Iterative scheme for the observation of a competitive Lotka-Volterra system

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Abstract

In this work, in terms of the model parameters, sufficient conditions are established to construct a sequence of approximate observers for a two-species competitive Lotka-Volterra system. This iterative approach makes it possible to localize the solution of the system, and reveal its long-term behaviour. The main results are also illustrated by numerical simulations.

Key words: iterative scheme; local observer; competitive Lotka-Volterra system.

1 Introduction

As a classical description of a simple population system, the first continuous-time deterministic model of a predator-prey interaction was proposed by Lotka (1925) and Volterra (1931). This model explained certain qualitative features of the behaviour of predator-prey systems that had been observed empirically (such as the periodic change of densities with a certain delay; or the increase and decrease in the time-mean density of predator and prey fish populations, respectively, due to harvesting activity.) In Volterra (1931) a first detailed stability analysis of a generalized n-species model was given for the basic classes of conservative and dissipative systems. Later on, the study was extended to non Lotka-Volterra type population systems. On the other hand, in a large number of papers, in-depth analysis of particular types Lotka-Volterra systems were given. For a collection of other classical works on the subject we refer the reader to Scudo and Ziegler (1978). For further references see e. g. Freedman (1980) and Yodzis (1989).

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In this work, the Lotka-Volterra model of two competing species will be considered. For some classical results on such systems see e.g. Freedman (1980). As a more recent development, in Tineo (1996) an iterative scheme has been proposed for the study of competitive systems, see also Gámez (1999), Tineo (2001) and Carreño et al. (2002). Now we shall apply a similar iterative scheme for the construction and analysis of approximate observers for the monitoring of the state process in a two-species competitive system.

Considering a two-species competitive Lotka-Volterra model with a partially monotonous observation, first we find sufficient conditions which make it possible to construct an observer for each member of the sequence of simple auxiliary observation systems, applying the result recalled from Sundarapandian (2002). For earlier applications of observers to systems of population biology see López et al. (2007a,b,c).

Finally, we shall present results and numerical calculations for a concrete two-species competitive system. The simulation results provide a "stripe" between two functions that is attractive for the original system in the sense that any positive solution of the latter tends to this stripe. This fact also makes it possible to make predictions on the long-term behaviour of the population system, such as coexistence (persistence) or extinction of one of the species.

2 Preliminaries

In this section we introduce some notation and recall some known concepts and results to be used in the paper.

2.1 Sub-and supersolutions

For the construction of the iterative scheme we shall use the following concepts and results. Let us consider system

$$x' = f(t, x), \tag{2.1}$$

where $f : W \to \mathbb{R}^n$, is a locally Lipchitzian and continuous function in x, with an open subset W of $\mathbb{R} \times \mathbb{R}^n$.

Definition 2.1 *A continuously differentiable function $u : [a, b] \to \mathbb{R}^n$ is called supersolution (respectively subsolution) of system (2.1) if $(t, u(t)) \in W$ and $u'(t) \geq f(t, u(t))$ (respectively $u'(t) \leq f(t, u(t))$) for all $t \in [a, b]$. (Here $\leq$ and $\geq$ mean inequalities by coordinates.)*

Now we recall a lemma of technical character for the scalar case.

Lemma 2.2 *(Gámez, 1999, Corollary 1.3.8). Let $u, v : [a, b] \to \mathbb{R}$ be continuously differentiable functions such that u (respectively v) is a subsolution (respectively supersolution) of (2.1) for $n = 1$. If $u(a) \leq v(a)$ then $u \leq v$.*

2.2 Construction of local observers

Given positive integers m, n, we suppose that the following functions

$$F : \mathbb{R}^n \to \mathbb{R}^n, \qquad H : \mathbb{R}^n \to \mathbb{R}^m$$

are $\mathcal{C}^1$ and for some $x^* \in \mathbb{R}^n$ we have that $F(x^*) = 0$ and $H(x^*) = 0$. We consider the following observation system

$$x' = F(x) \tag{2.2}$$

$$y = H(x) \tag{2.3}$$

where H is called observation function.

Definition 2.3 *A matrix $A \in \mathbb{R}^{n \times n}$ is said to be stable, if all its eigenvalues have negative real parts.*

In case of 2×2 matrix the general Routh-Hurwitz criterion (see e.g. Chen (2004)) reduces to the following simple conditions, given in terms of the characteristic polynomial coefficients.

