Pedagogical knowledge of numbers and operations: an international comparison

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Abstract

Introduction. In this paper, Spanish future primary teachers' pedagogical knowledge of numbers and operations, as revealed through their results in the TEDS-M (*Teacher Education and Development Study in Mathematics*), is compared with that of two other groups. The first group are the future primary teachers of other OECD countries that also participated in the study: Norway, Germany, Chile and Poland. The second group are the future primary teachers of participating countries where preservice teachers receive training similar to Spain's teacher training, namely, China-Taipei, Singapore, United States, Philippines and Switzerland.

Method. To meet this objective, data was analyzed and parameters were calculated based on categories that characterize the pedagogical knowledge required to correctly answer the questions in this conceptual domain, and using response assessment criteria.

Results. We found that Spanish prospective teachers have lower results, in general, than those of the other OECD countries and of countries in the group with similar training programs. The Spanish scores are lower than Norwegian, Swiss and Singaporean scores in all aspects considered. For most categories, Spain obtained higher scores than Philippines and Chile, and similar scores to the U.S.A.

Discussion: When comparing the results from the OECD countries that participated in the two international studies, TEDS-M and TIMSS 2011 (Spain, Poland, USA, Norway, Germany and Chile), we find that their relative positions are maintained in the ranking of assessed mathematical knowledge of numbers and operations, whether in preservice teachers or in primary students. Spanish results in the TIMSS are just above Poland and Chile and below the rest. The results obtained in this study may be useful in the current syllabus design process for subjects in the Elementary Teacher Education degree.

Keywords: Mathematics Pedagogy, numbers, pedagogical content knowledge, pre-service teacher education, primary education, TEDS-M.

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Conocimiento didáctico sobre números y operaciones: una comparación internacional

Resumen

Introducción. En este trabajo se presentan los resultados de comparar el conocimiento didáctico sobre números y operaciones manifestado por los profesores de primaria españoles en formación en el estudio TEDS-M *(Teacher Education and Development Study in Mathematics)* con el conocimiento manifestado, tanto por los futuros profesores de los países de la OCDE que participaron en el estudio —Noruega, Alemania, Chile, España y Polonia—, como por los futuros profesores de los países participantes receptores de un programa de formación inicial similar al de España —China-Taipéi, Singapur, Estados Unidos, Filipinas y Suiza—.

Método. Para lograr el objetivo propuesto se analizaron los datos y calcularon los parámetros, con base en categorías, que caracterizan el conocimiento didáctico requerido para responder correctamente las preguntas de este dominio conceptual y a partir de criterios de valoración de las respuestas de los futuros profesores.

Resultados. Los futuros maestros presentan, en general, resultados inferiores al del resto de países de la OCDE y al de los países de su grupo de programa de formación. Los resultados españoles son inferiores en todos los aspectos considerados a los resultados de Noruega, Suiza y Singapur. Para la mayoría de las categorías, los resultados españoles son superiores a Filipinas y Chile y próximos a los de EE.UU.

Discusión. Si comparamos los resultados de los países de la OCDE que han participado en los dos estudios internacionales, TEDS-M y TIMSS 2011, —España, Polonia, Estados Unidos, Noruega, Alemania y Chile— podemos comprobar que se conserva la posición relativa de los países en el ranking de rendimientos tanto de profesores en formación como de alumnos de primaria cuando se evalúa su conocimiento matemático sobre números y operaciones. Los resultados españoles en TIMSS son solo superiores a Polonia y Chile e inferiores al resto. Los resultados obtenidos en este trabajo pueden ser de utilidad en el proceso actual de diseño de las asignaturas de Grado de Maestro de Primaria.

Palabras Clave: conocimiento didáctico del contenido, Didáctica de la Matemática, formación inicial de profesores, contenido Números, educación primaria, TEDS-M.

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Introduction

Investigating the initial training of primary school mathematics teachers in Spain has become a topic of current interest due to the publication of Spain's primary and secondary school results in international assessment reports such as the 2012 PISA study (*Program for International Student Assessment*) (OECD, 2013; INEE, 2013*a*; INEE, 2013*b*), TIMSS 2011 (*Trends in Mathematics and Science Study)* (Martin & Mullis, 2013; INEE, 2012*a*; INEE, 2012*b*), and TEDS-M 2008 *(Teacher Education and Development Study in Mathematics)* (Tatto, Sharon, Senk, Ingvarson & Rowley, 2012; INEE, 2012*c*; INEE, 2012*d*). These studies concur and underscore the ongoing interest shown by the Spanish educational administration in recent years in reforming study programs for the initial training of primary education teachers (ANECA, 2005; MEC, 2007*a*). These studies demonstrate sustained attention to the evaluation of education, termed *evaluative culture*, which involves identifying the strengths and weaknesses of school mathematics training and the mathematical and pedagogical training of teachers in Spain, as well as to comparison of Spain with other countries.

The goal of this study is to describe and characterize the pedagogical knowledge of numbers and operations shown by Spanish preservice math teachers according to the TEDS-M study and to compare this knowledge with that of preservice teachers in some of the other countries that participated in this study. Specifically, we compare Spain's results to those of other TEDS-M participating countries who have preservice training programs similar to Spain's, and to those that are usually considered to be close to Spain's socioeconomic and cultural environment within the OECD. To achieve this goal, we begin by describing the TEDS-M study and its participants in order to develop a deeper understanding of the international results in the content domain of numbers and operations. Next, we describe how the TEDS-M study characterizes and assesses knowledge of teaching school mathematics, in order to then describe and characterize the knowledge exhibited by preservice teachers in the domain of numbers and operations. We then present the international results on pedagogical content knowledge and interpret them, making the appropriate comparisons, and conclude by showing the relevance of the results to current syllabus design in the new degree in Primary Education.

