$

Finally, letting $b \to 0$:

$$\delta(s,c) \geq \delta(t,a)$$

and so, as $s+c<t+a$, the instantaneous discount rate $\delta(t,a)$ is decreasing with respect to $t+a$. The demonstration of the converse statement is obvious. □

Corollary 1. *If a discount function* $F(t,a)$ *is subadditive of the second order then* $D_{(1,1)}\delta(t,a) \leq 0$*, where* $D_{(1,1)}\delta(t,a)$ *is the directional derivative of* $\delta(t,a)$ *according to vector* $(1,1)$*. Reciprocally, if* $D_{(1,1)}\delta(t,a) < 0$*, there exists a neighborhood of 0,* $E(0)$*, such that* $F(t,a)$ *is subadditive of the second order, for every a and b in* $E(0)$.

Proof. In effect, making the change of variable $z=t+a$, by the Chain Rule of derivation, one has:

$$\frac{\partial\delta(t,a)}{\partial z} = \frac{\partial\delta(t,a)}{\partial t}\frac{\partial t}{\partial z} + \frac{\partial\delta(t,a)}{\partial a}\frac{\partial a}{\partial z} = \frac{\partial\delta(t,a)}{\partial t} + \frac{\partial\delta(t,a)}{\partial a}.$$

Observe that, by Theorem 2, the left-hand side of the former equation is negative and that the right-hand side is the directional derivative of $\delta(t,a)$ according to vector $(1,1)$. □

From now on, we will only enunciate necessary conditions. The following result characterizes the delay effect.

Corollary 2. *A necessary condition for a discount function* $F(t,a)$ *satisfying the delay effect is that, for every t,* $\delta(t,a)$ *is decreasing with respect to a.*

Proof. It is obvious taking into account Theorem 2 and that, in this case, t is constant. □

The following three results characterize subadditive discounting.

Corollary 3. *A necessary condition for a discount function* $F(t,a)$ *being subadditive is* $\delta(t+a,0)\geq\delta(t,a)$.

Proof. It is obvious taking into account Theorem 2 and that, in this case, $s=t+a$ and $c=0$. □

Theorem 3. *A necessary condition for a discount function* $F(t,a)$ *being subadditive is* $\delta(t,0)\geq D_{(1,-1)}\ln F(t,a)$, *where* $D_{(1,-1)}\ln F(t,a)$ *is the directional derivative of* $\ln F(t,a)$ *according to vector* $(1,-1)$.

Proof. Taking natural logarithms in the following inequality:

$$F(t,b)<\frac{F(t,a+b)}{F(t+b,a)}$$

and dividing by *b*, one has:

$$\frac{\ln F(t,b)}{b}<\frac{\ln F(t,a+b)-\ln F(t+b,a)}{b}.$$

Observe that the right-hand side of the former inequality can be written as follows:

$$\frac{\ln F(t,b)}{b}<\frac{\ln F(t,a+b)-\ln F(t,a)+\ln F(t,a)-\ln F(t+b,a)}{b}.$$

Finally, letting $b\to 0$:

$$-\delta(t,0) \le \frac{\partial \ln F(t,a)}{\partial a} - \frac{\partial \ln F(t,a)}{\partial t}$$

or, equivalently,

$$\delta(t,0) \ge D_{(1,-1)} \ln F(t,a) \,. \; \square$$

Corollary 4. *A necessary condition for a discount function* $F(t,a)$ *being subadditive is*

$$-\frac{\partial \ln F(t,a)}{\partial t} \ge \left.\frac{\partial \delta(t,a)}{\partial a}\right|_{a=0} .$$

Proof. Observe that, according to Theorem 3,

$$\delta(t,0) \ge \frac{\partial \ln F(t,a)}{\partial t} + \delta(t,a)$$

or, equivalently,

$$-\frac{\partial \ln F(t,a)}{\partial t} \ge \delta(t,a) - \delta(t,0) \,.$$

Finally, letting $a \to 0$:

$$-\frac{\partial \ln F(t,a)}{\partial t} \ge \left.\frac{\partial \delta(t,a)}{\partial a}\right|_{a=0} . \; \square$$

Another case arises when comparing the values of the discount function for intervals $[t, t+a]$ and $[t+b, t+a+b]$:

$$F(t,a) < F(t+b,a) \,,$$

that is to say, the discount function is *contractive* (CONTR) (see [9]). Otherwise, the discount function is said to be *expansive* (EXP). A characterization of contractive discounting is provided by the following theorem.

Theorem 4. *A necessary condition for a discount function* $F(t,a)$ *being contractive is* $\frac{\partial \ln F(t,a)}{\partial t} \geq 0$. □

Chart 2 clarifies the involved implications:

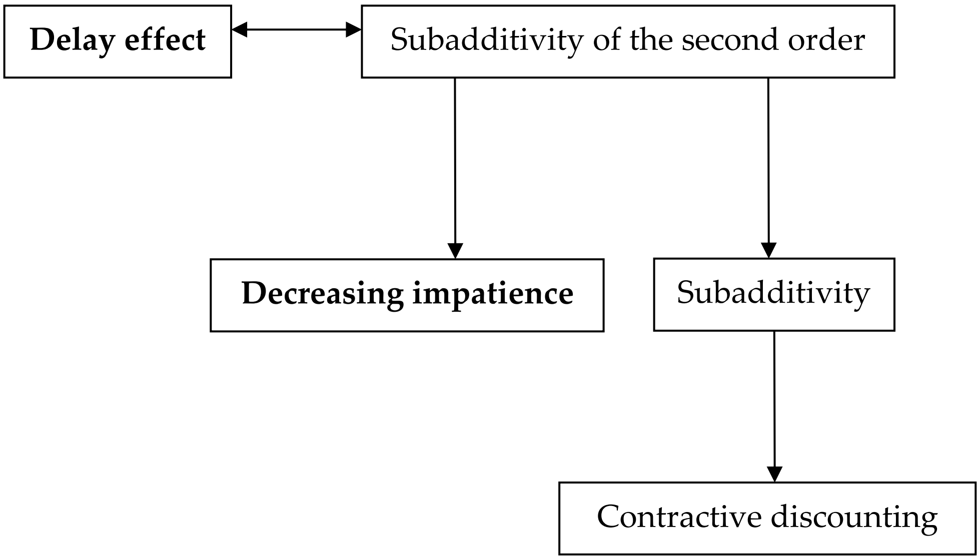

Chart 2. Delay effect and subadditivity in a dynamic context. **Source:** Own elaboration.

4. Proposal of a New Discounting Function: The Asymmetric Exponential Discounting

Let us consider the following discount function:

$$F(t,a) = \exp\{-k[(t+a^{\alpha})^{\beta} - t^{\beta}]\}, \quad (5)$$

where k, α, and β are strictly positive real numbers. To calculate the instantaneous discount rate, take into account that:

$$-\ln F(t,a) = k[(t+a^{\alpha})^{\beta} - t^{\beta}].$$

Thus,

$$\delta(t,a) = k\alpha\beta(t+a^{\alpha})^{\beta-1}a^{\alpha-1}. \quad (6)$$

Next, we are going to apply the sufficient conditions deduced in Section 3 to the new discount function proposed in this section. First, to analyze the *subadditivity of the second order*:

$$\frac{\partial \delta(t,a)}{\partial t} + \frac{\partial \delta(t,a)}{\partial a} < 0, \tag{7}$$

it is necessary to previously determine the partial derivative of $\delta(t,a)$ with respect to t and with respect to a:

$$\frac{\partial \delta(t,a)}{\partial t} = k\alpha\beta(\beta-1)(t+a^{\alpha})^{\beta-2}a^{\alpha-1}$$

And

$$\frac{\partial \delta(t,a)}{\partial a} = k\alpha^2\beta(\beta-1)(t+a^{\alpha})^{\beta-2}a^{2\alpha-2} + k\alpha(\alpha-1)\beta(t+a^{\alpha})^{\beta-1}a^{\alpha-2}.$$

Therefore,

$$\frac{\partial \delta(t,a)}{\partial t} + \frac{\partial \delta(t,a)}{\partial a} = k\alpha\beta(t+a^{\alpha})^{\beta-2}a^{\alpha-2}[(\beta-1)a + \alpha(\beta-1)a^{\alpha} + (\alpha-1)(t+a^{\alpha})]$$

or, equivalently,

$$\frac{\partial \delta(t,a)}{\partial t} + \frac{\partial \delta(t,a)}{\partial a} = k\alpha\beta(t+a^{\alpha})^{\beta-2}a^{\alpha-2}[(\beta-1)(a+\alpha a^{\alpha}) + (\alpha-1)(t+a^{\alpha})].$$

Consequently, $\frac{\partial \delta(t,a)}{\partial t} + \frac{\partial \delta(t,a)}{\partial a} < 0$ if, and only if, the following inequality holds:

$$\beta < 1 - (\alpha-1)\frac{t+a^{\alpha}}{a+\alpha a^{\alpha}}.$$

In particular, this inequality holds if $\alpha = 1$ or $0 < \alpha < 1$, and $0 < \beta < 1$. Table 1 summarizes all possible cases according to the values of α and β, where SUB2 means subadditivity of the second order, and RSUB2 reverses superadditivity of the second order:

$$\frac{\partial\delta(t,a)}{\partial t}+\frac{\partial\delta(t,a)}{\partial a}>0\,. \qquad (8)$$

Table 1. Delay effect according to the values of α and β. **Source**: Own elaboration.