The Routh-Hurwitz criterion for $n = 2$: Let $\lambda^2 + a_1 \lambda + a_2$ be the characteristic polynomial of $A \in \mathbb{R}^{2 \times 2}$. Then A is stable if and only if a_1, $a_2 > 0$.

Now, the construction of an observer system will be based on Sundarapandian (2002).

Definition 2.4 *A $\mathcal{C}^1$ dynamical system described by*

$$z' = G(z, y), \qquad (z \in \mathbb{R}^n) \tag{2.4}$$

is called a local asymptotic (respectively, exponential) observer for observation system (2.2)-(2.3) if the composite system (2.2)-(2.3),(2.4) satisfies the following two requirements.

i) If $x(0) = z(0)$, then $x(t) = z(t)$, for all $t \geq 0$.
ii) There exists a neighborhood V of equilibrium x^* such that for all $x(0), z(0) \in V$, the estimation error $z(t) - x(t)$ decays asymptotically (respectively, exponentially) to zero.

Theorem 2.5 *(Sundarapandian, 2002). Suppose that the observation system (2.2)-(2.3) is Lyapunov stable at equilibrium, and that there exists a matrix K such that matrix $A - KC$ is stable, where $A := F'(x^*)$ and $C := H'(x^*)$. Then dynamic system defined by*

$$z' = F(z) + K[y - H(z)] \tag{2.5}$$

is a local exponential observer for observation system (2.2)-(2.3).

Remark 2.6 *It is known (Sundarapandian, 2002) that under the condition*

$$rank[C|CA|CA^2|\dots|CA^{n-1}]^T = n$$

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for local observability, the existence of such an observer is guaranteed. The above theorem provides an efficient method to construct this observer.

3   Iterative scheme

We shall consider a so-called competitive Lotka-Volterra system

$$x_i' = x_i \left(a_i - \sum_{j=1}^{2} b_{ij} x_j \right) =: x_i f_i(x_1, x_2), \qquad i = 1, \ 2 \tag{3.1}$$

with $a_i, \ b_{ij} > 0$ for all $i, \ j = 1, \ 2$, satisfying condition,

$$a_1 b_{22} - b_{12} a_2 > 0 \ ; \ a_2 b_{11} - b_{21} a_1 > 0 \ ; \ b_{11} b_{22} - b_{21} b_{12} > 0. \tag{C1}$$

Let $h : \mathbb{R}^2 \to \mathbb{R}$ be a locally bounded function with partial derivatives $h_{x_i} > 0$ for $i = 1, \ 2$, defined and continuous in $\mathbb{R}^2$. Let us consider the following observation system

$$
\begin{aligned}
x_1' &= x_1(a_1 - b_{11}x_1 - b_{12}x_2) \\
x_2' &= x_2(a_2 - b_{21}x_1 - b_{22}x_2) \\
y &= h(x_1, x_2) - h(x_1^*, x_2^*).
\end{aligned}
\tag{3.2}
$$

A straightforward calculation shows that under condition $(C1)$, system (3.1) has an asymptotically stable positive equilibrium $x^* = (x_1^*, x_2^*)$ (see e.g. Freedman, 1980).

Our objective is to determine sufficient conditions, in order to find a sequence of observation systems, which provides a long-term upper and lower estimation of the solutions of system (3.1). To this end, let us introduce the following concepts and notation. Given a natural number N, let x^{*i} be the positive equilibrium of system

$$
\begin{aligned}
x_1' &= x_1 f_1(x_1, x_2^{*(i-1)}) \\
x_2' &= x_2 f_2(x_1^{*(i-1)}, x_2)
\end{aligned}
\quad i = 1, 2, \ldots, N
\tag{3.3}
$$

with $x^{*0} = (0, 0)$. For $i = 1, 2, \ldots, N$ we define, the following observation systems (OS_i):

$$
\begin{aligned}
x_1' &= x_1 f_1(x_1, x_2^{*(i-1)}) \\
x_2' &= x_2 f_2(x_1^{*(i-1)}, x_2) \\
y^i &= h^i(x_1, x_2) := h(x_1, x_2) - h(x_1^{*i}, x_2^{*i}).
\end{aligned}
\tag{OS_i}
$$

We note that here and throughout the paper, upper indices are not exponents.