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The TEDS-M study

The TEDS-M study was carried out in 2006-2010 with 17 participating countries, and an international report was published in 2012 (Tatto et al., 2012). The study responded to the international interest in the initial training of preservice mathematics teachers in compulsory education, both primary and secondary. It was sponsored by the International Association for the Evaluation of Educational Achievement (IEA) and is based on the assumption that one important factor in explaining the differences in the capabilities, knowledge, and attitudes of primary and secondary school students in compulsory education, as shown by international studies (such as PISA and TIMSS), is the variation in approaches to initial training of mathematics teachers (Tatto et al., 2012).

One of Spain's goals in participating in this study was to evaluate the initial training of mathematics teachers and mathematics pedagogy and to obtain information for describing the mathematical and pedagogical content knowledge of preservice teachers when they finished their studies. In Spain, the TEDS-M study was coordinated at the national level by the Secretary of State for Education and Advanced Vocational Training, of the Ministry of Education, through the Institute for Evaluation. National coordination of the research was directed by Professor Luis Rico of the University of Granada (INEE, 2012*c*, p.131).

Given the diversity of training programs, and to facilitate international comparisons, the TEDS-M team established distinctive characteristics to identify the initial training programs for primary school teachers from the participating countries, classifying them into four groups. Spain was assigned to Group 2, along with Chinese Taipei, Singapore, Switzerland, the United States, and the Philippines, countries that are similar in that they offer generalist programs to prepare preservice teachers to teach students up to 12 years of age (INEE, 2012*c*, p. 24).

The implications of the TEDS-M study are shown in secondary studies that are being carried out. The issues that these studies address, in relation to initial training of primary school teachers, include: the relationship between the grades obtained in secondary school and results in the TEDS-M study (Montalvo & Gorgels, 2013); the influence of universities on the results of preservice teachers, through a comparative study of Spain and the U.S. (Cebolla-Boado and Garrido-Medina, 2013); the role of Spain's practicum in teacher training (Egido & López, 2013); the beliefs preservice teachers hold about the nature of mathematics

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(Felbrich, Kaiser & Schmotz, 2012), etc. Studies are also beginning to be published about the mathematical and pedagogical content knowledge shown in the TEDS-M. These studies make international comparisons based on the results. One example is the study by Blömeke, Suhl and Döhrmann (2013), who attempt to find relationships between training programs and the items answered correctly. Our study follows this line of research.

Choice of the content domain of numbers and operations

We chose the content domain of numbers and operations for this study for several reasons. One is the importance of numeracy on an international level, currently recognized in several studies (e.g., Hardy, 2014). In the case of Spain, this importance can be seen in the Spanish primary school curriculum, where a unit on numbers and operations is found in several grades of primary school. Primary education seeks to achieve effective numeracy, understood as the capability of successfully handling situations involving numbers and their relationships, enabling one to obtain effective information, either directly or by comparison, estimation, or mental or written calculation (MEC, 2007b, p. 31555). Furthermore, the content of the numbers unit is included in all Spanish institutions of initial teacher training, and its sections form part of the common curriculum of these institutions (Rico, Gómez and Cañadas, 2014).

On the other hand, the international TEDS-M report (Tatto et al., 2012, p. 143), like the national report (INEE, 2012*c*, p. 85), presents Spain's results in mathematical and pedagogical knowledge of school mathematics in global terms. These results indicate only the overall result for Spanish preservice teachers in mathematical knowledge (481) and pedagogical content knowledge (492), over an international mean of 500. There is thus a pressing need to describe and characterize the knowledge that Spanish preservice primary school teachers have about numbers and how to teach them, and to situate this knowledge on an international level.

Pedagogical knowledge of numbers and operations

Following the ideas of Shulman (1987), the TEDS-M study considered that mathematical knowledge for teaching has two components —knowledge of mathematical content and pedagogical knowledge of mathematical content— and a questionnaire was designed to evaluate preservice teachers' mastery of each kind of knowledge separately. At the same time, based on the conceptual framework of TIMSS 2007, TEDS-M organized the items into four content domains: numbers and operations, geometry and measurement, algebra and functions, and data and chance (Mullis, Martin, Ruddock, O'Sullivan, Arora & Erberber, 2007).

The TEDS-M study assessed preservice primary school teachers' pedagogical knowledge of school mathematics based on 22 items about how to tackle various tasks and problems in school mathematics. Some items in the questionnaire came from studies such as *Learning Mathematics for Teaching Projects* (Hill & Ball, 2004) and *Mathematics Teaching for the 21st Century Project* (Schmidt, Blömeke & Tatto, 2011). The other items were developed by the TEDS-M team.

This study focuses on the 8 items that assess pedagogical knowledge of the mathematics content of numbers and operations. In previous studies, we concluded that "TEDS-M did not propose a complete category structure for establishing a definition of the pedagogical knowledge of school mathematics that the items address" (Gutiérrez-Gutiérrez, Gómez and Rico, 2014, p. 283). We therefore established a set of categories with which to characterize the pedagogical knowledge required to correctly answer the items in this content domain. The categories, presented below, emerged from analyzing the texts of the item statements and from the item scoring guides, and are derived from the categories established in the model of didactic analysis (Rico, 2013).