		Values of β		
		$0<\beta<1$	$\beta=1$	$\beta>1$
Values of α	$0<\alpha<1$	SUB2	Generalized exponential discounting	?
	$\alpha=1$	SUB2	Exponential discounting	RSUB2
	$\alpha>1$	?	Generalized exponential discounting	RSUB2

Now, to analyze the *subadditivity*, we will take into account the following sufficient condition:

$$\delta(t,0)>\frac{\partial\ln F(t,a)}{\partial t}+\delta(t,a), \qquad (9)$$

it is necessary to take into account that:

$$\frac{\partial\ln F(t,a)}{\partial t}=k\beta(t+a^{\alpha})^{\beta-1}-k\beta t^{\beta-1}.$$

Therefore,

$$\frac{\partial\ln F(t,a)}{\partial t}+\delta(t,a)=k\beta(t+a^{\alpha})^{\beta-1}-k\beta t^{\beta-1}+k\alpha\beta(t+a^{\alpha})^{\beta-1}a^{\alpha-1},$$

or, equivalently,

$$\frac{\partial\ln F(t,a)}{\partial t}+\delta(t,a)=k\beta[(t+a^{\alpha})^{\beta-1}(1+\alpha a^{\alpha-1})-t^{\beta-1}].$$

On the other hand,

$$\delta(t,0)=\begin{cases}0, & \alpha>1\\ \infty, & 0<\alpha<1\end{cases}$$

Consequently, $\delta(t,0)\geq\frac{\partial\ln F(t,a)}{\partial t}+\delta(t,a)$ if, and only if, $0<\alpha<1$.

Finally, the condition of *contractiveness* is satisfied by requiring:

$$\frac{\partial\ln F(t,a)}{\partial t}>0 \tag{10}$$

holds if, and only if, $\beta>1$.

Example 4. *Let us consider, for example, k = 0.1, t = 2, and a = 2 to calculate the subadditivity and superadditivity of the second order of the proposed model depending on the different values of α and β.*

- Superadditivity of the second order: $\frac{\partial\delta(t,a)}{\partial t}+\frac{\partial\delta(t,a)}{\partial a}>0$.
- Subadditivity of the second order: $\frac{\partial\delta(t,a)}{\partial t}+\frac{\partial\delta(t,a)}{\partial a}<0$.

In Table 2, we can see the same conclusions obtained in Table 1.

Table 2. Example of the delay effect according to the values of *α* and *β*.
Source: Own elaboration.

	$0<\alpha<1$ $0<\beta<1$	$\alpha=1$ $0<\beta<1$	$\alpha>1$ $0<\beta<1$	$0<\alpha<1$ $\beta>1$	$\alpha=1$ $\beta>1$	$\alpha>1$ $\beta>1$
α	0.5	1	2	0.5	1	2
β	0.5	0.5	0.5	2	2	2
Result	−0.004	−0.006	?	?	0.400	6.400
	SUB2	SUB2	?	?	RSUB2	RSUB2

Example 5. *Let us considerer, for example, k = 0.1, t = 2, and a = 2 to calculate the subadditivity of the proposed model depending on the different values of α and β.*

$$\text{Subadditivity: } \delta(t,0) > \frac{\partial \ln F(t,a)}{\partial t} + \delta(t,a)$$

Table 3. Example of subadditivity according to the values of α and β. **Source**: Own elaboration.

	$0<\alpha<1$ $0<\beta<1$	$\alpha=1$ $0<\beta<1$	$\alpha>1$ $0<\beta<1$	$0<\alpha<1$ $\beta>1$	$\alpha=1$ $\beta>1$	$\alpha>1$ $\beta>1$
α	0.5	1	2	0.5	1	2
β	0.5	0.5	0.5	2	2	2
Result	∞	-	**0**	∞	-	**0**
	SUB	-	SUB	SUB	-	SUB

This example confirms the conclusions obtained above.

Example 6. *If k = 0.1, t = 2, and a = 2, the condition of contractiveness is the following:*

$$\frac{\partial \ln F(t,a)}{\partial t} > 0$$

In Table 4, we can see that the condition of contractiveness holds if, and only if, $\beta > 1$

Table 4. Example of contractiveness according to the values of α and β. **Source**: Own elaboration.

	$0<\alpha<1$ $0<\beta<1$	$\alpha=1$ $0<\beta<1$	$\alpha>1$ $0<\beta<1$	$0<\alpha<1$ $\beta>1$	$\alpha=1$ $\beta>1$	$\alpha>1$ $\beta>1$	$\alpha>1$ $\beta=1$
α	0.5	1	2	0.5	1	2	2
β	0.5	0.5	0.5	2	2	2	1
Result	−0.008	−0.010	−0.015	0.283	0.400	0.800	0
	Exp	Exp	Exp	Contr	Contr	Contr	-

5. Conclusions

This paper has dealt with the topic of the delay effect and decreasing impatience in intertemporal choice, one of the most important anomalies of the Discounted Utility model. This paradox has been treated in a static framework, by using a stationary discount function, and in a dynamic setting with the use of a general discount function. The delay effect means a strong preference for small sooner rewards instead of larger later outcomes. In a stationary context, this is equivalent to requiring that the discount ratio corresponding to the underlying discount function is increasing with respect to the delay of a given interval.

In a dynamic framework, the delay effect can be analyzed following three methodologies:

1. By comparing the discount ratios corresponding to two delayed intervals of the same length. This case gives rise to the concept of the subadditivity of the second order.
2. By comparing the value of the discount function with the discount ratio corresponding to a delayed interval with the same amplitude. This is a condition weaker than the former one and gives rise to the concept of subadditivity.
3. By comparing the values of the discount function in two intervals with the same amplitude. This situation gives rise to a contractive discount function.

This manuscript has presented mathematical characterizations of these three types of the delay effect, which will be useful in understanding the mode of decreasing impatience exhibited by a given discount function. Finally, we have provided a new function, the so-called asymmetric exponential discount function, which shows all types of decreasing impatience defined in this paper.

A further research line is to relate the delay effect with another very similar anomaly, the so-called interval effect, and analyze the adequacy of this novel discount function to explain both paradoxes.

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CAPÍTULO IV.

EL EFECTO INTERVALO

Are Delay and Interval Effects the Same Anomaly in the Context of Intertemporal Choice in Finance?

Artículo publicado en 2021 en Symmetry.

Factor de Impacto (2019) 2,645 (Journal Citation Reports-Thomson-Reuters)

Are Delay and Interval Effects the Same Anomaly in the Context of Intertemporal Choice in Finance?

Abstract: Traditionally; the interval and delay effects have been identified and considered as the same anomaly in the context of intertemporal choice; when individuals or groups of individuals make their decisions about reward preferences. This has supposed that most studies on this topic have been focused on the delay effect and; consequently; that the discount functions provided by the existing literature have considered only this effect. This is the case of hyperbolic discounting; which has been used to describe the delay, but not the interval effect. Therefore, the main objective of this paper is to carry out a detailed analysis of both anomalies, which will allow us to mathematically relate them, thus finding their analogies and differences. To do this, we will first analyze the concept of delay effect and later the different definitions of the interval effect. The main conclusion of this paper is twofold. On the one hand, if the benchmark for valuation is fixed, the delay effect coincides with the so-called decreasing interval effect. On the other hand, if the assessment reference point is the beginning of each interval, both anomalies are different. These findings make necessary to redefine the concept of interval effect. Finally, we will analyze the relationship between the interval effect, the delay effect and the subadditivity

Keywords: interval effect; delay effect; impatience; discount function; subadditivity; managerial decision making

1. Introduction

Intertemporal decisions refer to the choice of a reward among a series of alternative actions available at different moments of time so that the made decision is the most profitable for the individual. This is because, continuously, all individuals are immersed in a great dilemma: to obtain less benefit and pay less immediately, or to obtain greater benefits and pay more after a period of time

[1–3]. Samuelson [4] was one of the first scholars to describe this phenomenon through his discounted utility (DU) model, which has been used up to now as the prominent discount model.

