

An experimental study on preventing first graders from finger counting in basic calculations

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Abstract

Introduction: When counting is taught to students at primary stage of schooling, they are generally allowed to use their fingers as a counting tool. Therefore, some students continue using their fingers to count, while others stop this habit later. The students who have the habit of using their fingers to count have difficulty when their fingers are not enough for mathematical calculations. The purpose of this experimental study was to prevent students from finger counting and enable students who already have a habit of finger counting to quit this habit.

Method: Since the interest and the intent of the researcher was to compare the differences between students' pre and post test scores from the control and experimental groups receiving different instructional process, this study is static-group pretest-posttest design. In each group there were 33 students who were mostly seven years old first graders. The experimental group received the remedial procedures while the control group was receiving traditional counting instruction for two whole semesters in an academic year.

Results: The study revealed that the students in the experimental group were more successful without using their fingers in basic calculations due to the remedial program implemented in this group.

Discussion and Conclusion: It is believed that better results in terms of improving students' counting skills can be achieved if the content of the remedial program is expanded and appropriate environment for the application of the program is provided

Keywords: Rhythmic counting, rote counting, semi-concrete or semi-abstract objects, abstract counting, rote counting, counting skills

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Estudio experimental entre alumnado de primer curso para prevenir la resolución de operaciones básicas contando con los dedos

Resumen

Introducción: Cuando se enseña a sumar a los niños de primaria se les permite, de forma generalizada, el usar sus dedos como herramienta para contar. Así, algunos de ellos continúan utilizando los dedos para contar mientras que otros abandonan este hábito más tarde. El alumnado con el hábito de contar con los dedos tienen dificultades cuando los dedos no son suficientes para realizar cálculos matemáticos. El propósito de este estudio experimental es prevenir que los estudiantes utilicen los dedos para contar así como facilitar que estudiantes que ya tienen este hábito puedan abandonarlo.

Método: Dado que el interés del investigador fue comparar las diferencias entre estudiantes, en las puntuaciones pre y post test, del grupo experimental y grupo control que habían recibido diferente formación, se utilizó un diseño pretest-posttest con grupo estático. En cada grupo había 33 estudiantes mayoritariamente de 7 años de edad. Durante dos semestres académicos, el grupo experimental recibió procedimientos de remediación mientras que el grupo control recibió la instrucción tradicional para el aprendizaje del conteo.

Resultados: Los resultados muestran que los estudiantes del grupo experimental tuvieron más éxito sin usar los dedos en cálculos básicos debido al programa implementado en este grupo.

Discusión y conclusiones: Creemos que se pueden conseguir mejores resultados en relación con la mejora de las habilidades de conteo de los estudiantes si los contenidos del programa remediativo se extienden y se proporciona un ambiente más apropiado para la aplicación del programa.

Palabras Clave: conteo rítmico, conteo rutinario, objetos semiconcretos o semiabstractos, conteo abstracto, habilidades de conteo.

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Introduction

Mathematics which has unique contribution to cultural, social and cognitive development of students is included in formal curriculum in every country in the world (Lupiañez, 2009). As was also stated in literature, numbers and counting are among the first and key concepts that are taught to students in improving mathematical skills in mathematics education (Baroody 1987; Bashash, Shiraz, Outhred, & Bochner 2003; Hughes, 1989).

Counting is the foundation of many skills which students learn in mathematics education. Therefore, counting exercises such as classifying, ordering, and matching, constitute a basis for students' learning arithmetical knowledge in the following years (Hohmann & Weikart 2000). In addition to this, the concept of numbers and counting plays an important role in helping students to comprehend the relations between numbers (few-many, big-small, order of numbers and so on) and to understand the numerical system (Maclellan 1997).

Counting based approach is always used when teaching students four basic calculations (Sarama & Clements 2003). According to Cobb (1987), counting styles and thinking strategies significantly contribute to students' problem solving skills. Pepper and Hunting (1998), who emphasize the importance of meaningful counting in problem solving process, state that in order to participate in problem solving process actively, students need to acquire counting skills and learn counting accurately in their minds without using counting tools. Besides, there is an important relationship between counting skills and understanding numbers (Bashash, Outhred & Bochner 2003).