Proposition 3.1 *Given $k_1,\ k_2 > 0$ we have*

$$0 < U^2 \le U^4 \le \ldots \le U^{2N} \le U^{2N-1} \le \ldots \le U^3 \le U^1, \tag{3.4}$$

where U^N ($N \in \mathbb{N}$) is the solution of the initial value problem

$$
\begin{aligned}
z_1' &= z_1 f_1(z_1, x_2^{*(N-1)}) + k_1(y^N - h^N(z_1, z_2)) \\
z_2' &= z_2 f_1(x_1^{*(N-1)}, z_2) + k_2(y^N - h^N(z_1, z_2)) \\
z^0 &= (z_1^0, z_2^0).
\end{aligned}
\tag{3.5}
$$

Proof. We shall follow an inductive process:

Step 1. In system (3.1), we set $x_2 := 0$ in f_1 and $x_1 := 0$ in f_2, and consider the observation system (OS_1)

$$
\begin{aligned}
x_1' &= F_1^1(x) = x_1 f_1^1(x_1, x_2) := x_1 f_1(x_1, 0) = x_1(a_1 - b_{11}x_1) \\
x_2' &= F_2^1(x) = x_2 f_2^1(x_1, x_2) := x_2 f_2(0, x_2) = x_2(a_2 - b_{22}x_2) \\
y^1 &= h^1(x_1, x_2) := h(x_1, x_2) - h(x_1^{*1}, x_2^{*1})
\end{aligned}
\tag{3.6}
$$

where $x_1^{*1} = \dfrac{a_1}{b_{11}}$; $x_2^{*1} = \dfrac{a_2}{b_{22}}$. It is easy to check that $x^{*1} > x^*$. Now, with matrices

$$A^1 := \frac{\partial F^1}{\partial x}(x^{*1}) = \begin{pmatrix} a_1 - 2b_{11}x_1^{*1} & 0 \\ 0 & a_2 - 2b_{22}x_2^{*1} \end{pmatrix}$$

$$C^1 = (c_1^1, c_2^1) := \left(\frac{\partial h^1}{\partial x_1}(x^{*1}), \frac{\partial h^1}{\partial x_2}(x^{*1}) \right)$$

and $K = col(k_1,\ k_2)$ we calculate the characteristical polynomial of the auxiliary matrix $A^1 - KC^1$:

$$
\begin{aligned}
p^1(\lambda) &= \lambda^2 + (a_1 + a_2 + k_1 c_1^1 + k_2 c_2^1)\lambda + (a_1 + k_1 c_1^1)(a_2 + k_2 c_2^1) - k_1 k_2 c_2^1 c_1^1 \\
&= \lambda^2 + (a_1 + a_2 + k_1 c_1^1 + k_2 c_2^1)\lambda + a_1 a_2 + a_1 k_2 c_2^1 + a_2 k_1 c_1^1.
\end{aligned}
$$

As $h_{x_i} > 0$ for $i = 1, 2$ and $a_1,\ a_2,\ k_1,\ k_2 > 0$ we obtain that

$$a_1 + a_2 + k_1 c_1^1 + k_2 c_2^1 > 0 \quad ; \quad a_1 a_2 + a_1 k_2 c_2^1 + a_2 k_1 c_1^1 > 0$$

therefore, according to the Routh-Hurwitz criterion, matrix $A^1 - KC^1$ is stable, and by Theorem 2.5 we can construct the observer of system (3.6) in the following way:

$$
\begin{aligned}
z_1' &= z_1(a_1 - b_{11}z_1) + k_1(y^1 - h^1(z)) \\
z_2' &= z_2(a_2 - b_{22}z_2) + k_2(y^1 - h^1(z))
\end{aligned}
\tag{3.7}
$$

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Given an initial condition $z^{01} = (z_1^{01}, z_2^{01})$ near x^{*1}, we can find a solution of (3.7) denoted by U^1.