However, from the 1980s onward, a series of counterexamples of the DU model began to emerge in the context of what is currently known as behavioral finance (see, for example, [5–7]). This new setting favored a new way of studying finance since, after numerous empirical studies, it was demonstrated that people make irrational decisions [8] in that they do not fit the DU model initially provided by Samuelson [4]. In effect, this model does not explain certain behaviors of the decision maker, known as anomalies or paradoxes in intertemporal choice: the delay effect [9,10], the magnitude effect [9–11], speedup–anticipation asymmetry [10,12] and the improving sequence effect [1,13–16], among others.

Over the last 30 years, attempts have been made to find some mathematical solutions able to cover the deficiencies presented by the DU model. Among them, we can find the hyperbolic discount model [2,17], in which the discount rate decreases with the passage of time, thus solving the anomaly called the delay effect or decreasing impatience, demonstrated for monetary decisions [9,10] and non-monetary decisions [18,19].

Later, some scholars detected that the delay effect was sometimes identified with another paradox: the so-called interval effect [20]. This confusion has resulted in very little research on this anomaly. In effect, most proposed functions aim to solve the delay effect, but few of them characterize the interval effect [21,22]. As a result, Read [23], when describing the interval effect, points out that "A systematic analysis of the relative contributions of delay and interval to discounting is yet to be done". Therefore, the objective of this paper is to analyze the concept of the interval effect and analyze the similarities and differences between this anomaly and the delay effect. Moreover, we will study the relationship between both effects and the concept of subadditivity [24,25].

Some of these anomalies have been widely analyzed in different fields of research, such as psychology, medicine, finance, economics, marketing and even in business decisions. However, this has not been analysis on the case of the interval effect, which has begun to be studied in medicine and finance but has hardly been developed in other disciplines. That is why this paper intends to change this trend and open a new field of research in managerial decision-making.

If the interval effect means that the discount rate tends to be higher the closer the reward is to its equivalent amount [26], we could extrapolate this definition to managerial decisions in order to choose between three investment strategies (A, B and C) of a company, whose profits will be obtained in the short, medium and long term, respectively (assume that the short, medium and long term are equidistant from each other, e.g., 6, 12 and 18 months). In the beginning, a rational manager (constant discount rate) could be indifferent to the choice of any of the three former strategies. However, if the interval effect is present in managerial decisions, according to its definition, the manager could prefer to implement strategy A over strategy B and could prefer strategy B over strategy C (he prefers the closest option in small intervals). Nevertheless, by using the definition provided by Read [20] and Scholten and Read [26], the manager would choose strategy C over strategy A (he chooses the latest option for wider intervals). Observe that this leads to a contradiction in his decision-making; that is to say, it leads to an inconsistency.

The study of the interval effect, as well as the delay effect in making decisions, can help to understand the behavior of managers and to answer some questions such as the following: Is there an interval effect or a delay effect in managerial decisions? Are there any differences in short- and long-term managerial decision-making between small businesses and large companies? Is the net present value (NPV) based on Samuelson's exponential discount [4] a good tool for business decision-makers? The answers to these questions and many others can help to open a wide field of research in the strategic direction of a company.

This paper is structured as follows. In Section 2, we will define the concept of the delay effect while, in Section 3, we will focus on clarifying the concept of the interval effect. Section 4 will provide the joint mathematical analysis of the interval and delay effects. Finally, Section 5 summarizes and concludes the paper.

2. The Delay Effect

The delay effect, or common difference effect, means that the discount rate decreases as the delay increases; that is to say, the discount rate is lower for intervals of the same length starting later. This effect is one of the most studied anomalies. The first authors, who analyzed this effect, were Prelec and Loewenstein [12]. Let us see an example to explain this concept [22].

Example 1. *A person may prefer receiving EUR 50 in a month to EUR 75 in two months. However, this same person may prefer EUR 75 within 13 months to EUR 50 within 12 months. Observe that, between the two rewards, there is a difference of one month (from 1 to 2 and from 12 to 13). However, the preferences of the decision-maker have changed, resulting in a time inconsistency which is not compatible with the exponential discount function, since we have gone from preferring the EUR 50 reward in a month to preferring the EUR 75 reward in the thirteenth month (in both cases, there was an increase of the delay in 11 months).* See Figure 1.

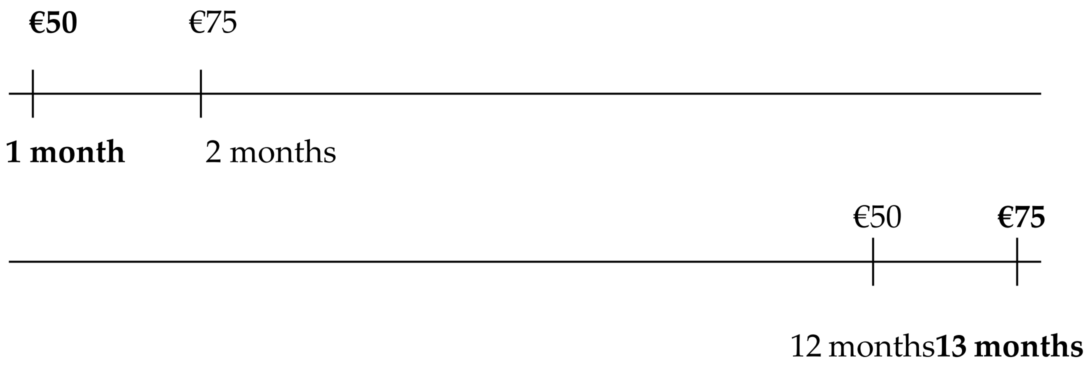

Figure 1. Delay effect (Example 1) (in bold, the chosen option). **Source**: Own elaboration.

This would be equivalent to an Example 2 beginning with the following statement: **Example 2.** *A person may be indifferent between receiving EUR 50 in a month and receiving EUR 75 in two months. However, this same person may prefer EUR 75 within 13 months to EUR 50 within 12 months.* Therefore, mathematically, this effect can be formalized as follows:

$$(x,s)\sim(y,t) \text{ implies } (x,s+\varepsilon)\prec(y,t+\varepsilon)$$

where x and y $(x<y)$ represent the rewards equivalent at instants s and t, respectively, and $\varepsilon>0$ denotes the incremental delay (in Examples 1 and 2, ε is 11 months) applied to each reward. Specifically, the mathematical expression of the immediacy effect is a particular case of the delay effect and would remain in the following form [27]:

$$(x,s)\sim(y,t) \text{ implies } (x,s+\varepsilon)\prec(y,t+\varepsilon)$$

where x and y $(x<y)$ represent the rewards equivalent at instants $s=0$ and t, respectively, and $\varepsilon>0$ denotes the incremental delay.

3. The Interval Effect

The interval effect, also called the interval length effect, was demonstrated by Read [20]. This scholar distinguished between the delay and interval effects, thus opening a new field of research between two anomalies which, traditionally, have been studied as only one, namely the delay effect. Read [20] stated that the discount rate depends on the length of the interval in such a way that the larger the interval, the smaller the discount rate.

Later, Read and Roelofsma [28] identified the interval effect with subadditive discounting ("for a given delay, the total discounting is greater when it is broken into intervals, and discounting measured separately for each interval, than when it is left unbroken") and Read [23] completed the definition of the interval effect as "shorter intervals lead to more discounting per-time-unit".

The following works based their definitions on previous studies. Thus, Scholten and Read [26] provided another definition of this effect: "the discount rate will tend to be higher the closer the rewards are to each other". On the other hand, Kinari et al. [29] stated that the interval effect is a more general concept than subadditive time discounting; that is, the longer the interval, the lower the per period time discount rate. Moreover, the delay effect leads to an examination of the interval effect as a by-product.

As indicated in the former definitions, there is unanimity in that the interval effect means that the larger the interval, the smaller the discount rate. However, Read [20] and Read and Roelofsma [28] identified this concept with that of subadditivity, and later, Kinari et al. [29] stated that the interval effect is a more general concept than subadditivity time discounting, in the same way that it regards the interval effect as a by-product of the delay effect. However, none of these statements have been mathematically shown, as there is not a mathematical concept of the interval effect or of the different situations in which this anomaly can appear. Later, Cruz Rambaud and Ortiz Fernández [22] mathematically demonstrated that, from a dynamic point of view, it can be deduced that subadditivity is a particular case of the delay effect. Therefore, the relationship between the interval effect, the delay effect and subadditivity remains to be demonstrated. Table A1 (see Appendix A) summarizes the characteristics of the papers analyzing the interval effect.

Next, we are going to analyze the mathematical concept of the interval effect, as well as the possible situations in which it can occur. First, let us see an example (Figure 2).