- Identify and distinguish variables that affect the difficulty of a problem (IDD).
- Recognize and describe the errors students make in doing an activity or their incorrect conceptions of a specific concept or procedure (RE).
- Represent mathematics concepts and procedures in alternative ways in the teaching process (REP).
- Recognize the mathematics concepts and procedures involved in teaching a school mathematics topic and the relationships between them (MCP).

The first two categories refer to the preservice teacher's knowledge of the limitations in students' learning —the difficulties related to mathematical knowledge or errors that the student may commit in approaching mathematical tasks. The other two categories indicate knowledge of school mathematics content and its representations, and the conceptual structure of this content.

Study Objective

As stated above, the goal of this study is to describe and characterize the pedagogical knowledge of numbers and operations exhibited by Spanish preservice primary school teachers who participated in the TEDS-M study and to compare this, on the one hand, with the knowledge demonstrated by preservice teachers in the OECD countries that participated in the study (Norway, Germany, Chile, and Poland); and, on the other, with that of the preservice teachers from participating countries belonging to the same group as Spain —Chinese Taipei, Singapore, the U.S., the Philippines, and Switzerland— labelled Group 2. Ours is thus a comparative study based on data from the TEDS-M study questionnaire.

Method

Participants

The fieldwork followed the design established by the TEDS-M (Tatto, Schwille, Senk, Ingvarson, Peck & Rowley, 2008). For each country, we chose representative samples from the institutions that provided training to the target population of preservice teachers (those who were preparing to teach primary school mathematics and who were in their last year of training). The selection was performed using probability sampling proportional to the size of the institution, where size was defined as number of students in the Primary Education program who were in their last year of study in 2008. At each institution, we chose a random sample of 30 preservice teachers, or the full population if the program size was below 30. A total of 483 institutions participated, with their respective training programs, including 13,871 preservice primary school teachers from these institutions (Tatto et al., 2012).

The sample of Spanish institutions was composed of 50 from a total of 73 institutions that provided initial training to preservice primary school teachers. Two preservice teachers declined the invitation to participate. A total of 1093 Spanish preservice teachers in the training program established by Royal Decree 1440/1991 (MEC, 1991) (forerunner of the current undergraduate degree), answered the questionnaire (INEE, 2012*c*).

Information sources

This study is based on the information provided by the TEDS-M questionnaire, the item scoring guides designed for scoring open response items, and the responses of the preservice teachers who completed this questionnaire.

Procedure

To achieve the proposed objective, we followed a three-step procedure:

- Selection, analysis, and classification of the items in the domain of numbers and operations
- Establishment of evaluation criteria for the preservice teachers' responses and calculation of parameters
- Summary of the data in terms of measurement and comparison of these measurements

1) Selection, analysis, and classification of the items for the domain of numbers and operations

In previous studies, we presented the process of identifying, analyzing, and classifying the items (Gutiérrez, 2012; Gutiérrez-Gutiérrez, Gómez & Rico, 2014). The results of this process are shown in Table 1, which includes the 8 items used for assessing the pedagogical knowledge of numbers and operations content in the TEDS-M study. For each item, the table shows the specific mathematical content, the specific pedagogical content knowledge required to answer the item correctly (identified by the categories established), and the type of answer. As is common in this kind of study, the questionnaire items took several forms: simple multiple choice —in the case of the numbers and operations domain, where 4 response options were provided for each item, and only one was correct; complex multiple choice —each response option for the problem (4) was presented as a heading with two more response options; and open response. For the last two types, the TEDS-M study included the corresponding item scoring guides and coding procedure.

***Table 1.* Items regarding pedagogical knowledge in the numbers subdomain.**

Item number	Content type	Pedagogical knowledge	Response type
1	Arithmetic problems	IDD	OR
2	Direct proportionality	IDD	OR
3	Decimal numbers	RE	OR
4	Representation of decimal numbers	REP	OR
5	Ordering fractions	MCP	M
6	Graphic meaning of the division of fractions	REP	CM
7	Operations with mixed numbers	RE	M
8	Algorithms for subtraction	MCP	M

Note: *OR* = open response; *M* = multiple choice; *CM* = complex multiple choice
Source: Gutiérrez-Gutiérrez, Gómez and Rico (2014, p. 284)

Figure 1 shows an example of an item taken from the TEDS-M questionnaire on pedagogical knowledge of operations with decimal numbers. This item is composed of two parts, which are classified and corrected independently. According to our classification of the items, Part (a) was identified as Item 3 and Part (b) as Item 4.

[Jeremy] notices that when he enters 0.2×6 into a calculator his answer is smaller than 6, and when he enters 6÷0.2 he gets a number greater than 6.
He is puzzled by this, and asks his teacher for a new calculator!

(a) What is [Jeremy's] most likely misconception?

(b) Draw a visual representation that the teacher could use to model 0.2× 6 to help [Jeremy] understand WHY the answer is what it is.

***Figure 1*. Item on decimal numbers**

We now offer a concise presentation of the knowledge preservice teachers should demonstrate in order to correctly answer each part according to the item scoring guides. We propose some examples that show the presence of the knowledge assessed in the literature on the initial teacher training, specifically through certain documents from Spain. Finally, we will summarize the pedagogical knowledge of the mathematical content that is assessed through each item.

