



Cognition in children's mathematical processing: bringing psychology to the classroom

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Abstract

Introduction. The cognitive processes that underpin successful mathematical processing in children have been well researched by experimental psychologists, but are not widely understood among teachers of primary mathematics. This is a shame, as an understanding of these cognitive processes could be highly useful to practitioners. This paper focuses on ‘working memory’, a cognitive system responsible for the concurrent storage *and processing* of information, which has been shown to be highly important in children’s mathematical processing.

Method. This paper describes two experiments, both using a correlational design, undertaken with groups of 9 and 10 year old children. The first experiment compared performance on all elements of the Baddeley and Hitch working memory model with performance on simple addition and multiplication. The second experiment developed the first by taking a more detailed look at central executive functioning and comparing it with performance on addition and multiplication. Both experiments also looked at the children’s predominant calculation strategy to see if there was any evidence that working memory performance affects strategy development.

Results. Results from the first experiment suggested that addition and multiplication make different cognitive demands on children, with multiplication drawing more heavily on phonological working memory and addition drawing more heavily on executive processes. The second experiment suggests that the involvement of executive processes in multiplication may be inhibiting incorrect answers, whereas in addition they are linked to the ability to maintain and process information concurrently.

Discussion and Conclusion: The results are discussed in terms of their usefulness for classroom practitioners.

Keywords: working memory, mathematics, addition, multiplication, executive processes, classroom

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Cognición en el procesamiento matemático de los niños: psicología en el aula

Resumen

Introducción. Los procesos cognitivos que subyacen bajo el éxito en el tratamiento matemático con niños han sido bien investigados por los psicólogos experimentales, pero no están ampliamente reconocidos entre los profesores de matemáticas de primaria. Esto es una pena porque una comprensión de estos procesos cognitivos podría ser de gran utilidad para los profesionales. Esta publicación se centra en 'la memoria de trabajo', un sistema cognitivo responsable del almacenamiento y el procesamiento concurrente de información que ha demostrado ser muy importante en el procesamiento matemático de los niños.

Método. Este artículo describe dos experimentos realizados con grupos de niños de 9 y 10 años de edad. Ambos experimentos usan un diseño correlacional. El primero comparó el rendimiento en todos los elementos del modelo de memoria de trabajo de Baddeley y Hitch con el rendimiento en la adición y multiplicación de números simples. El segundo experimento desarrolló el primer y es una exploración más detallada del funcionamiento ejecutivo central y lo comparó con el rendimiento en la suma y la multiplicación. Los dos experimentos también examinaron la estrategia de cálculo predominante para ver si había evidencia de que el rendimiento de la memoria de trabajo afecta al desarrollo de la estrategia.

Resultados. Los resultados del primer experimento sugieren que la adición y la multiplicación demandan diferentes procesos cognitivos en los niños, con la multiplicación se utiliza en mayor medida la memoria fonológica de trabajo y la adición se centra en mayor medida en los procesos ejecutivos. El segundo experimento sugiere que la participación de los procesos ejecutivos en multiplicación puede crear la inhibición de respuestas incorrectas, mientras que en la adición tienen una relación con la capacidad de mantener y procesar información.

Discusión y conclusiones: Los resultados son tratados en términos de utilidad ~~euanto~~ para los profesionales de la clase.

Palabras Clave: memoria de trabajo, las matemáticas, adición, multiplicación, los procesos ejecutivos, la clase.

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Introduction

There is a lot more to working memory than the simple short-term storage of information. Working memory refers to a complex cognitive system that is responsible for the storage *and concurrent processing* of information in the short term. Although there are several models of working memory, the most widely known and the one that has proved most robust in the face of research evidence is that first proposed by Baddeley and Hitch (1974). This model consists of four parts. Two ‘slave’ systems, the phonological loop and the visuo-spatial sketchpad, are thought to be responsible for the short-term storage of phonological and visuo-spatial information respectively. The episodic buffer (Baddeley, 2000) is thought to integrate information in various forms into an integrated whole for a short period. These elements are connected and co-ordinated by the ‘central executive’, responsible for controlling and directing attention. The central executive component is thought to monitor cognitive processes, inhibit unwanted information from current processing and to control the complex processes involved in the concurrent storage and processing of information. Clearly a mathematical task such as mental computation would make demands of such a cognitive system as partial results are stored while other parts of the computation are calculated, information from previous calculations is inhibited and the whole calculation process is monitored. Practitioners may well be familiar with children who appear to lose their way during mental calculations forgetting which parts of the calculation they have completed and what the next step in the process is.

Despite its obvious importance, there is little consensus about the precise role of working memory in adults’ mathematical processing (DeStefano & LeFevre, 2004) and the situation is likely to be more complex among children whose working memory is still developing. Various working memory processes have been linked to mathematical processing, e.g. inhibitory processes (Passolunghi & Siegel, 2004), updating (Passolunghi & Pazzaglia, 2004), phonological loop and visuo-spatial sketchpad (McKenzie Bull, & Gray, 2003; Ramirez, Arenas, & Henao, 2005), central executive and visuo-spatial sketchpad (D’Amico & Guarnera, 2005) and visuo-spatial working memory (Reuhkala, 2001).