Step 2. Now, in system (3.1), we set $x_2 := x_2^{*1}$ in f_1 and $x_1 := x_1^{*1}$ in f_2, defining observation system (OS_2)

$$x_1' = F_1^2(x) = x_1 f_1^2(x_1, x_2) := x_1 f_1(x_1, x_2^{*1}) = x_1(a_1 - b_{11}x_1 - b_{12}x_2^{*1})$$

$$x_2' = F_2^2(x) = x_2 f_2^2(x_1, x_2) := x_2 f_2(x_1^{*1}, x_2) = x_2(a_2 - b_{21}x_1^{*1} - b_{22}x_2) \quad (3.8)$$

$$y^2 = h^2(x_1, x_2) := h(x_1, x_2) - h(x_1^{*2}, x_2^{*2})$$

where $x_1^{*2} = \dfrac{a_1 - b_{12}x_2^{*1}}{b_{11}} > 0$; $x_2^{*2} = \dfrac{a_2 - b_{21}x_1^{*1}}{b_{22}} > 0$, which are positive by $(C1)$, moreover, inequalities $x^{*1} > x^* > x^{*2}$ hold.

Again, with matrices

$$A^2 := \frac{\partial F^2}{\partial x}(x^{*2}) = \begin{pmatrix} a_1 - 2b_{11}x_1^{*2} - b_{12}x_2^{*1} & 0 \\ 0 & a_2 - b_{21}x_1^{*1} - 2b_{22}x_2^{*2} \end{pmatrix}$$

$$C^2 := (c_1^2, c_2^2) := \left(\frac{\partial h^2}{\partial x_1}(x^{*2}), \frac{\partial h^2}{\partial x_2}(x^{*2}) \right),$$

if we calculate the characteristical polynomial of $A^2 - KC^2$,

$$p^2(\lambda) = \lambda^2 + (b_{11}x_1^{*2} + b_{22}x_2^{*2} + k_1c_1^2 + k_2c_2^2)\lambda +$$

$$+ b_{11}b_{22}x_1^{*2}x_2^{*2} + b_{11}x_1^{*2}k_2c_2^2 + b_{22}x_2^{*2}k_1c_1^2$$

by Routh-Hurwitz criterion, we again have that $A^2 - KC^2$ is stable and we can construct the observer of system (3.8)

$$z_1' = z_1(a_1 - b_{11}z_1 - b_{12}x_2^{*1}) + k_1(y^2 - h^2(z))$$
$$z_2' = z_2(a_2 - b_{21}x_1^{*1} - b_{22}z_2) + k_2(y^2 - h^2(z)). \quad (3.9)$$

Now, for an initial condition $z^{02} = (z_1^{02}, z_2^{02})$ near x^{*2} with $z^{02} \leq z^{01}$, we find a solution U^2 of (3.9). Moreover, by the partial monotony of h, we have

$$y^1 - h^1(z) > y^2 - h^2(z)$$

which permit us to compare the solution of systems (3.7) and (3.9) with initial conditions $z^{02} \leq z^{01}$. Hence we get

$$U^2 \leq U^1.$$

Step 3. In system (3.1), we set $x_2 := x_2^{*2}$ in f_1 and $x_1 := x_1^{*2}$ in f_2. Hence we obtain observation system (OS_3)

$$x_1' = F_1^3(x) = x_1(a_1 - b_{11}x_1 - b_{12}x_2^{*2}) = x_1 f_1(x_1, x_2^{*2})$$

$$x_2' = F_2^3(x) = x_2(a_2 - b_{21}x_1^{*2} - b_{22}x_2) = x_2 f_2(x_1^{*2}, x_2) \qquad (3.10)$$

$$y^3 = h^3(x_1, x_2) := h(x_1, x_2) - h(x_1^{*3}, x_2^{*3})$$

where $x_1^{*3} = \dfrac{a_1 - b_{12}x_2^{*2}}{b_{11}} > 0$; $x_2^{*3} = \dfrac{a_2 - b_{21}x_1^{*2}}{b_{22}} > 0$, which are positive by $(C1)$, moreover, inequalities $x^{*1} > x^{*3} > x^* > x^{*2}$ are verified.