Example 3. *A subject faces three intertemporal choices: the first two will be separated by intervals of the same length, and the length of the third interval is the sum of the lengths of the former intervals.*

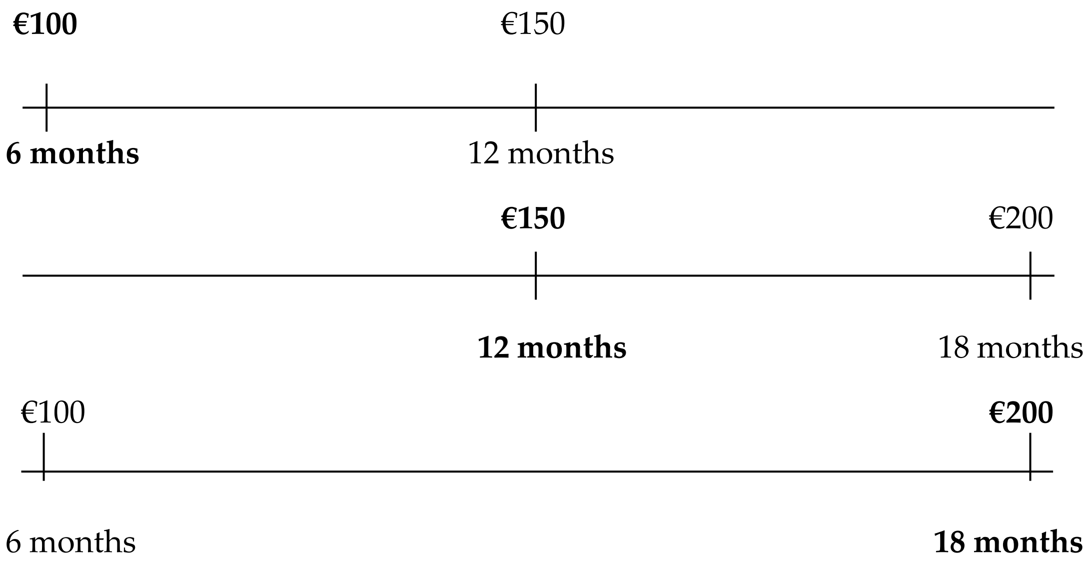

Figure 2. Interval effect. (The chosen option is in bold. **Source:** Own elaboration.

Under the interval effect, a decision-maker could prefer EUR 100 to EUR 150 and, moreover, he or she could prefer EUR 150 over EUR 200. Both choices are separated by a time horizon of 6 months or, in other words, they are separated by an interval of a length equal to 6 months. However, we can find a third intertemporal choice, in which the decision-maker must choose between EUR 100 within 6 months and EUR 200 12 months later (that is, in 18 months). In this case, the decision-maker could opt to choose EUR 200 and wait for 12 months more. As indicated, in the 6 months intervals, the decision-maker could prefer the earliest option while, in the 12 months interval, the decision-maker could prefer to wait. It can be observed that, for the smallest interval, the earliest option is chosen, and for the largest interval, the latest option is preferred, giving rise, as in Example 1, to time inconsistency.

Mathematically, this effect can be formalized as follows [29]:

$$(x,s)\sim(y,t) \text{ but } (x,s')\prec(y,t')$$

where $t-s<t'-s'$.

Example 3 is based on quantities, but what about the discount rates? Observe that, in the 6 month intervals, the discount rates are greater than in the 12 month interval. From a theoretical point of view, we can write this as follows:

$$r_{is}>r_{ij}$$

and

$$r_{sj}>r_{ij}$$

where r_{is} is he discount rate in the interval $[i,s]$, r_{sj} the discount rate in the interval $[s,j]$ and r_{ij} the discount rate in the interval $[i,j]$ ($i<s<j$). Definitively, joining the two former inequalities into one yields

$$\min\{r_{is},r_{sj}\}>r_{ij}$$

As stated in [21], the discount rate depends on the length of the interval in such a way that the larger the interval, the smaller the discount rate. On the other hand, we can introduce the following definition.

Definition 1. *Given a stationary (resp., dynamic) discount function, the average discount ratio associated with the interval* $[t,t+a]$ *(resp.,* $[t+a,t+b]$*), denoted by* $\overline{f}(t,a)$ *(resp.,* $\overline{f}(t,a,b)$*), is defined as the geometric mean of the corresponding discount ratio* $f(t,a)=\dfrac{F(t+a)}{F(t)}$ *(resp.,* $f(t,a,b)=\dfrac{F(t,a+b)}{F(t,a)}$*), which is to say that*

$$\overline{f}(t,a)=\left[\frac{F(t+a)}{F(t)}\right]^{1/a}$$

(resp., $\overline{f}(t,a,b)=\left[\dfrac{F(t,a+b)}{F(t,a)}\right]^{1/b}$*), where a and b are non-negative real numbers.*

The following proposition gives two basic properties of the average discount ratio for the stationary case (the statements for the dynamic case are analogous).

Proposition 1. *The following equalities hold:*

$\overline{f}(t,a)=\exp\{-\overline{\delta}(t,a)\}$, *where* $\overline{\delta}(t,a)$ *is the mean discount rate in the interval* $[t,t+a]$;

$\lim_{a\to 0}\overline{f}(t,a)=\exp\{-\delta(t)\}$, *where* $\delta(t):=\lim_{a\to 0}\frac{F(t+a)-F(t)}{aF(t)}$ *is the instantaneous discount rate at time t.*

Proof. In effect, the following can be said:

The general expression of a discount function, according to its instantaneous discount rate, leads to $F(t+a)=\exp\left\{-\int_0^{t+a}\delta(x)dx\right\}$ and $F(t)=\exp\left\{-\int_0^{t}\delta(x)dx\right\}$.

Therefore, as $\frac{1}{a}\int_t^{t+a}\delta(x)\mathrm{d}x$ is the average of function δ in the interval $[t,t+a]$, one has

$$\overline{f}(t,a)=\left[\frac{\exp\left\{-\int_0^{t+a}\delta(x)\mathrm{d}x\right\}}{\exp\left\{-\int_0^{t}\delta(x)\mathrm{d}x\right\}}\right]^{\frac{1}{a}}=\exp\left\{-\frac{1}{a}\int_t^{t+a}\delta(x)\mathrm{d}x\right\}==\exp\{-\overline{\delta}(t,a)\},$$

which is the required equality;

$\lim_{a\to 0}\overline{f}(t,a)=\lim_{a\to 0}\left[\frac{F(t+a)}{F(t)}\right]^{1/a}=1^{\infty}$, which is an indetermination. Let us solve this indetermination by using the well-known formula to solve this type of indetermination:

$$\lim_{a\to 0}\overline{f}(t,a)=\exp\left\{\lim_{a\to 0}\frac{1}{a}\left[\frac{F(t+a)}{F(t)}-1\right]\right\}=\exp\left\{\lim_{a\to 0}\frac{F(t+a)-F(t)}{aF(t)}\right\}=$$

$$=\exp\{-\delta(t)\}.\ \square$$

4. Mathematical Analysis of the Delay and Interval Effects

As formerly indicated, the delay and interval effects are different in spite of the fact that, in some specific cases, they coincide. This is the reason why they have been traditionally confused. In effect, the difference between them is based on the difference between time as a delay and time as an interval.

In Figure 3, we can see that the interval a is the difference between the delays s and t, which is to say that

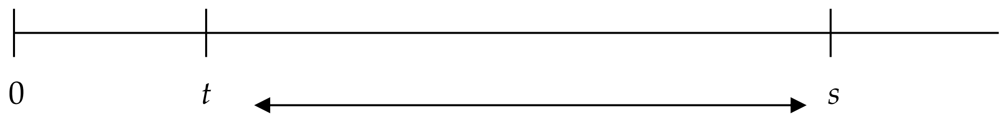

Figure 3. Time as a delay and time as an interval. **Source:** Own elaboration.

$$a = s - t$$

This can also be expressed as

$$s = t + a$$

In a beginning, Table 1 clarifies the difference between both concepts.

Table 1. Differences between the delay and interval effects. **Source**: Own elaboration.

	Delay	**Interval**
Delay effect	Different	Equal
Interval effect	Equal	Different

However, the definition of the interval effect provided by Kinari et al. [29,30] does not consider the restriction of equal delays of the intervals involved in the definition. For this reason, we are going to analyze all possible situations with different intervals independently of the delays associated with the intervals involved in the analysis. These scholars even consider that the interval effect is a particular case of the delay effect. In effect, the following subsections demonstrate that the delay effect can be derived as a particular case of the interval effect. Table 2 summarizes the definitions of the interval effect analyzed in this paper.

Table 2. Some definitions of the interval effect. **Source:** Own elaboration.

Ref.	Definition
[20]	"The discount rate will be greater the shorter the interval"
[28]	"Shorter intervals lead to more discounting per-time-unit"
[29,30]	"The longer the interval, the lower the per-period time discount rate" "The per-period time discount rate decreases as the interval lengthens"

The definitions by Read [4] and Read and Roelofsma [28] state that shorter intervals will exhibit greater discount rates; that is to say, the longer interval, the lower the per period time discount rate, which corresponds to the definitions by Kinari et al. [29,30], stating that the per period time discount rate decreases as the interval lengthens.