Possibly important relationships between mathematical performance and working memory could be obscured by the use of compound measures for either working memory or mathematics, or by collapsing the results of different tests to give a single measure. The picture might be further complicated by the extrapolation of findings from adults to children who

are still learning mathematics and whose working memory may not be fully developed. This is further complicated by findings (e.g. MacKenzie, Bull & Gray, 2003) that boys and girls may carry out mathematical calculations in different ways, possibly drawing on different cognitive skills as they do so. Other studies (e.g. Trbovich & LeFevre, 2003) suggest that the surface form of the mathematical calculations may affect working memory processes.

The role of working memory in children's addition

Addition is usually met early in children's mathematical education and is initially performed using counting procedures. An ability to visualise (and mentally manipulate) sets of objects is an advantage in building early addition skills (Rasmussen & Bisanz, 2005). Recent research has indicated that children use both direct retrieval and procedural strategies to solve simple addition problems (Imbo & Vandierendonck, 2007; 2008). Calculations that are not solved by direct recall, but by a counting-based or other method are vulnerable to deficiencies in working memory. Impaired counting speeds may make algorithmic solution of addition problems so slow that the memory trace of the operands decays (Hopkins & Lawson, 2005). Using a relatively simple strategy such as counting on from the larger of the two addends requires a lot of working memory capacity. The child must recall the larger addend, remember how much to count on and keep track of the count (Berg, 2008). There is evidence that deficits in children's visual memory may lead to problems with planning strategies related to recalling stored information (Ramirez, Arenas & Henao, 2005). Children often seek to reduce the working memory load by using their fingers as representations of some of this information (Geary, Hoard, Byrd-Craven & DeSoto, 2004). However, it would seem from this that the ability to store information accurately in the face of concurrent processing demands (a feature of the central executive of working memory) will mediate success with simple addition calculations.

The role of working memory in children's multiplication

Multiplication facts are often committed to long-term memory by the rote learning of pairs of numbers and their resulting product in multiplication tables (see Becker, McLaughlin, Weber & Gower, 2009 for an interesting discussion of methods of facilitating the learning of multiplication facts). The nature of the facts to be learned makes this an efficient strategy (Dehaene, 1998) although children do solve multiplication problems using a variety of strate-

gies. In order for children to establish a robust link between the two multiplicands and the resulting product, they need to retain the two multiplicands for long enough that they are associated with the product.

It has been widely suggested (e.g. Galfano, Rusconi & Umiltà, 2003) that multiplication facts are stored in networks of associated facts and that the mere presentation of two numbers is sufficient to activate other multiples of the multiplicands. This explains the predominance of incorrect answers in single-digit multiplication being multiples of one of the operands (e.g. a child answering $6 \times 7 = 49$), so-called 'associative intrusion errors' or 'operand errors' (e.g. Barrouillet, Fayol & Lathulière, 1997). Such errors are explained by the activation of a network of associations with each of the operands and the need to select the appropriate answer.

What distinguishes those children who make associative intrusion errors from those who do not may be the ability to inhibit unwanted (and recently activated) information from working memory. The ability to inhibit unwanted information is thought to be a function of the central executive of the working memory model (Passolunghi & Siegel, 2004). It is difficult to predict whether a specific piece of unwanted information might 'contaminate' current processing, as the likelihood of an intrusion error may depend on how recently and how strongly the unwanted information was activated and whether it was the correct answer to a previous problem or from a previous division calculation (Campbell, 1997).

The experiments described here sought to explore some of the possible confounds in existing research on working memory and arithmetic by looking at the relationships between specific elements of the Baddeley and Hitch (1974) working memory model (outlined above) and specific mathematical operations. It is hoped that a better understanding of the way that working memory affects children's addition and multiplication will be of benefit to primary mathematics teachers who are seeking to find ways to help children carry out these calculations.

Method (Experiment 1)

Participants

The 32 participants in this study were selected from two classes in state primary schools in the west of England. The mean age of the participants was 10 years and 2 months (SD = 4.1, range 9 years 10 months to 10 years 8 months) and there were 8 males and 24 females in the sample. None of the children had identified special educational needs. All had normal or corrected to normal vision.

Instruments

Phonological Working Memory

Simple phonological working memory was measured using the Non-word List Recall subtest from the Working Memory Test Battery for Children (WMTB-C, Pickering and Gathercole, 2001). Non-words are used in this task in order to minimise the support for recall from long-term memory, as would be the case for real words or digits. The task required participants to repeat back sequences of single-syllable non-words. The sequences increased in length until the participant was unable to recall them accurately. The sequences of non-words were presented in blocks of six trials. The number of non-words in each trial was increased by one for each new block. If a participant scored 4 correct trials in any one block, he/she moved on to the next block. If a participant made three errors in any one block, the task was terminated.