Part (a) evaluates preservice primary teachers' knowledge concerning limitations in students' learning. The item scoring guide establishes that a correct answer only requires the preservice teachers to suggest that a student with an incorrect conception would believe that multiplication of decimal numbers always gives a result greater than the numbers proposed, and that division always gives a result smaller than the dividend. There is extensive literature on the errors that students make when working with decimal numbers. For example, Castro (2001) includes the incorrect conception that is assessed in this problem, referring to the case where children believe that "multiplying is always increasing and dividing is always decreasing. Children persist in this misconception with positive decimals less than one" (p. 335).

Part (b) evaluates one aspect of the preservice teachers' pedagogical knowledge of mathematical content. The preservice teacher was asked to represent the product 0.2 x 6 graphically so that the student could understand the foregoing conceptual error about multiplying with decimal numbers. That is, Part (b) evaluates the preservice teacher's general knowledge of graphic representation of decimal numbers and specifically of multiplying a decimal number between 0 and 1 by a natural number. According to the item scoring guide, to answer this part correctly, the preservice teacher must know that "decimal numbers provide an extension of the decimal number system: natural numbers are used to represent whole quantities; decimal numbers are also used to express different parts of the unit" (Castro, 2001, p. 320). Figure 2 shows two representations proposed as examples of correct responses, as they appear in the item scoring guide. The item scoring guides do not consider the implicit use of the commutative property of the product, that is, the representations that correspond to the product 6 x 0.2 and not the product 0.2 x 6.

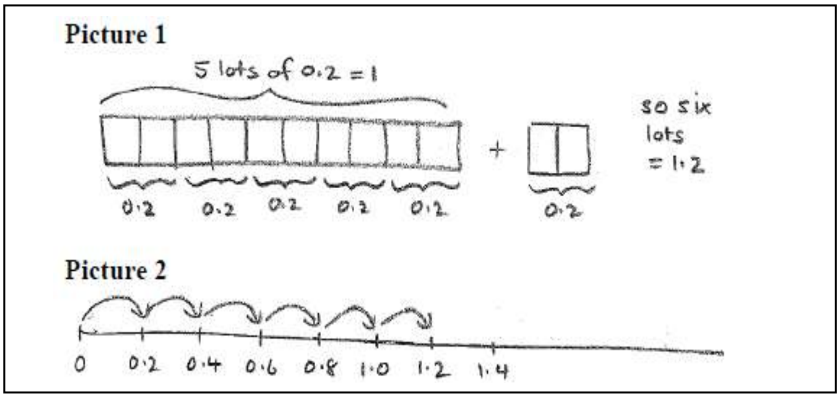

***Figure 2*. Examples of correct answers for Part (b)**

Source: Original figure from item scoring guide designed by TEDS-M

In the first figure (Picture 1), the preservice teacher shows that he/she recognizes the graphic representation of the decimal numbers as subareas of a region taken as a unit. In this case, the example of 0.2 is represented as 2/10. In the second figure (Picture 2), the preservice teacher considers the decimals as points on the number line. Both representations identify that 0.2 x 5 corresponds to the unit.

With this result, we conclude that it is not enough for preservice primary school teachers to have mathematical knowledge of decimal numbers and operations to answer the item proposed in Figure 1 correctly. They must also have pedagogical knowledge of the teaching of decimal numbers, their graphic representation, the most frequent errors students make, and the application of graphic representations in teaching. Parts (a) and (b) give information on the preservice teachers' pedagogical knowledge of decimal numbers, but each part provides information on a specific aspect of this pedagogical content knowledge.

2) Criteria for evaluating the preservice teachers' answers

The study participants' answers were coded according to the item scoring guide developed by TEDS-M (Tatto et al., 2008). The point system for each open response item is a two-digit code. The first digit, 2 or 1, indicates a correct or partially correct response, respectively, and also signifies the value assigned to the answer. Incorrect answers were coded using 7 as the first digit. The second digit refers to different focuses used by the preservice teachers in their answers.

In our case, which is similar to the calculation of general results in the TEDS-M (Tatto et al., 2008, p. 42), we evaluated the preservice teachers' answers to each item taking into account the different types of response.

- If the item is multiple choice, we assigned the value 1 if the preservice teacher answered correctly and 0 if he/she answered incorrectly.
- If the item is open response, we assigned the value 1 if the preservice teacher answered correctly, 0 if he or she answered incorrectly, and 0.5 if the answer was partly correct. For this last case, based on the analysis of the item scoring guides performed by Gutiérrez (2012) and Gutiérrez-Gutiérrez, Rico and Gómez (2014), we believe that the knowledge shown in the different kinds of partially correct answers merits the same score.
- If the item was complex multiple choice, we assigned the value 1 if the preservice teacher answered all response options correctly, 0.5 if he or she answered all but one correctly, and 0 for other cases. Tatto et al. (2008, p. 76) provide an example of the evaluation analysis for correcting an item of this type.
- Illegible, crossed-out, or blank answers were considered lost values.

3) Summary of the data

The following are the parameters we used to perform comparisons for the set of items that assessed pedagogical knowledge of numbers and for each of the established categories of pedagogical knowledge.

- Mean and standard deviation of each country.
- Mean and standard deviation of total number of countries participating in TEDS-M.

- Mean and standard deviation of the set of participating countries that form part of the OECD.
- Mean and standard deviation of the set of participating countries that form part of Group 2.