4.1. Assessment at a Given Benchmark (Time 0)

Let s denote the left endpoint of the shorter interval (of length a), and let t denote the left endpoint of the larger interval (of length b). Therefore, $a < b$. If, moreover, $s \leq t$, then we can provide the following definition.

Definition 2. A stationary discount function is said to be subadditive of the second order if, for every $a \geq 0$, it satisfies the following inequality [27]:

$$F(t)F(s+a) < F(s)F(t+a)$$

Specifically, if $s = 0$, then

$$F(t)F(a) < F(t+a)$$

which is subadditivity. If F is differentiable, the following theorem holds.

Theorem 1. *The following conditions are equivalent:*

(i) If $s = t$ *, then* $\overline{\delta}(s,a) > \overline{\delta}(t,b)$*;*

(ii) The instantaneous discount rate is strictly decreasing;

(iii) If $s \leq t$*, then* $\overline{\delta}(s,a) > \overline{\delta}(t,b)$*;*

(iv) The delay effect holds;

(v) The subadditivity of the second order holds.

Proof. (i) $\Rightarrow$ (ii). Assume that, if $s = t$, then $\overline{\delta}(s,a) > \overline{\delta}(t,b)$. In this case, the intervals exhibit equal left endpoints and different lengths ($a < b$) (see Figure 4). □

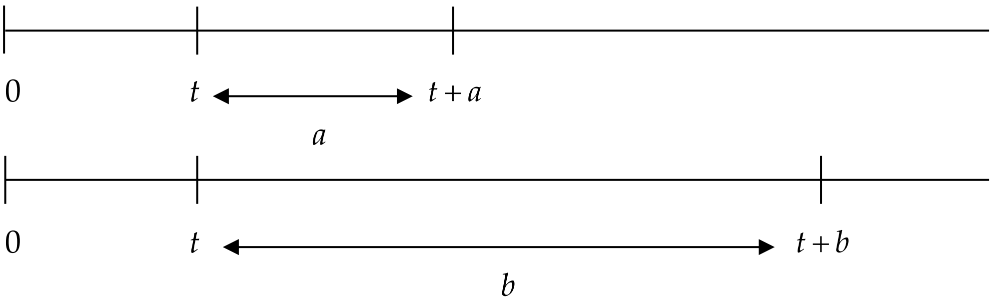

Figure 4. Condition (1) of Theorem 1. **Source**: Own elaboration.

In effect, assume that there exist r and s and $r < s$, such that $\delta(r) \leq \delta(s)$. If $\delta(r) < \delta(s)$, by continuity, there exists a neighborhood of r, $E(r) := [r_0, r_1]$, and a neighborhood of s, $E(s) := [s_0, s_1]$, with $r_0 < r_1 < s_0 < s_1$, such that for every $x \in E(r)$ and every $y \in E(s)$, the inequality $\delta(x) < \delta(y)$ holds. If we now consider the intervals $[r_0, s_0]$ and $[r_0, s_1]$, it is easy to verify that $\overline{\delta}(r_0, s_0) < \overline{\delta}(r_0, s_1)$ and, consequently, $\overline{f}(r_0, s_0) > \overline{f}(r_0, s_1)$, in contradiction with the hypothesis. On the other hand, if $\delta(r) = \delta(s)$, we could consider two cases:

- The instantaneous discount rate is constant in the interval $[r,s]$. This is not possible because by taking $a=\frac{s-r}{2}$ and $b=s-r$, one has $\overline{\delta}(t,a)=\overline{\delta}(t,b)$, in contradiction with (i).
- The instantaneous discount rate is not constant in the interval $[r,s]$. In this case, there is a subinterval of $[r,s]$, where the instantaneous discount rate is increasing and, as such, the reasoning is the same as the case in which $\delta(r)<\delta(s)$.

(ii) $\Rightarrow$ (iii). This implication is obvious (see figures 5 and 6).

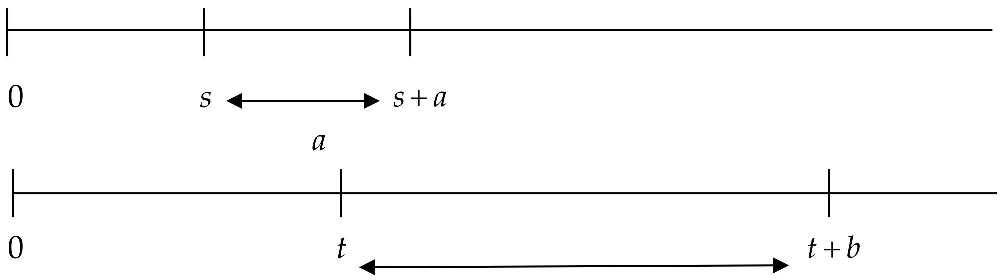

Figure 5. Condition (iii) of Theorem 1 (I). **Source**: Own elaboration.

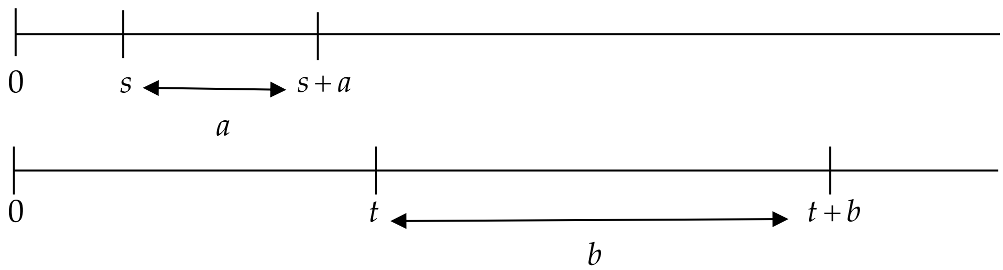

Figure 6. Condition (iii) of Theorem 1 (II). **Source**: Own elaboration.

(iii) $\Rightarrow$ (iv). This is obviously letting $a\to 0$ and $b\to 0$ in the inequality $\overline{\delta}(s,a)>\overline{\delta}(t,b)$, which leads to $\delta(s)\geq\delta(t)$. However, the case $\delta(s)=\delta(t)$ is not possible (see the last paragraphs of the implication (i) $\Rightarrow$ (ii)).

(iv) $\Rightarrow$ (v). Assume that $a<c$ and $a+b=c+d$ (which implies $b>d$ and $b-d=c-a$). By the delay effect, one has

$$\frac{F(c)}{F(a)} > \frac{F(b)}{F(d)}$$

or, equivalently,

$$F(a)F(b) < F(c)F(d)$$

which is subadditivity of the second order.

(v) $\Rightarrow$ (i). Assume that $a < c$. In the definition of subadditivity of the second order, take $b = c + \varepsilon$ and $d = a + \varepsilon$. Therefore, in this case, one has

$$F(a)F(c+\varepsilon) < F(c)F(a+\varepsilon)$$

or, equivalently,

$$\frac{F(a)}{F(c)} < \frac{F(a+\varepsilon)}{F(c+\varepsilon)}$$

Taking Napierian logarithms in both sides of the former inequality, dividing by ε and letting $\varepsilon \to 0$, one has

$$\delta(a) \geq \delta(c)$$

If $\delta(a) = \delta(c)$, as the former inequality is valid for every a and c, the instantaneous discount rate would be constant in the interval $[a,c]$. However, this is not possible, as there would be two amounts m and n such that

$$(m,a) \sim \left(n, \frac{a+c}{2}\right) \text{ but } (m, a+\varepsilon) \sim \left(n, \frac{a+c}{2} + \varepsilon\right)$$

where $\varepsilon < \frac{c-a}{2}$. Observe that (1) follows immediately. This completes the proof.

Corollary 1. The delay effect implies subadditivity.

Proof. This is an immediate consequence of Theorem 1, as subadditivity of the second order implies subadditivity (see the remark after Definition 2). □

Analogously, we can enunciate the following theorem. Before we do, we need the following definition.

Definition 3. *A stationary discount function is said to be superadditive of the second order if, for every* $a \geq 0$*, it satisfies the following inequality [21]:*

$$F(t)F(s+a) > F(s)F(t+a)$$

Specifically, if $s = 0$*, then*

$$F(t)F(a) > F(t+a)$$

which is superadditivity.

Theorem 2. *The following conditions are equivalent:*

(i) If $s+a = t+b$*, then* $\overline{\delta}(s,a) > \overline{\delta}(t,b)$*;*

(ii) The instantaneous discount rate is strictly increasing;

(iii) If $s+a \geq t+b$*, then* $\overline{\delta}(s,a) > \overline{\delta}(t,b)$*;*

(iv) The reverse delay effect holds;

(v) The superadditivity of the second order holds.