If we calculate the characteristic polynomial of matrix $A^3 - KC^3$ with

$$A^3 := \frac{\partial F^3}{\partial x}(x^{*3}) = \begin{pmatrix} a_1 - 2b_{11}x_1^{*3} - b_{12}x_2^{*2} & 0 \\ 0 & a_2 - b_{21}x_1^{*2} - 2b_{22}x_2^{*3} \end{pmatrix}$$

$$C^3 := (c_1^3, c_2^3) := \left(\frac{\partial h^3}{\partial x_1}(x^{*3}), \ \frac{\partial h^3}{\partial x_2}(x^{*3}) \right),$$

according to the Routh-Hurwitz criterion we get that $A^3 - KC^3$ is stable, therefore we can construct the observer of system (3.10) in the following way:

$$z_1' = z_1(a_1 - b_{11}z_1 - b_{12}x_2^{*2}) + k_1(y^3 - h^3(z))$$

$$z_2' = z_2(a_2 - b_{21}x_1^{*2} - b_{22}z_2) + k_2(y^3 - h^3(z)), \qquad (3.11)$$

and for an initial condition $z^{03} = (z_1^{03}, z_2^{03})$ near x^{*3}, such that $z^{02} \le z^{03} \le z^{01}$, we can find a solution denoted by U^3. Moreover, again by the strict partial monotony of h we have

$$y^1 - h^1(z) > y^3 - h^3(z) > y^2 - h^2(z).$$

Now comparing the solutions of systems (3.7), (3.9) and (3.11), we obtain

$$0 < U^2 \le U^3 \le U^1.$$

The process can be continued in an iterative way, which completes the proof. ∎

From the above construction we have two sequences (U^{2N-1}) and (U^{2N}) for $N \in \mathbb{N}$ that are monotonous and bounded, therefore, convergent point-wise. Hence, for each $t \in \mathbb{R}$, we define:

$$\overline{U}(t) = \lim_{N \to +\infty} U^{2N-1}(t) \quad , \quad \underline{U}(t) = \lim_{N \to +\infty} U^{2N}(t). \qquad (3.12)$$

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Moreover, as

$$
\begin{aligned}
(U_1^N)' &= U_1^N(a_1 - b_{11}U_1^N - b_{12}x_2^{*(N-1)}) + k_1(y^N - h^N(U^N)) \\
(U_2^N)' &= U_2^N(a_2 - b_{21}x_1^{*(N-1)} - b_{22}U_2^N) + k_2(y^N - h^N(U^N)),
\end{aligned}
\tag{3.13}
$$

and h is bounded on compact sets, sequence $((U^N)')$ is uniformly bounded on compact sets. Applying the Ascoli Theorem to a fixed compact subset of $[0, +\infty)$, this sequence has a uniformly convergent subsequence denoted in the same way. By (3.4) we conclude that the convergence expressed in (3.12) is uniform on compact sets of $\mathbb{R}$.

On the other hand, the idea developed in the above iterative process, also applies to a positive solution of (3.1). In this way we can also establish a relation between the two iterative schemes and as a result, we shall conclude that every solution of system (3.1) is attracted by the "stripe" $[\underline{U}, \overline{U}]$.

Indeed, let $u = (u_1, u_2)$ be the positive solution of system (3.1), satisfying the initial condition $x^0 > 0$, near x^*. Define $x_2^{*0} := 0$ and substitute it as x_2 in $f_1(x_1, x_2)$ of system (3.1), to obtain

$$
x_1' = x_1(a_1 - b_{11}x_1) \quad x_1(0) = x_1^0.
$$

This initial value problem has a solution u_1^1 defined and bounded in $[0, +\infty)$. Moreover, by the monotony of f_1, we have $f_1(x_1, u_2) \leq f_1(x_1, 0)$ implying u_1^1 is a supersolution of $x_1' = x_1 f_1(x_1, u_2)$, and by Lemma 2.2 we have $u_1 \leq u_1^1$.

We define $x_1^{*0} := 0$ and substitute it as x_1 in $f_2(x_1, x_2)$ of system (3.1) to obtain

$$
x_2' = x_2(a_2 - b_{22}x_2) \quad x_2(0) = x_2^0
$$

which has a solution u_2^1 defined and bounded in $[0, +\infty)$. Moreover, by the monotony of f_2, we have $f_2(u_1, x_2) \leq f_2(0, x_2)$, therefore u_2^1 is a supersolution of $x_2' = x_2 f_2(u_1, x_2)$, and by Lemma 2.2, we obtain $u_2 \leq u_2^1$.