We present the parameters in two tables. Table 2 shows the mean and standard deviation of Spain —for the total of items that assessed pedagogical knowledge of numbers and operations (PK) and for each category— relative to the mean and standard deviation of the set of participating countries and the mean and standard deviation of the set of OECD countries (including Spain) that participated in TEDS-M. Table 3 presents the same information for Spain and internationally, but in this case relative to the mean and standard deviation of countries in Group 2. In both cases, we indicate the countries that have results higher and lower than Spain for the set of items considered and for each category of pedagogical content knowledge. The table does not include the countries whose means are not statistically higher or lower than Spain's with a significance level of 0.05 in each case. For countries with more than one training program, we identify the specific program.

For example, Figures 3, 4, and 5 in the appendix show the mean and standard deviation of each participating country for the set of items that assess pedagogical knowledge in the domain of numbers, as well as the categories DIF and RE. In these figures, we represent the corresponding confidence interval for each country by a segment situated within its real mean point value at a confidence level of 95%. The greater or smaller interval depends on the size of the sample and the dispersion of the results. In each figure, we identify the international mean and standard deviation around it with a red border.

These and the other results that appear in this study should be interpreted taking into account the limitations in the participation of Chile, the U.S., Norway, Russia, Poland, and Switzerland (INEE, 2012*c*, p.136).

Table 2. **Summary comparing Spain's results to those of OECD countries**

CAT	SPAIN		International		OECD COUNTRIES			
	Me	SD	Me	SD	Me	SD	Higher	Lower Me
PK	0.39	0.01	0.44	0.00	0.43	0.00	Norway Switzerland Germany Poland Sp	Poland Gen
IDD	0.70	0.02	0.72	0.01	0.63	0.00	Norway Switzerland Germany United States Poland Gen	Poland Sp
RE	0.23	0.02	0.31	0.01	0.37	0.01	Poland Norway Germany Switzerland	Chile
REP	0.24	0.02	0.30	0.01	0.31	0.01	Germany Switzerland Norway Poland	Chile
MCP	0.33	0.01	0.36	0.01	0.35	0.01	USA Gen Norway+ Switzerland Poland Sp	Poland Gen USA Gen Chile

Nota: *CAT* = categories; *Me*: mean; *SD*: standard deviation; *Norway+*: general teacher education with mathematics option; *Gen* = generalist program; *Sp* = specialist program.

Table 3. **Summary comparing Spain's results to those of Group 2 countries**

CAT	SPAIN		INTERNATIONAL		GROUP 2			
	Me	SD	Me	SD	Me	SD	Higher Me	Lower Me
PK	0.39	0.01	0.44	0.00	0.44	0.01	Chinese Taipei Singapore Switzerland	Philippines
IDD	0.70	0.02	0.72	0.01	0.69	0.01	Singapore U.S. Switzerland	Chinese Taipei Philippines
RE	0.23	0.02	0.31	0.01	0.32	0.01	Singapore Chinese Taipei Switzerland	Philippines
REP	0.24	0.02	0.30	0.01	0.33	0.01	Chinese Taipei Singapore Switzerland United States	Philippines
MCP	0.33	0.01	0.36	0.01	0.39	0.01	Chinese Taipei Switzerland Singapore	United States Philippines

Nota: *CAT* = categories; *Me*: mean; *SD*: standard deviation; *Gen* = generalist program; *Sp* = specialist program.

We now indicate the most salient issues that emerge from interpreting the information in Tables 2 and 3 and the figures included in the appendix.

Results

The information provided in tables and figures enables us to determine the following results.

Results for Spain relative to OECD countries

Spain's mean is only higher than the OECD mean in the category DIF. It is lower in all other categories. Spain obtains results lower than Norway and Switzerland in all aspects considered. Spain's results are also lower than Germany's for both German programs, except in the category CPM. Poland's specialist program has higher results than Spain in all categories. The Polish mean for the generalist program is lower than Spain's for the set of items that evaluate the domain of numbers except in the category CPM. Spain's results are higher than Chile's in all categories except DIF and CD, where there are no significant differences. The country whose results are closest to Spain's is the U.S., with its two programs.

The best results for the group of OECD countries for the set of items on the domain of numbers are obtained by Norway (ALU+) (0.59) and Germany and Poland (0.58). Their difference from the Spanish mean is 0.20 and 0.19, respectively. The difference in scores between the OECD countries with the highest and lowest scores in the domain of numbers and operations is 0.22 for the set of countries.

The small variation in Spain's results for each issue studied in this paper is striking. The OECD countries with the greatest dispersion of results are the specialist programs in Germany and the U.S. and the generalist program in Poland. This variation is related to the size of the respective samples, which range from 85 to 135 for the cases mentioned, while Spain's sample is composed of 1093 preservice teachers.

Results for Spain relative to countries in Group 2

Spain's mean is lower than the Group 2 mean except in the category DIF, where there is no significant difference. Spain's results are lower than those of Switzerland and Singapore in all aspects considered. It is worth noting that Spain's mean in the category DIF is much higher than that of Chinese Taipei, although lower in all other categories. Spain's results are higher than those of the Philippines in all categories. There are no significant differences relative to the U.S. except in the category CPM, where Spain's average is higher than that of the

U.S. The results for the countries in Group 2 generally show slight variation. Only Singapore and the Philippines occasionally score higher than the standard deviation of 0.02, whereas Spain always scores lower than or equal to this value.