The development of the counting concept and numbers is still a popular research topic. Latest research indicates that children start using numbers meaningfully before the age of seven (Baroody 1987; Diezmann and English 2001; Fuson 1988; Irwin 1996; Viadero 1994). Therefore, students have different skills related to counting when they start primary education. Regarding this, there are various views about how students obtain these skills. According to Hunting (1999), children learn counting randomly by imitating their parents or the people around them. It is beneficial to eliminate students' differences and false habits related to counting.

A number of research has been carried out about how students learn the concept of numbers and how students acquire counting skills in the last 30 years (Davis, Maher, & Nodding 1990; Goldin, DeBellis, DeWindt-King, Passantino, & Zang 1993; Goldin & Herscovics 1991; Hannula & Lehtinen 2005; Hannula, Räsänen, & Lehtinen 2007; Herscovics, 1996; Goldin 1990; Peper&Hunting 1998; Reynolds &Wheatley 1994). In these studies, counting is first taught by matching objects and then, students are expected to learn the names and order of numbers in time. Studies carried out on teaching counting by matching are in accordance with the principles recommended by Gelman and Gallistel (1978), and Steffe and Cobb (1988). The study done by Thomas, Mulligans and Goldin (2002) is another example. Some studies also focused on using counting in calculations taught in the pre-school period (Baroody 1987, 1999; Bashash, Quthred, & Bochner 2003; Bruce 2003; Clements 1999a; Fuson 1988; Maclellan 1997; Sophian 1995; Steffe & Cobb 1988; Steffe, Glasersfeld, Richards, & Cobb 1983; Thompson 1997; Wright 1994; Wynn 1990). The followings are common in the studies on counting seen in related literature:

- Starting to count by matching one by one and teaching rote and meaningful counting together
- The representation and usage of concrete, semi-concrete and abstract objects that are used in counting
- The usage of hands in starting from a number and counting forward and backward or in doing basic calculations (Burton,1985; Esther, Schopman & Van Luit, 1999; Gray, 1991; Johansson, 2005; MacLellan,1997; Sarama & Clements, 2004).

Students use their fingers while counting and doing basic calculations due to their habits of finger counting or being afraid of making mistakes. When counting fast, some students even need to touch the objects or match them with their fingers because they worry about not counting them (Brias & Siegerler 1984; Wilkonson 1984). In order to familiarize students with counting or prevent them from miscounting, it is necessary to use concrete objects, such as beans, marbles, buttons, beads, matchsticks, and students (Busbridge & Womack 1991; Hopkins, Gifford, & Pepperell 1996; Nair & Pool 1991). Students also worry about miscounting when they solve problems based on counting. Counting by matching with fingers is considered as a useful approach in order to stop students from getting worried about making mistakes. However, finger counting should not be turned into an indispensable habit. If finger counting has become a habit, students have trouble when their fingers are not enough

in counting. Besides, using fingers is not an effective way of doing calculations with big numbers (Burton 1985; Johansson 2005).

Although using fingers is the first method used in teaching counting to students, finger counting has negative effects on students. Therefore, finger counting should be discontinued in meaningful counting exercises (Clements, 1999b; Sarama & Clements, 2004). Pepper, and Hunting (1998) have also emphasized the importance of meaningful counting in problem solving. According to them, meaningful counting is related to active participation of students in problem solving processes, acquiring counting skills and being able to count without using any matching tools. Moreover, Cobb (1987) goes on to state that counting styles and thinking strategies contribute significantly to students' problem solving processes.

Counting is a cognitive activity requiring eye and hand coordination and verbal skills (Wilkinson 1984). In learning how to count, knowing the names and the order of numbers is an important stage. One of the purposes of counting is going from the ordinal value of a number to the cardinal value (Orton & Frobisher 1996). In order to reach that aim, students need to continue counting on concrete objects. While counting objects, using the fingers as a matching tool is not necessary at all. In order to understand it is necessary or unnecessary, the researcher considered the challenging questions of "if finger counting is used as a matching tool, can it be stopped before becoming a habit? What can be used as a matching tool instead of fingers? How can meaningful counting be performed without using the objects that are used as a matching tool? "

Considering these challenges, following research question were tested in this research:

- Is there a significant difference between the pre test and post test scores of experimental group implemented "counting without finger program" and control group implemented traditional method, "counting with finger"?