Now we take x_2^{*1} defined in (3.3) and substitute it into $f_1(x_1, x_2)$, obtaining the initial value problem:

$$
x_1' = x_1(a_1 - b_{11}x_1 - b_{12}x_2^{*1}) \quad x_1(0) = x_1^0,
$$

which has a solution u_1^2 defined and bounded in $[0, +\infty)$. Moreover, by the monotony of f_1, we have $f_1(x_1, u_2) \geq f_1(x_1, x_2^{*1})$, therefore u_1^2 is a subsolution of $x_1' = x_1 f_1(x_1, u_2)$, and by Lemma 2.2, we have $u_1 \geq u_1^2$. If we take x_1^{*1} defined in (3.3), and substitute it into $f_2(x_1, x_2)$, we get the initial value problem

$$
x_2' = x_2(a_2 - b_{21}x_1^{*1} - b_{22}x_2) \quad x_2(0) = x_2^0,
$$

which has a solution u_2^2. Now, similarly to the reasoning made for u_1^2, we obtain $u_2 \geq u_2^2$. Therefore,

$$
u^2(t) \leq u(t) \leq u^1(t), \qquad \forall t \geq 0.
$$

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If we continue the process, we get a sequence of solutions, that in general terms we can express by the following iterative scheme:

$$(u_1^N)' = u_1^N(a_1 - b_{11}u_1^N - b_{12}u_2^{*(N-1)}) \quad u_1^N(0) = x_1^0$$
$$(u_2^N)' = u_2^N(a_2 - b_{21}u_1^{*(N-1)} - b_{22}u_2^N) \quad u_2^N(0) = x_2^0$$
(3.14)

with $N \in \mathbb{N}$ and $u_1^{*0} = u_2^{*0} = 0$. Moreover, it is easy to check that the following monotony conditions are satisfied,

$$0 \leq u^2 \leq u^4 \leq \ldots \leq u^{2N} \leq u \leq u^{2N-1} \leq \ldots \leq u^3 \leq u^1.$$

From the definition of observer, we immediately obtain

Proposition 3.2 *Given $N \in \mathbb{N}$, for u^N, U^N defined in the iterative schemes (3.14) and (3.5), we have*

$$u^N(t) - U^N(t) \to 0 \quad when \quad t \to +\infty.$$

Remark 3.3 *Although from the iterative scheme Ascoli's Theorem provides only a sequence (U^N) uniformly convergent on compact sets, our simulation results suggest that this convergence is uniform on $[0, +\infty)$, too. As a matter of fact, as we illustrate in the next section, we also observe that if u is a positive solution of system (3.1), then for all $\epsilon > 0$ there exists $t_0 \geq 0$ such that*

$$\underline{U}_i(t) - \epsilon \leq u_i(t) \leq \overline{U}_i(t) + \epsilon \quad i = 1, 2; \quad t \geq t_0.$$

4   Example

For system (3.1) we consider the observed function $h : \mathbb{R}^2 \to \mathbb{R}$ defined by $h(x_1, x_2) = x_1 + x_2$, which is increasing in x_1 and x_2. Therefore, if we take $k_1, k_2 > 0$, the hypothesis of Proposition 3.1 is verified and the inequalities of (3.4) hold. We take the observation system

$$x_1' = x_1(1 - 0.9x_1 - 0.5x_2)$$
$$x_2' = x_2(1 - 0.5x_1 - 0.7x_2)$$
$$y = h(x_1, x_2) - h(x_1^*, x_2^*),$$
(4.1)

where $x^* = (x_1^*, x_2^*)$ is the equilibrium of system

$$x_1' = x_1(1 - 0.9x_1 - 0.5x_2)$$
$$x_2' = x_2(1 - 0.5x_1 - 0.7x_2).$$
(4.2)

For $k_1 = k_2 = 0.5 > 0$ and $z^0 = (1.4, 3.9)$, we solve numerically observation systems OS_i for $i = 1, \ldots, 4$, observing how inequalities of (3.4) hold (see