Within the set of countries in Group 2, Singapore and Switzerland have the best results in all categories. Chinese Taipei is also among the best, except in the category DIF. For the set of items that comprise the domain of numbers and operations, the highest score is obtained by Chinese Taipei (0.55), and Spain differs from it by 0.16.

Results for Spain relative to the set of participating countries

For the set of all categories, the mean of Spain's results is always lower, and it is closer to the international mean of the set of participating countries than to the mean of Group 2, except in the category DIF, where Spain obtains its best results.

Spain obtains the lowest results for categories RE and REP; the same occurs for the set of Group 2 countries and the set of all participating countries. Among the OECD countries, however, Spain's lowest results are in categories REP and CPM. The country with the highest mean is Singapore, for its specialist program, differing from Spain's mean by 0.21.

Given the quality of Spain's sample, the Spanish results present little variation for the set of all aspects considered. Great variation, however, can be seen in countries whose sample was smaller than 140 —as in the case of specialist programs from Germany and the U.S., and generalist programs from Botswana, Switzerland, and Norway (ALU+).

Discussion

This study contributes to a deeper understanding of Spanish preservice teachers' pedagogical knowledge of numbers and operations, relative to the other countries participating in the study. The results presented here identify teacher training deficiencies that concur with the place of Mathematics Pedagogy in initial teacher training programs for the period 1991-2010, during which "Spain was situated in the lowest positions relative to other countries, both on the global level and relative to the countries in its group, for proportion of topics studied in Mathematics Pedagogy" (Rico, Gómez and Cañadas, 2014). The community of teachers and experts in this discipline already perceived that the training was patently insufficient for exercising the role of a teacher responsible for teaching mathematics to children in primary education. There is “a deep contradiction between the importance citizens assign to mathematical competence and the training of the professionals responsible for mathematics education in the decisive first levels of schooling” (Rico, Sierra & Castro, 2002, p. 43).

The quality of the Spanish sample attests to the interest in analyzing the results obtained by Spanish preservice teachers (INEE, 2012C, p.17). In making international comparisons, however, we should take into account the limitations of the participation by Chile, the U.S., Norway, Russia, Poland, and Switzerland (INEE, 2012*c*, p.136).

Spain obtains the lowest mean (0.23) in the category of recognizing the errors committed by students. We stress this fact for initial teacher training, since the LOMCE *(Organic Law for Quality Improvement in Education*, which takes effect in 2015-2016*)* sets an objective of early detection of learning disabilities, as part of personalized attention (MECD, 2013*b*). At the same time, we would point out that knowledge of errors as assessed in the TEDS-M questionnaire is included in the manuals for training Spanish primary school teachers, such as Castro (2001). Spain also obtains a low mean (0.24) in the category of representing mathematical concepts and procedures in alternative ways during the teaching process. This fact is also interesting since “representations play a fundamental role in mathematical thinking, encourage comprehension of mathematics concepts, and stimulate the development of flexible, versatile thinking in problem solving” (Villegas, Castro & Gutiérrez, 2009, p. 280).

We have seen from certain items on the TEDS-M questionnaire that a preservice teacher’s mathematical knowledge may not be sufficient to recognize the errors that a student may commit or to understand graphic representations for teaching a specific topic. This fact underscores the need to change not only the preservice teacher’s mathematical knowledge but also their knowledge of the teaching and learning of school mathematics.

If, on the other hand, we compare the results from the OECD countries that participated in the two international studies, TEDS-M and TIMSS 2011 (Spain, Poland, the U.S., Norway, Germany, and Chile), we can confirm that Spain holds the same rank for both preservice teachers and primary school students in terms of their assessed mathematical knowledge of numbers and operations. Spain’s results in TIMSS are only higher than those of Poland and Chile, and they are lower than those of the other countries (INEE, 2013*a*, p. 62).

Although it was not part of the goal of this study, we found that the best results by far, for all countries, were obtained on the items that assessed preservice teachers' knowledge of the variables that affect the difficulty of problems. As we can confirm from the tables and figures in the appendix, the best results in the four categories of pedagogical knowledge (and also the greatest dispersions) were seen in Chinese Taipei, Norway (ALU+), and the countries with specialist training programs (Germany, Poland, and the two Singapore programs), with few exceptions. At the same time, if we examine Tables 2 and 3 and the graphs in the appendix figures, we find great variation in the results, not only as a function of the programs but also of the type of mathematics content and category of pedagogical content knowledge being assessed in the items.

Measures taken by the different countries to improve teachers' professional aptitude, as in the case of Finland, Poland, and Singapore, show shared concern for expanding and improving the level of education required to be a teacher. This should be the general objective of the reforms undertaken in Spain: "to improve the initial training our teachers receive in order to respond to the new demands of a changing society" (Esteve, 2006, p.19). Although the study is partial in terms of the domain considered and the categories that can be identified, these categories represent a cross-section of all domains (Gómez & Gutiérrez-Gutiérrez, 2014). The results obtained in this study can thus be useful in the current process of designing and implementing subjects within the new degree in Primary Education.