Corollary 2. The reverse delay effect implies superadditivity.

Proof. This is an immediate consequence of Theorem 2, as superadditivity of the second order implies superadditivity (see the remark after Definition 3). □

Figures 7–9 illustrate different situations, collected in Theorem 2.

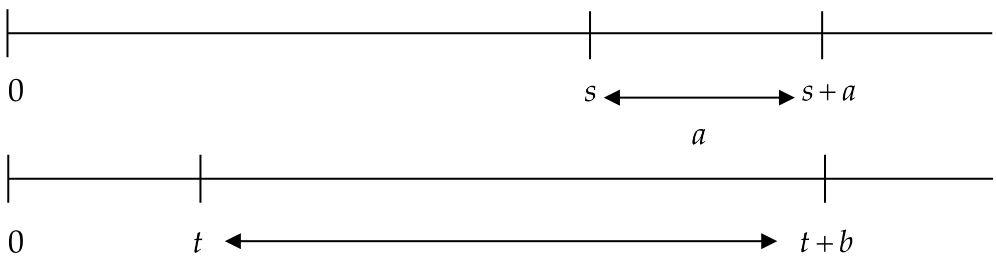

Figure 7. Condition (i) of Theorem 2. **Source**: Own elaboration.

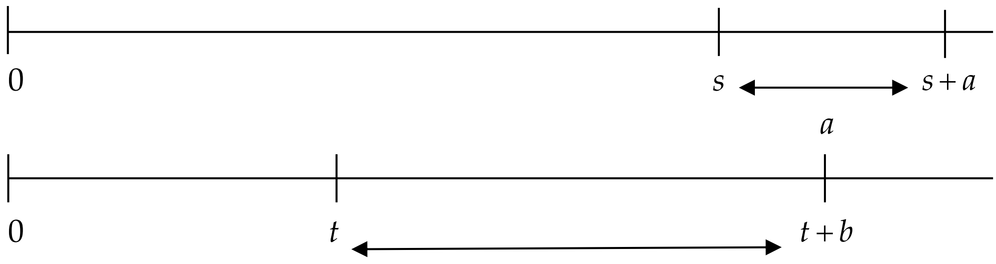

Figure 8. Condition (iii) of Theorem 2 (I). **Source**: Own elaboration.

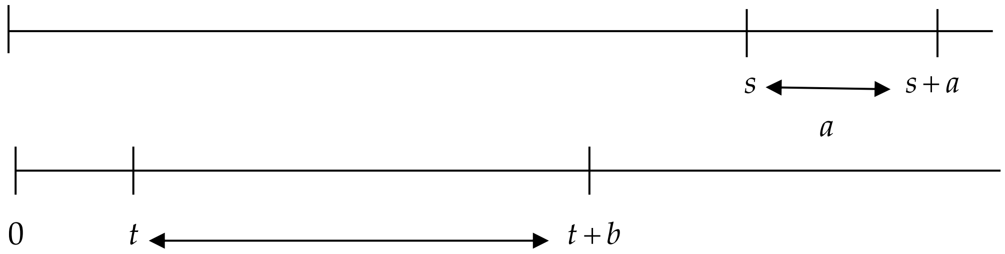

Figure 9. Condition (iii) of Theorem 2 (II). **Source**: Own elaboration.

To summarize, in the cases displayed in Figures 5 and 6, the instantaneous discount rate is decreasing, while in the cases displayed in Figures 7–9, the instantaneous discount rate is increasing. Finally, the case shown by Figure 10 ($s+a<t+b$ and $t<s$) is not mathematically possible.

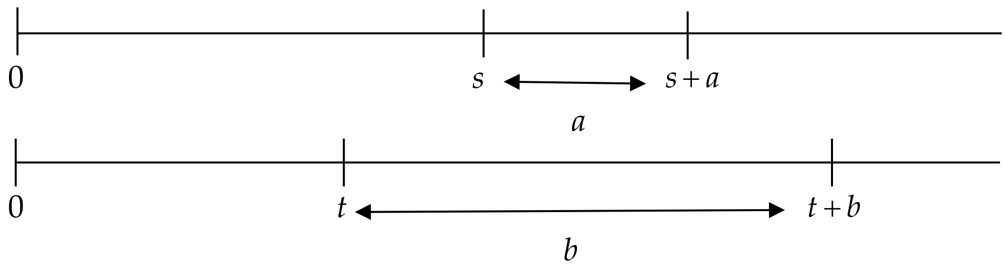

Figure 10. Case in which $s+a<t+b$ and $t<s$. **Source**: Own elaboration.

In effect, by letting $s \to t$ by Theorem 1, the instantaneous discount rate has to be decreasing, while by letting $s+a \to t+b$ by Theorem 2, the instantaneous discount rate has to be increasing. However, both situations are not simultaneously possible. This allows us to conclude that, in the cases in which the short interval begins before or at the same time as the larger interval, the instantaneous discount rate is decreasing and, contrarily, in all cases in which the short interval ends after the larger interval, the instantaneous discount rate is increasing. Obviously, both results are not consistent, and then we have to redefine the interval effect in this context.

In effect, the previous analysis allows us to claim that the former analyzed cases could be descriptive of the so-called interval effect, regardless of whether the discount rate increases or decreases. However, it is necessary to make a distinction, as a given discount function cannot simultaneously fit both situations. Therefore, the interval effect could be classified as follows:

- The decreasing interval effect, wherein the so-called front-end delay (FED) of the short interval is less than or equal to the FED of the greater interval. FED is defined as the delay between zero and the beginning of the interval [31]. This would correspond to figures 4–6.
- The increasing interval effect, wherein the FED of the larger interval is less than the FED of the shorter interval. This would correspond to figures 7–9.

Chart 1 clarifies the involved implications.

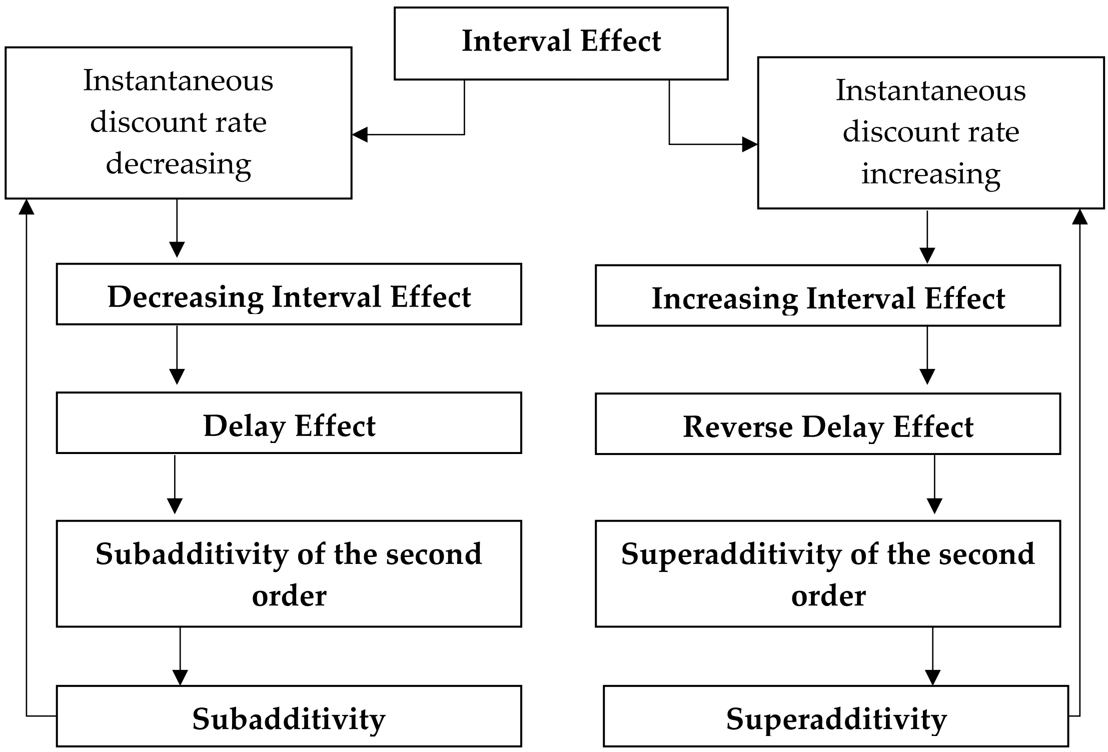

Chart 1. The interval effect. **Source:** Own elaboration.