Method

This research was carried out by taking account of the procedures in static group pre-test post-test experimental study design (Büyüköztürk, Çakmak, Akgün, Karadeniz & Demirel, 2008). The researcher studied in public primary school classrooms, since he had no

chance to assign or pair the students, two first grade classrooms were randomly assigned in a primary school in the city of Ağrı, located in the east part of Turkey. There were very few institutions providing preschool education in the city and most of the parents cannot contribute to their children's preschool education quality. Therefore, observations in the primary schools in this city showed that children had difficulties in developing counting and basic calculation skills. The rationale for choosing such a school was that because the children would get little help from their environment outside the classroom and thus the success of the program applied in the study would be observed easily.

Participants

There were 33 students (19 male, 14 female), aged mostly seven years in each class and the study continued for two semesters in an academic year. The teachers of the two classes were male and both had six years of teaching experience. The study was completed with the same teachers.

Instruments and procedure

First, each student's level of counting forward by ones was determined by a pre-test in the two first grade classes. In the test, whether students knew the names and order of the numbers were determined. Then, students were asked to count forward on concrete objects, such as students, beans, and buttons. In this way, students' skill to associate objects with numbers was measured.

Although the studies of Fuson (1988) and Wynn (1990) whose ideas were benefited in this study have similarities, they are different in terms of the following points: i) teaching only counting forward, ii) the order and types of counting, iii) the tools used in counting, iv) counting exercises, v) the readiness and ages of the participants, and vi) evaluation style.

Before the lessons, the teacher of the experimental group was specifically trained on the method that he would use and the researcher was always in contact with the teacher in order to carry out the study successfully. So, this close partnership and training process provided objectivity in the evaluation stage.

The mathematics syllabus followed in both groups was the same, but in the experimental group additional remedial applications were used. While having the students do the exercises, the teacher tried to prevent exam anxiety by informing them about all exercises beforehand. The assessment tools used in evaluating the activities were prepared in advance and during the evaluation, only how high students can count and how they count was observed and recorded on an observation summary sheet. The evaluations were done in the last two months of each academic year.

Counting apparatus:

The activities used in the remedial program that was applied together with the primary school mathematics curriculum in the experimental group as follows:

- The counting tools were composed of concrete representations of objects such as abacus, beads, marbles, beans, students, and semi-concrete and abstract representations of objects which can be found inside or outside the classroom.
- Teachers draw the shapes below and associated them with the problem.
- After doing the activities in stated in the Figure 1 and 2, the activity in the Figure 4 was done. The numbers above the line were read loudly and the numbers below the line were read silently.
- Figure 3 shows counting of disarranged objects. The counting style is eye fixation and grouping.