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Appendix

FIGURE 3. Pedagogical knowledge of number and operations

Program-Group	Country	Sample Size	Valid Data	Percent Missing Weighted	Mean	Standard Deviation
Group 1	Georgia	506	390	23.03	0.34	0.01
Generalist teacher (Students under 10 years of age)	Germany	907	717	23.83	0.47	0.02
	Poland	1,799	1413	21.25	0.37	0.01
	Russian Federatio	2,260	1800	20.61	0.45	0.02
	Switzerland	121	98	18.55	0.49	0.03
Group 2	Chinese-Taipei	923	742	19.74	0.55	0.01
Generalist teacher (Students under 12 years of age)	Philippines	592	473	19.32	0.29	0.02
	Singapore	262	204	22.39	0.53	0.02
	Spain	1,093	875	20.10	0.39	0.01
	Switzerland	815	648	20.45	0.50	0.01
	United States	951	750	43.77	0.40	0.01
Group 3	Botswana	86	65	24.49	0.33	0.02
Generalist teacher (Students under 14 years of age)	Chile	654	518	21.11	0.39	0.02
	Norway (ALU)	392	127	20.39	0.59	0.02
	Norway (ALU +)	159	312	20.53	0.49	0.01
Group 4	Germany	97	81	17.58	0.58	0.04
Primary Mathematics Specialists	Poland	574	243	18.47	0.58	0.02
	Malaysia	300	452	21.61	0.38	0.01
	Singapore	117	101	13.66	0.60	0.03
	Thailand	660	529	19.88	0.41	0.01
	United States	132	104	47.82	0.41	0.02

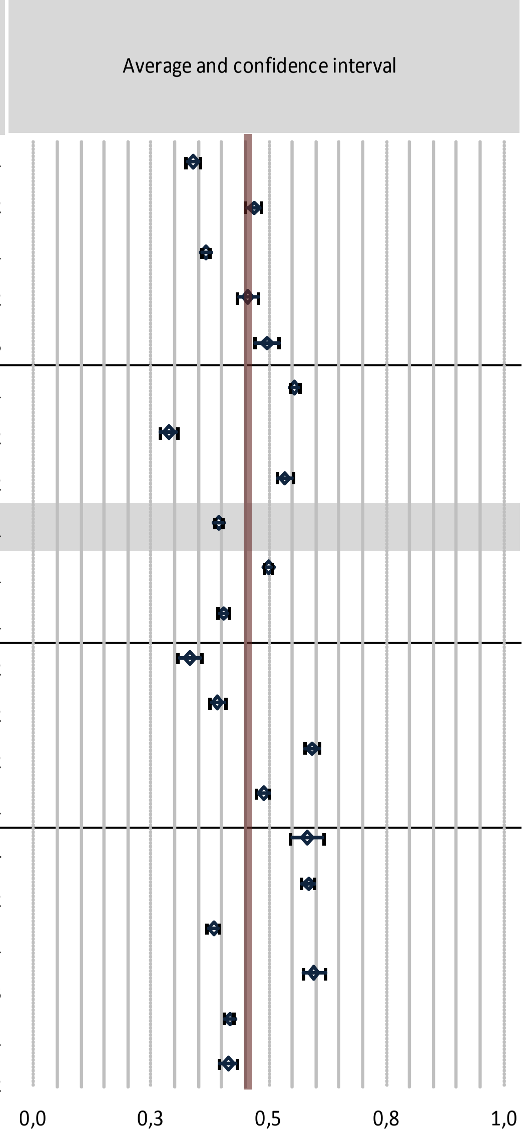
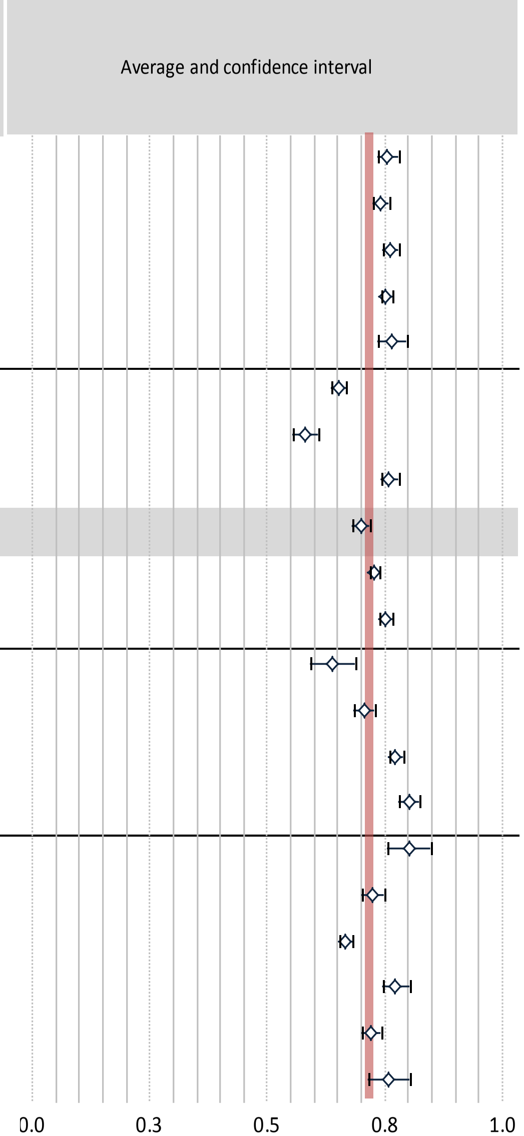