Once the concept of the interval effect has been clarified, another question arises: is the interval effect a by-product of the delay effect? This statement was introduced by Kinari et al. [29,30]. The delay effect depends on the FED of the intervals considered in the analysis, whereas the interval effect depends on length of the involved intervals. However, from a stationary point of view, when the length of the intervals is the same (Figure 1), it can be stated that the delay effect is a by-product (a particular case) of the interval effect, specifically of the decreasing interval effect. This conclusion runs contrary to that stated by Kinari et al. [29,30].

4.2. Assessment at Variable Reference (at the Front-End Delay of the Interval)

In Section 4.1, we measured the instantaneous discount rates with reference to a given benchmark (labeled as time 0). However, the use of discount ratios implies that the process of intertemporal choice is transitive, and there is nothing further from the truth. In effect, the additive property of discount ratios

$$f(t,a)f(t+a,b)=f(t,a+b)$$

is not possible because, by the interval effect, the average instantaneous discount rate in the intervals $[t,t+a]$ and $[t+a,t+a+b]$ is greater than the corresponding mean in the interval $[t,t+a+b]$.

Therefore, we are going to measure the average instantaneous discount rate by using the discount function referenced at the front-end delay of the involved interval. However, it is necessary to take into account that the interval effect obviously implies that, for every *s*, *t* and *a*, the following equality holds:

$$F(t,a)=F(s,a)$$

That is to say, the discount function is stationary. In other words, the analysis of the interval effect with dynamic discount functions does not make sense. If the instantaneous discount rate is decreasing, it is immediate, showing that both the delay and the interval effect hold. However, our aim is to analyze if the interval effect is independent of the delay effect. To do this, we are going to consider the discount function whose instantaneous discount rate is

$$F(t) = 1 + \exp\{-t\}\cos t$$

Integration by parts leads to the following equality:

$$\overline{f}(t) = 1 + \frac{\exp\{-t\}(\sin t - \cos t) + 1}{2t}$$

Moreover, the derivative of $\overline{f}(t)$ is

$$\frac{\mathrm{d}}{\mathrm{d}t}\overline{f}(t) = \frac{[(2t+1)\cos t - \sin t]\exp\{-t\} - 1}{2t^2} < 0$$

This means that the average discount rate is lower for larger intervals. Moreover, as the discount function is stationary, the delay effect does not hold.

5. Conclusions and Future Research

In this paper, we clarified the concept of the interval effect which, traditionally, has been confused with the delay effect. The interval effect means that the discount rate is greater the shorter the interval, while the delay effect means that the discount rate is greater the shorter the delay. However, before jointly analyzing these two effects, it was necessary to study the possible cases in which the interval effect can appear. This analysis has allowed for redefining the concept of the interval effect by subdividing it into two sub-concepts:

- The decreasing interval effect, wherein the discount rate decreases (the FED of the short interval is less than or equal to the FED of the larger interval).
- The increasing interval effect, wherein the discount rate increases (the FED of the larger interval is less than the FED of the shorter interval).

From this distinction, we have been able to deduce that, from a stationary point of view, the delay effect and, therefore, the subadditivity are a particular case of the decreasing interval effect under certain conditions, and the reverse implications cannot be stated. In the same way, it has been found that, starting from the increasing interval effect, it is possible to deduce the concept of the reverse delay effect and, therefore, superadditivity.

Another contribution of this paper is that the interval effect does not make sense from a dynamic point of view, since this effect implies a stationary discount function for this effect to exist. Moreover, the interval and delay effects have shown to be independent of each other.

The classical methods included in the paper, such as discount functions, could be extended to include memory effects. In effect, it has been shown that fractional operators with memory modify the delay in biological and other systems, and that could be used to calculate adequate fractional discounts. In this way, [35] introduced the relationship between human decision-making, fractional memory and delays. On the other hand, given the parallelism between discount and probability functions [36], some distributions modeling the delay in human decisions could inspire new discount functions in the ambit of intertemporal choice. In this way, [37] showed that human decision delays present a gamma probability distribution, which can be used to adjust the discount factors weighting the value of different delays. These ideas are proposed as future research.

Finally, another further research line is to analyze the consequences of the delay effect and the interval effect in the field of managerial decision-making.

6. Appendix A

Table A1. The interval effect in the existing literature. **Source**: Own elaboration.

Ref	Term Used	Def.	Exp. Work?	Math. Definition?
[20]	Subadditive discounting	Yes	Yes	No
[28]	Interval effect and subadditive discounting	Yes	Yes	No
[23]	Interval effect	[28]	No	No
[21]	Interval effect	Yes	Yes	No
[32]	Interval effect	[20]	No	No
[26]	The effect of interval length	[20]	Yes	No
[30]	Interval effect	Yes	No	No
[27]	Interval effect	[20,23]	No	No
[33]	Interval effect	[23]	Yes	No
[34]	Interval effect	[20,28]	Yes	No

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CAPÍTULO V.

CONCLUSIONES

1. Conclusiones

El principal objetivo de esta Tesis Doctoral ha sido clarificar el concepto de efecto intervalo, así como realizar un análisis lo más completo posible del mismo desde un punto de vista teórico. Como ya se ha indicado anteriormente en las diversas secciones de este trabajo, el efecto intervalo es una de las anomalías menos estudiadas de la elección intertemporal, posiblemente causada por su tradicional identificación con el efecto plazo y, posteriormente, con la subaditividad.

El primer paso para conseguir el objetivo principal de esta Tesis ha sido llevar a cabo una revisión sistemática de la literatura sobre la mayoría de las anomalías en la elección intertemporal. Esto nos ha permitido conocer cuál es el estado actual de investigación en cada una de ellas y corroborar que el efecto intervalo es prácticamente desconocido en relación al resto de anomalías (véase el capítulo II). Las principales conclusiones de este capítulo son que el efecto magnitud y el efecto plazo son las anomalías más estudiadas tanto teórica como empíricamente. Estas dos anomalías han sido estudiadas en cuatro áreas, Economía, Medicina, Psicología y Ciencias Políticas. Sin embargo, la Neurología ha estudiado el efecto magnitud, el efecto signo y el efecto secuencia. Las anomalías menos estudiadas han sido el efecto asimetría plazo-anticipación, el efecto secuencia, el efecto fecha-aplazamiento y el efecto intervalo. De este último trabajo, solo disponíamos de dos trabajos anteriores, al que tenemos que añadir la aportación científica derivada de esta Tesis. Este efecto, a diferencia del resto, solamente se ha trabajado en el campo de la Economía, observándose así la necesidad de estudiar esta anomalía desde un punto de vista teórico. Esto permitirá asentar las bases para futuros estudios empíricos y, por tanto, para su desarrollo en el resto de las áreas, como se ha hecho con otros efectos. Otra de las conclusiones obtenidas en este capítulo es la variedad de denominaciones que reciben estas anomalías; en concreto, en el efecto plazo hemos encontrado catorce denominaciones distintas mientras que, en el resto de efectos, entre dos y cuatro nombres distintos.

Únicamente el efecto fecha-aplazamiento es el que ha recibido una única denominación. Esta variedad en la nomenclatura empleada dificulta notablemente las labores de investigación, y ésa es la razón por la que, en este capítulo, proponemos una serie de denominaciones que permita unificar criterios en todas las áreas, e incluso dentro de las mismas. Los nombres propuestos son los siguientes: *delay effect*, *magnitude effect*, *sign effect*, *sequence effect*, *delay/speed up asymmetry*, *date-delay effect* and *interval effect*. Este capítulo nos ha permitido alcanzar el primer objetivo secundario de esta Tesis, a saber, conocer el estado actual de la investigación de las principales anomalías presentes en la elección intertemporal.

En el tercer capítulo de la Tesis, hemos analizado el efecto plazo desde un punto de vista matemático, pues este paso ha sido completamente necesario para conocer su relación con el efecto intervalo. Para ello, se han deducido diferentes expresiones del efecto plazo y su caracterización matemática, utilizando funciones de descuento en un contexto estacionario y en un contexto dinámico, y se ha buscado su relación con la subaditividad y la impaciencia decreciente. En este sentido, hemos demostrado que, en el contexto estacionario, la subaditividad es un caso particular de efecto plazo, lo que se ha podido deducir directamente a partir de la expresión del efecto plazo e indirectamente partiendo de la expresión de la denominada subaditividad de segundo orden. Desde un punto de vista dinámico, el efecto plazo es un caso más general que la subaditividad y la impaciencia decreciente, al igual que en el contexto estacionario. Sin embargo, su demostración no es inmediata ya que debe utilizarse la subaditividad de segundo orden para llegar a estas implicaciones. Además, se comprueba teóricamente que el descuento contractivo es un caso particular de la subaditividad. En este capítulo también proponemos una nueva función de descuento, el modelo de descuento exponencial asimétrico (*Asymmetric Exponential Discounting*). Esta función representa un modelo de descuento dinámico que incorpora no solo la variable plazo, sino que también incluye a la variable intervalo. Esta función es capaz de explicar el efecto plazo, la subaditividad y la impaciencia decreciente. Este capítulo nos ha permitido alcanzar dos de los objetivos secundarios propuestos en esta Tesis: analizar el efecto plazo y proponer un modelo matemático que describa este efecto y la subaditividad.