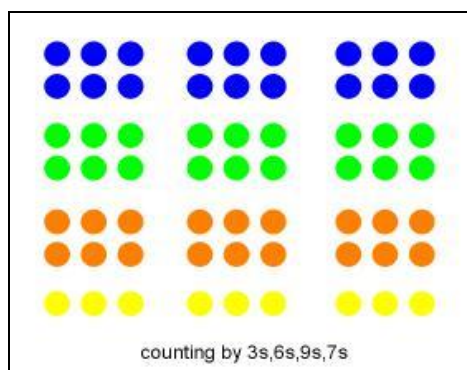


Figure 1. Counting panel 1

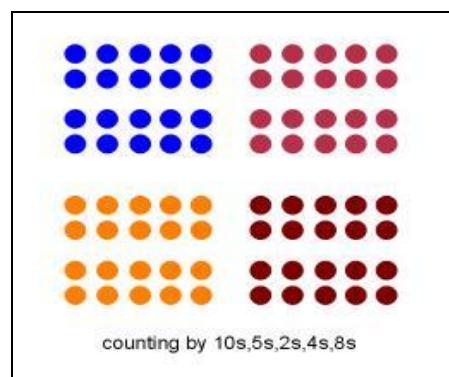


Figure 2. Counting panel 2

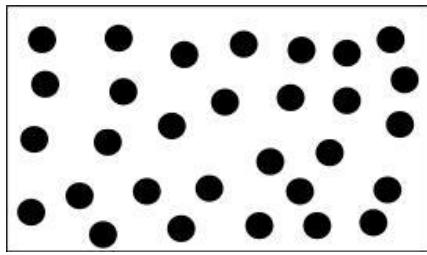


Figure 3. Counting panel 3

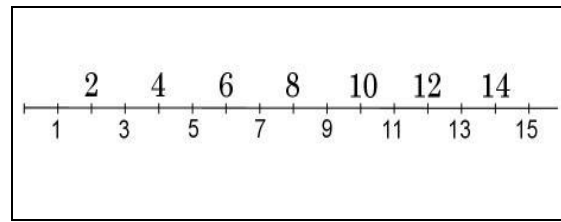


Figure 4. Counting panel 4

- Problems related to daily life were asked to students by associating counting with problem. For example: “Ali’s family consumes 3 loaves of bread every day. How many loaves of bread do they eat in a week?”
- Games, competitions, riddles, songs, tongue twisters and so on which were performed in the classroom or in the school garden. (Hide and seek game was often used in teaching and evaluating rote counting forward by ones).
- Using concrete and semi-concrete counting tools were used in counting activities. In such activities, students were asked to count using pointing strategy and eye fixation.
- Students were asked to guess the number of disarranged concrete or semi-concrete objects in the environment (Figure 3), to count them and to talk about how close they get to the correct answers. The activities were done with the help of objects like matchsticks and beans on a table, and the aim of these activities were to improve the students’ intuition and guessing and to contribute to their abstract thinking skills. Logical guessing is one of the objectives of the NCTM (2000).

The procedures applied in control and experimental groups

The control group received traditional way of instruction regarding teaching counting in basic calculations as was required in the first grade Mathematics curriculum which mainly employs several matching exercises containing mainly finger counting calculations.

Teachers of the experimental group were informed about the following steps which are the essence of the remedial program prepared by the researcher.

1. Students should not be asked to count in the whole lesson; only a part of the lesson should be allocated to counting. For example, counting should be done for 10 minutes or less in a lesson. The exercises should be done for short periods of time in most lessons throughout the semester.
2. All counting activities should be done in the classroom and they should not be given as homework.
3. Rote counting should be limited to counting by ones and tens.
4. In the counting activities, first the teacher should count loudly and then students should repeat what the teacher has said. Until students learn how to count, the teacher should do such activities often (Busbridge & Womack 1991; Hopkins, Gifford & Pepperell 1996; Nair & Pool 1991; Orton & Frobisher 1996).
5. Students should be asked to perform rote counting together (chorus counting)
6. (Rote or concrete) individual counting should be performed only for the purpose of controlling.
7. Students should be first asked to count forward by ones in order to help them perceive the name and order of numbers and the rhythm between the numbers.
8. While having students count, activities including games, songs, puzzles and jokes should frequently be used.
9. Rote counting should be made meaningful with the help of objects in the environment and counting tools by establishing one-to-one correspondence between objects and numerals (Busbridge & Womack 1991; Hopkins, Gifford & Pepperell 1996; Nair & Pool 1991; Orton & Frobisher 1996).
10. Students should be asked to count the objects on the counting boards by ones or by grouping. Counting boards should be put on one of the walls of the classroom and students should be asked to repeat the counting on these boards regularly.
11. The following order should be followed in the counting except for counting by ones: for example, the teacher who will teach counting by twos should start the lesson by asking these questions: “Does everybody have two eyes?” “How many eyes are there in the classroom?” While answering these questions, the teacher should count by ones. However, while counting by ones, he should emphasize even numbers by saying them loudly. He can also use gestures while saying even numbers. Students should do the same while counting. This method is called “counting the even numbers loudly”. Another lesson can start with a similar problem and the hands, legs or ears of the students in the classroom can be counted.

While the teacher and the students are counting by ones, they say the odd numbers silently and even numbers loudly. This method is called “counting the odd number silently”. In this activity, pointing strategy (pointing objects without touching them) can be used.

12. The same or similar activities should be done using semi-concrete objects. As such activities are done, students start to count faster. When students count faster, they will start counting in their minds instead of counting odd numbers loudly or silently. Counting in their minds makes meaningful counting not only faster but also easier.

Types of counting and the analysis of data

In this research, following counting types without using fingers were taught in three subsequent steps

1. *Teaching rote counting by ones and tens.* This step aimed to teach students the names and orders of the numbers, and the rhythm between the numbers, because some mathematics educators (Baroody 1999; Carr 1995; Gelman 1993) have claimed that students can succeed in rote counting before learning the basic counting principles.

2. *Changing rote counting by ones into meaningful counting.* This step aimed to teach students the relation between counting and the concept of number and the ordinal and cardinal values of numbers because there is a relationship between the ordinal value of the numbers and the counting style and Brannon and Vande Walle (1994) and Ausubel (1968) state that rote learning provides a good foundation for meaningful learning. The goal in the first two steps was to improve students' counting skills.