FIGURE 4. Identify variables that affect the difficulty of a problem

Program-Group	Country	Sample Size	Valid Data	Weighted Percent Missing	Mean	Standard Deviation
Group 1 Generalist teacher (Students under 10 years of age)	Georgia	506	238	53.22	0.75	0.02
	Germany	907	653	30.53	0.74	0.02
	Poland	1,799	1072	38.99	0.76	0.02
	Russian Federatior	2,260	1575	29.32	0.75	0.01
	Switzerland	121	95	21.63	0.76	0.03
Group 2 Generalist teacher (Students under 12 years of age)	Chinese Taipei	923	727	21.33	0.65	0.02
	Philippines	592	452	21.84	0.58	0.03
	Singapore	262	201	23.52	0.76	0.02
	Spain	1,093	837	23.69	0.70	0.02
	Switzerland	815	632	22.49	0.73	0.01
	United States	951	732	44.83	0.75	0.01
Group 3 Generalist teacher (Students under 14 years of age)	Botswana	86	60	30.89	0.64	0.05
	Chile	654	427	34.91	0.70	0.02
	Norway (ALU)	392	290	26.14	0.77	0.02
	Norway (ALU +)	159	124	22.32	0.80	0.02
Group 4 Primary School Mathematics Specialists	Germany	97	75	21.36	0.80	0.05
	Poland	574	228	23.81	0.72	0.02
	Malaysia	300	417	27.76	0.67	0.01
	Singapore	117	99	15.39	0.77	0.03
	Thailand	660	502	23.96	0.72	0.02
	United States	132	101	49.06	0.76	0.05



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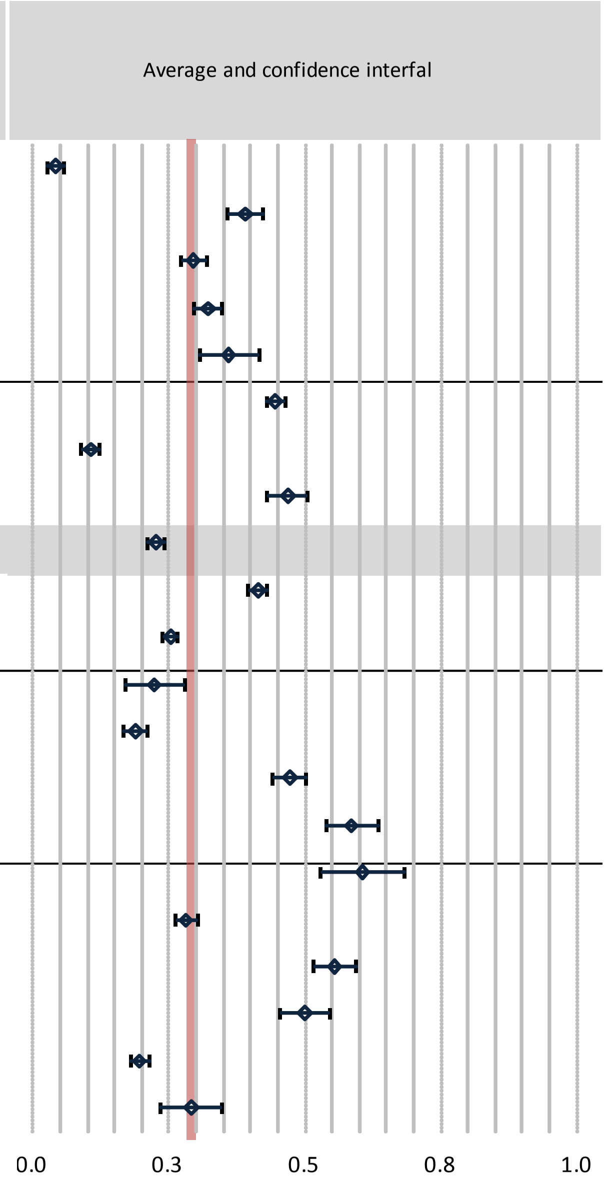
http://dx.doi.org/10.14204/ejrep.35.13136

FIGURE 5. Recognize the errors made by students

Program-Group	Country	Sample Size	Valid Data	Percent Missing Weighted	Mean	Standard Deviation
Group 1	Georgia	506	204	59.54	0.04	0.02
	Germany	907	592	36.96	0.39	0.03
Generalist teacher (Students under 10 of age)	Poland	1799	985	44.06	0.29	0.02
	Russian Federatior	2260	1499	34.25	0.32	0.03
	Switzerland	121	83	32.23	0.36	0.05
Group 2	Chinese-Taipei	923	730	21.10	0.44	0.02
	Philippines	592	447	25.21	0.10	0.02
Generalist teacher (Students under 12 years of age)	Singapore	262	201	23.54	0.47	0.04
	Spain	1093	768	30.96	0.23	0.02
	Switzerland	815	590	27.72	0.41	0.02
	United States	951	724	46.00	0.25	0.01
Group 3	Botswana	86	55	36.69	0.22	0.05
Generalist teacher (Students under 14 years of age)	Chile	654	336	48.61	0.19	0.02
	Norway (ALU)	392	276	31.10	0.47	0.03
	Norway (ALU +)	159	121	24.10	0.58	0.05
Group 4	Germany	97	69	26.43	0.61	0.08
	Poland	574	418	27.55	0.28	0.02
Primary Mathematics Specialists	Malaysia	300	226	23.29	0.55	0.04
	Singapore	117	98	16.21	0.50	0.05
	Thailand	660	485	26.44	0.20	0.02
	United States	132	98	49.48	0.29	0.06



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