En el cuarto capítulo de este trabajo, se ha analizado el efecto intervalo. En primer lugar, se analizaron las distintas definiciones de este efecto, lo que nos permitió conocer la evolución de este concepto y recopilar las conclusiones aportadas por algunos investigadores. En este sentido, Read (2001) hizo la primera distinción entre plazo e intervalo y, posteriormente, Read y Roelofsma (2003) identificaron el efecto intervalo con la subaditividad. Más adelante, Kinari et al. (2016) afirmó que el efecto intervalo era un concepto más general que la subaditividad y, además, que este efecto era un subproducto del efecto plazo. En este capítulo, hemos analizado la precisión de estas afirmaciones para lo que hemos partido de la formalización matemática del efecto intervalo, aportada por Kinari et al. (2016), en particular, de su definición de ratio de descuento, que no había sido tratada desde un punto de vista matemático. En este capítulo, basándonos en el enfoque de Kinari et al., analizamos las relaciones existentes entre el efecto intervalo, el efecto plazo y la subaditividad. En primer lugar, se pormenorizaron todas las posibles situaciones en las que puede presentarse el efecto intervalo y observamos que, cuando el inicio del intervalo (FED) más corto es menor que el comienzo del intervalo más largo, la tasa de descuento es decreciente, mientras que, en los otros casos, la tasa es creciente. Esto nos ha obligado a redefinir el concepto de efecto intervalo ya que hemos deducido que este efecto deriva, a su vez, en dos subefectos: el efecto intervalo decreciente, para aquellos casos en los que el FED del intervalo más corto se presenta antes que FED del intervalo más largo, y el efecto intervalo creciente, para aquellos casos en los que el FED del intervalo más corto se presenta después que el FED del intervalo más largo. A partir de esta definición, demostramos que el efecto intervalo decreciente es un concepto más general que el efecto plazo, y este último, un concepto más general que la subaditividad (esta última afirmación también fue demostrada en el Capítulo 3 de esta Tesis). Estas demostraciones nos permiten rebatir la afirmación de Kinari et al. (2016) en el que afirmaba que el efecto intervalo era un subproducto del efecto plazo. Además, demostramos que el efecto intervalo creciente, es decir, cuando las tasas de descuento instantáneo son crecientes, es un caso más general que la reversión del efecto plazo y éste, a su vez, un caso más general que la superaditividad. Por otro lado, en nuestro análisis, también deducimos que el estudio del efecto intervalo desde un punto de vista dinámico no tiene sentido, es decir, que el efecto intervalo solo puede ser estudiado con funciones estacionarias de descuento. Esta conclusión nos obliga a descartar la posibilidad de analizar este efecto mediante la función propuesta en el capítulo anterior, es decir, analizar el efecto intervalo con la función de Descuento Exponencial Asimétrica, al ser dinámica esta función. Este capítulo nos ha permitido alcanzar varios objetivos secundarios, a saber, definir el efecto intervalo y su relación con el efecto plazo, aunque principalmente nos ha permitido alcanzar el objetivo principal de esta Tesis: realizar un profundo análisis teórico del efecto intervalo. Esto permitirá incentivar el estudio del efecto intervalo, tanto desde un punto de vista teórico como empírico, en las diversas áreas de estudio que analizan la elección intertemporal como la Psicología, la Medicina o la Neurociencia.

En resumen, las principales contribuciones de esta Tesis son:

- Aportación de futuras líneas de investigación sobre las principales anomalías en la elección intertemporal.
- Propuesta unificada de denominaciones para las anomalías en la elección intertemporal, que faciliten la búsqueda de bibliografía para futuras investigaciones.

- Justificación de la necesidad de investigar el efecto intervalo.
- Análisis matemático del efecto plazo, desde un contexto estacionario y dinámico.
- Demostración matemática de que el efecto plazo es un concepto más general que la subaditividad, en ambos contextos.
- Propuesta de una función de descuento dinámica denominada "Función de Descuento Exponencial Asimétrica" (*Asymmetric Exponential Discounting*), que permite explicar el efecto plazo, la subaditividad y la impaciencia decreciente.
- Análisis de las distintas situaciones en las que se puede dar el efecto intervalo.
- Propuesta de una nueva definición de efecto intervalo, subdividiendo este efecto en dos subefectos, en función de si la tasa instantánea de descuento es creciente (efecto intervalo creciente) o decreciente (efecto intervalo decreciente).
- Relación matemática entre el efecto intervalo, el efecto plazo y la subaditividad a partir de la nueva definición aportada para el efecto intervalo.
- Análisis del efecto intervalo desde un punto de vista dinámico, comprobándose que este efecto no tiene sentido para funciones dinámicas sino estacionarias.

2. Limitaciones y Futuras Líneas de Investigación

A lo largo de la elaboración de esta Tesis Doctoral, nos hemos encontrado con varias limitaciones:

- En relación al Capítulo II, se ha realizado una revisión sistemática de las anomalías en la elección intertemporal utilizando dos bases de datos, la Web of Science (WOS) y Scopus, ambas por su relevancia científica. Sin embargo, una gran parte de las primeras aportaciones científicas sobre estas anomalías no se encontraban dentro de estas bases de datos, por lo que no han podido ser analizadas. Otra de las limitaciones encontradas en el Capítulo II ha sido la gran variedad de denominaciones que existen para referirse a estos efectos, lo que ha limitado enormemente las labores de búsqueda.

- En cuanto al Capítulo IV, la principal limitación que hemos tenido es la escasez de trabajos existentes sobre el efecto intervalo. Esto ha dificultado bastante las labores de comprensión de este efecto y, principalmente, su caracterización matemática debido a la ausencia de una base científica.

En cuanto a las futuras líneas de investigación, proponemos las siguientes:

- En primer lugar, es necesario encontrar una función de descuento que explique el efecto intervalo. En el Capítulo III, hemos aportado una función que explicaba el efecto plazo y que incorporaba una variable que representaba el intervalo. Sin embargo, en el capítulo IV demostramos que el efecto intervalo no tiene sentido analizarlo en funciones dinámicas, por lo que nos ha sido imposible analizar el efecto intervalo en dicha función. Por consiguiente, una futura línea de investigación sería encontrar una función estacionaria que explique adecuadamente este efecto.
- Otra futura línea de investigación sería constatar el efecto intervalo a partir de estudios empíricos, que podrían implementarse en diversas áreas:
    - Campo de la Medicina: Una posible línea de investigación podría ser estudiar el efecto intervalo en esta área; por ejemplo, ¿qué ocurre cuando se toman decisiones intertemporales sobre hábitos no saludables, pero cuyas consecuencias futura en nuestra salud se manifiestan en diferentes intervalos de tiempo? Otro posible estudio podría ser analizar dos hábitos no saludables (como fumar y drogarse), sabiendo que con el primero el resultado de muerte es más tardío que con el segundo. Por último, analizar el

comportamiento del decisor en función del intervalo en el que se manifiesta la consecuencia de un hábito permitirá conocer mejor el comportamiento de las personas y, por tanto, realizar un mejor diseño de las campañas de concienciación.

- En el campo de la Neurociencia sería interesante analizar esta anomalía en relación con las áreas de nuestro cerebro para conocer así qué tipo de sustancia puede interferir en el aumento o disminución de este efecto. Además, este tipo de experimentos puede analizarse diferenciando entre colectivos, es decir, personas sanas, personas con adicciones o incluso personas con alguna enfermedad. También puede analizarse cómo este efecto es afectado por la memoria, la imaginación o la fuerza de voluntad de las personas.
- El campo de la Psicología es el área que más estudios empíricos ha proporcionado al ámbito de la elección intertemporal; sin embargo, no hemos encontrado trabajos que analicen el efecto intervalo. Es, por ello, que proponemos que se estudie empíricamente este efecto en personas con TDAH (Trastornos de Déficit de Atención e Hiperactividad), con personas drogodependientes, fumadoras o con problemas alimenticios, tanto para decisiones monetarias como no monetarias, diferenciando los resultados obtenidos por edad y sexo.
- También proponemos que se estudien las anomalías en la elección intertemporal en las decisiones empresariales, pues consideramos que estas decisiones tienen una gran repercusión en el futuro de una empresa y, sin embargo, apenas han sido objeto de estudio.

- Otro aspecto novedoso que proponemos es aplicar las anomalías en la elección intertemporal a la inteligencia artificial. En efecto, si realmente se quiere predecir el comportamiento de los humanos, y sabemos que éstos no son racionales, se deberían estudiar los efectos en este ámbito.

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