3. *Enabling students to count concrete or semi-concrete objects without using their fingers.* In this step, students were taught pointing strategy or eye fixation instead of matching by touching fingers. These activities were not done in a hurry because counting by pointing is easier than eye fixation. Before being able to count using eye fixation, students needed to achieve a certain level of counting speed by pointing. Having students do a lot of counting activities made it easy to teach counting by grouping. There are similarities between this

study and the studies done by Ginsburg, Klein and Starkey (1998) in terms of the method applied.

All process in both groups were observed, recorded and evaluated individually. In the evaluation of the observation process, firstly students' rote counting activities counting up to 100 was observed. Secondly, the students were given concrete objects (beans, marbles, beads, buttons, match sticks etc.) or semi-concrete objects (pictures of objects) and asked to count them. While the students were counting, their behaviors related counting skills were observed carefully and recorded into the sheets. The evaluation criteria was only limited to counting forward on concrete and semi-concrete objects and counting styles. By using frequency analysis, three points were given to the students who counted correctly without using their fingers, 2 points were given to the students who received help less than three times or used their fingers, 1 point was given to the students who received help three times or more, or used their fingers while counting. All process were observed and recorded in the observation sheets with the researcher and the teachers in both classes.

The data obtained in this study was analyzed using SPSS 10.00 software, and (Chi-Square) χ^2 was used as the statistical analysis. ($p < .05$).

Results

Before starting the research, differences between the groups in terms of counting were determined through a pre-test. In addition, in the pre-test, readiness level of the students was accepted as the ability to memorize counting forward by ones up to 30 accurately and to count forward by ones accurately on objects up to 15. Observations done by the researcher and teachers showed that the students in both experimental and control groups could do rote counting from 1 to 100 by ones accurately.

The differences between the pre-test scores of both experimental and control groups were not statistically significant ($\chi^2=3,137$, $p=.069$). However, as was seen from the Table 1 and Graph 1, there was statistically significant difference between the post-test scores of the students in experimental and control groups.

Table 1. *The values regarding the results of both groups*

	Experimental Group Pre-test 1-A			Control Group Pre-test 1B			Experimental. Group Post-test 1-A			Control Group Post-test 1-B			χ^2	p
	P	G	K	P	G	K	P	G	K	P	G	K		
	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)	(%)		
Grades	69,7	18,1	12,2	72,7	15,1	12,2	12,1	30, 3	57,6	42,4	30,3	27,3	9,13	,010*

P: Counting by touching (one-to-one),

G: Counting by pointing or eye-fixation,

K: Other counting styles (novel, stable order)

p<.05

The frequency of the undesired “counting with fingers” behaviors shown by the students in the experimental group radically decreased (from %69,7 to %12,1) after the remedial program, as compared to the students receiving traditional program in control group. The results also showed that there is statistical significance ($\chi^2=9,127$, $p=.10$) between the post-test scores concerning acquired counting abilities of the students in the experimental and Control groups, in favor of experimental one.