Figures 1-2) , that is

$$U^1 \geq U^3 \geq U^4 \geq U^2.$$

Moreover, as we have mentioned in the previous section, we can check that from certain $t \in \mathbb{R}^+$ we have that the solution u of system (4.2) verifies
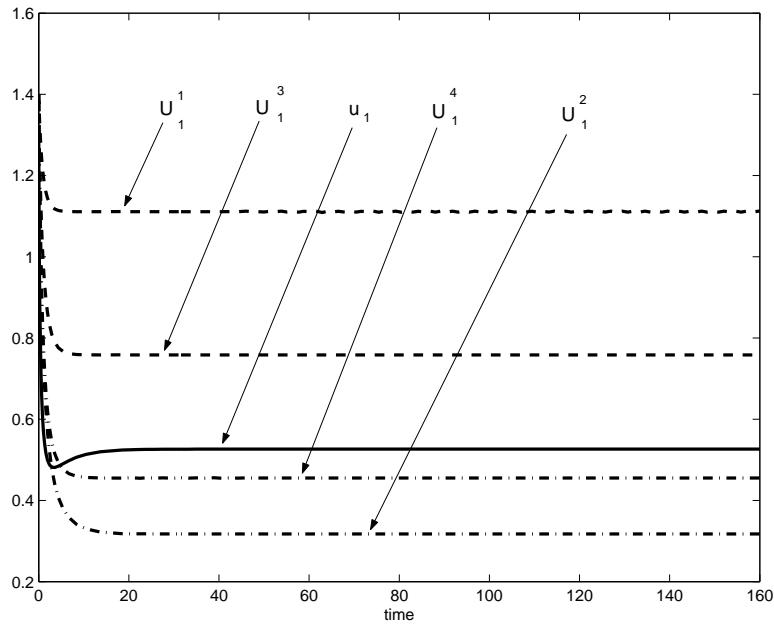
$$U^1 \geq U^3 \geq u \geq U^4 \geq U^2.$$

Fig. 1. Iterative scheme for $z^0 = (1.4, 3.9)$

If we solve the above observation system numerically for any other positive initial condition and arbitrary coefficients from the parameter space of the model, we obtain the same convergence result.
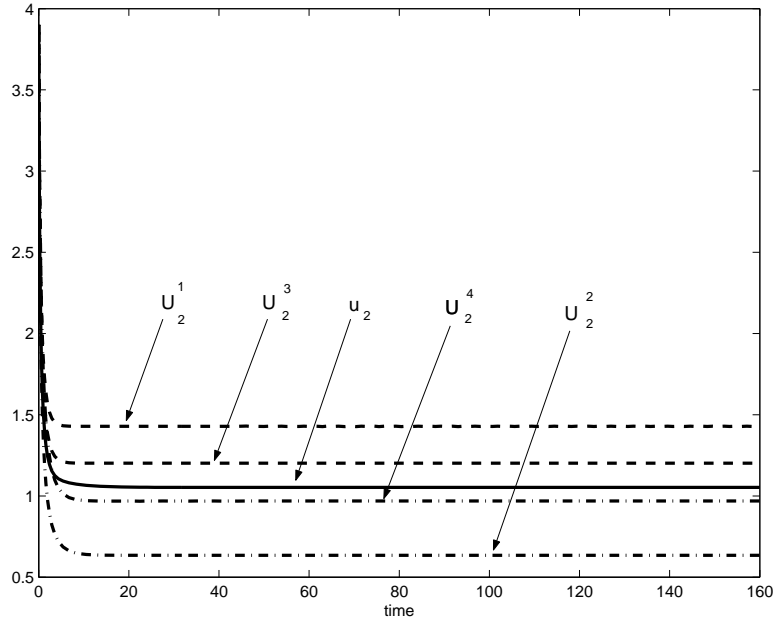
Fig. 2. Iterative scheme for $z^0 = (1.4, 3.9)$

5 Discussion

The sufficient conditions we proposed in the paper make it possible to develop an iterative scheme for a competitive Lotka-Volterra system, providing a sequence of observer systems for the localization of the solution of the original system. Although theoretically only a convergence on compact intervals can be guaranteed, the numerical simulations confirm convergence on infinite time intervals, as well. Hence, in practice, by this method the long-term behaviour of the population system can also be analyzed.

Since the condition of partially monotony of the observation function is also fulfilled for phenotypic observation of genetic processes, calculated in terms of the Hardy-Weinberg proportions (see López et al. 2007a), an extension of the above methodology to frequency-dependent models of population genetics may be straightforward.

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