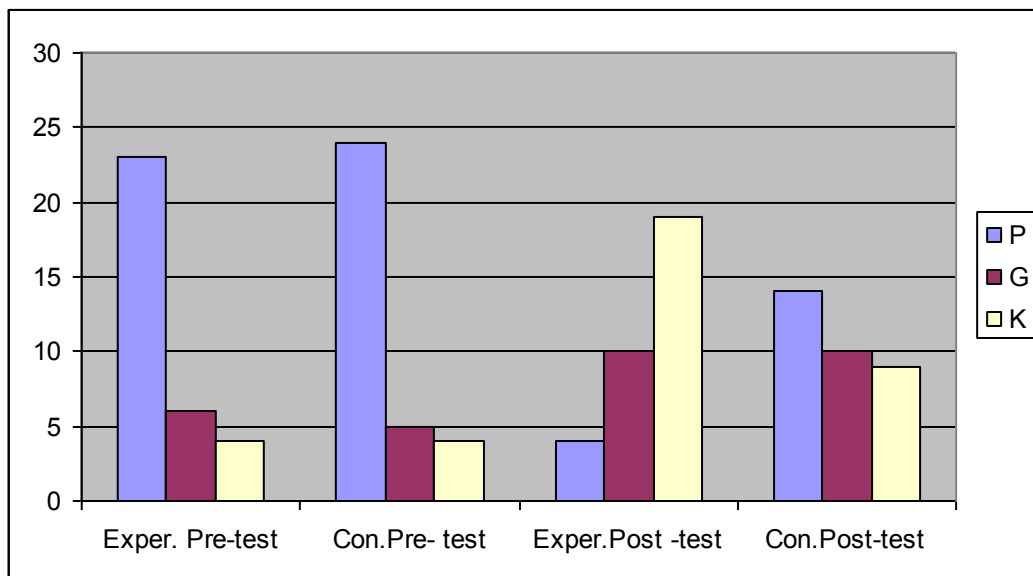


Figure 1. Evaluation in the first grade

While in the experimental group (P) type counting is 12,1, in the control group it is 42,4. It is significant that the difference between the groups is more than three folds. While the students who can use (K) type counting is 57.6 in the experimental group, the students who can use (K) type counting in the control group is 27,3. The significance of the results

($\chi^2= 9,127, p=.010$) is related to (P) type and (K) type counting, because the results of (G) type counting is the same for each group.

Discussion and conclusion

At the end of the first class, the students in both classes were able to memorize the sequence of numbers up to 100 and count forward by ones accurately. This result is parallel to the results of the previous research done on this subject (Baroody 1999; Carr 1995; Gelman 1993). Therefore, it can be said that since all students were able to count to 100 accurately without using their fingers showed that the method applied in the study was successful.

Since children start counting before going to primary school (e.g., Baroody 1987; Fuson 1988; Hunting 1999; Irwin 1996), many children come to school with a habit of using their fingers for counting. This study aimed to enable students to break their habits, such as finger counting acquired before starting school. In order to reach this goal, it was important that counting activities should be “short, frequent, and continuous”; that is, they should be done for short periods of time in most lessons throughout the semester. Furthermore, when counting is associated with daily life, students become more interested in counting. Games and competitions including counting also increase students’ interest. As Mar Garcia (2009) found in a study that students’ numeric competencies enhanced when working with real life problems.

In the first grade, significant differences were observed between control and experimental groups in terms of counting concrete and semi-concrete objects. The rate of students’ using (P) type counting in the control group was 42,4, whereas the rate of students’ using (K) type counting was 57,6. The difference between the groups in terms of counting style was significant. Moreover, this difference observed at the end of the first grade resulted from the application of remedial program in the experimental group. According to these results, it can be said that finger counting is a habit that can be broken. Owing to the remedial program and the age of the participants, the researchers didn’t encounter counting difficulties mentioned in the previous studies (Fuson 1988; Gelman 1993; Stefan, 2007).

Applying the remedial program enabled the students in the experimental group to use (K) type counting instead of (P) and (G) type counting. In the counting competitions done between individuals or groups in the two groups or between the two groups, the students who used (K) type counting finished counting more quickly than those who used (G) type counting. The students who observed that the students who used (K) type counting were more advantageous tried to use that kind of counting.

According to the findings obtained in this study, it was observed that students' habit of using fingers as a tool to match while counting was mostly broken. That finger counting was a breakable habit indicates that other wrong habits related to counting which are acquired informally can also be broken. The researcher believes that the research regarding the influence of meaningful counting on problem solving process (Clements 1999a; Pepper & Hunting 1998; Sarama & Clements 2004; Thomas, Mulligan, & Goldin 2002) will be significantly affected by the results of this study. Moreover, the researcher believes that this study will positively influence students' ability to do calculations in their minds and guess correctly in a short time, which were recommended in NCTM (2000) and Van de Walle (1994). Actually, the findings of this experimental study were not surprising for the researcher. However, this study constitutes the first stage of a larger project aiming at removing the failure of calculation skills of pupils related to one and two-digits arithmetical activities. For example, if the students calculate $13+5$ or $18-5$, their fingers will not be enough when they use them, and they may be unsuccessful at the end of this activity. So, this experimental study clarified that when the students are taught without fingers in basic arithmetical calculations, their abstract thinking and problem solving skills will also be developed easily. Their failure in these basic calculations will be decreased through this approach and the calculations will be done in shorter time by the students.

This study can also be applied to any classroom environment. The remedial program implemented in the study may provide a basis for future studies in this field and it can easily be added to the primary school mathematics curriculum. For example, specific units about counting can be introduced in mathematics syllabus of the first three grades of primary schools. In the specific units, counting can be associated with the problems encountered in daily life. It is believed that better results in terms of improving students' counting skills can be achieved if the content of the remedial program is expanded and appropriate environment for the application of the program is provided.

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