Inhibition

The children were shown red shapes in order to establish that they were able to identify a square, circle, triangle and rhombus. All the shapes in this task were intentionally red in order to establish in the children a pre-potent tendency to name the red shape. The children were then shown various shapes superimposed on other shapes using PowerPoint on a laptop computer. The shapes were either red or green. Sometimes the larger shape was red; sometimes the smaller shape was red. The children's task was to name the green shape and to ignore

the red shape. Each screen of shapes was visible for two seconds. Each child was shown twenty shapes and the number of intrusion errors made was recorded.

Visual-Spatial Working Memory

Visuo-spatial working memory was measured using a visual patterns test (Della Sala, Gray, Baddeley & Wilson, 1997). The test used semantically neutral matrices of filled and blank squares to minimise any contribution from information held in long-term memory and was intended to be a measure of pure visuo-spatial storage. Participants were shown a series of matrices with some of the squares filled in, for a period of 3 seconds. The target matrix was then removed from view and the child asked to recall the location of the filled squares in an identical, but empty matrix. The children were shown 16 matrices with between 2 and 6 filled squares. They scored one point for each trial on which they were able to recall the location of all the filled squares accurately.

Addition and multiplication

For the purposes of this study, we simply looked at the time taken to answer simple addition and multiplication questions rather than try to define when a certain fact has been 'mastered'. For all of the mathematical measures, the children were asked to carry out the calculations mentally. The children were presented with twenty addition and twenty multiplication questions. The digits were presented horizontally (see Trbovich & Lefevre, 2003 for a discussion) in font size 72. The questions remained visible until the children provided a spoken answer. All the questions were in the form $a+b$ or $a*b$ where both a and b were single digits. The children were asked to respond verbally as quickly as possible while still being accurate. Any errors were noted. The time taken to complete all twenty questions correctly was recorded. As it was not possible to obtain standardised scores on all the tests, raw scores were used throughout.

Calculation Strategy

We wanted to see if there was any connection between the children's working memory performance and their choice of strategy for the different calculations. We were less interested in comparing calculation speed with performance on individual trials. In order to do

this, we needed to get a feel for the *predominant* strategy that each child was using for the addition and multiplication questions. This was established by observing the children as they completed the calculations and then talking to them about strategy use. From this we put them into one of two categories. For addition the two categories were 'counters' (those children who were predominantly using counting strategies) and 'mixed' (those children who were using recall strategies or other, more sophisticated strategies such as adding ten and adjusting). For the multiplication questions the children were categorised as 'recallers' (using a direct recall strategy) and 'calculators' (those children who did not use a direct recall strategy and had to work out the multiplication calculations e.g. solving 3×5 by counting in fives).

All the tasks were administered individually in a quiet place within the school. The testing took place over two sessions. The children did the mathematical tasks during the first session and the working memory tasks during the second session.

Results

Independent samples t-tests showed that there were no gender significant differences on any of the cognitive or mathematical tasks. There was concern that some children would sacrifice accuracy for speed, while other, perhaps more cautious children, would sacrifice speed for accuracy. The tests for correlation revealed a positive correlation between multiplication times and the number of errors made ($r=.374$, $p<.05$). This shows that there was no trade-off between speed and accuracy, as the fastest children were also the most accurate. None of the children made addition errors. The correlations between the working memory and mathematical measures are summarised in Table 1 below:

Table 1. *Correlations between the working memory and the mathematical measures.*

Measures	Non-word List Recall		Visual Patterns		Shapes Inhibition	
	r	p	r	p	r	p
Addition (time)	-.324	n/s	-.065	n/s	-.621	<.001
Multiplication (time)	-.405	<.05	-.336	n/s	-.396	<.05
Multiplication Int Errors	-.207	n/s	-.238	n/s	-.267	n/s
Multiplication Total Errors	-.307	n/s	-.188	n/s	-.380	<.05

Note: The correlations are negative because higher scores for the mathematical tasks (completion times and error rates) indicate poorer performance, whereas higher scores for the working memory tasks indicate better performance.

Pearson correlation tests revealed moderate but statistically significant relationships between performance on the Non-word List Recall task and multiplication speed ($r=-.453$, $p<.05$). No such correlation was seen between performance on the Non-word List Recall task and the measure of addition speed. The Visual Patterns Test scores were not significantly related to either addition or multiplication speed.

The test of shapes inhibition showed moderate to strong correlations with addition speed ($r=-.504$, $p<.001$). It was also weakly but statistically significantly related to the total number of multiplication errors ($r=-.380$, $p<.05$), but not to the total number of associative intrusion errors, nor to multiplication speed. However, if ‘operation errors’ (i.e. giving the correct answer as the sum rather than the product of the multiplicands) are included as ‘intrusion errors’ then the inhibition score was weakly, but significantly correlated with the total number of intrusion errors ($r=-.371$, $p<.05$).

The comparison of the children using different addition strategies showed, as expected, significant differences between the two groups in terms of addition speed, but no significant differences in working memory scores. However, as well as significant differences in both speed and accuracy between the two multiplication strategy groups, the t-test analysis revealed a significant difference between the two groups’ scores on the non-word recall task. The differences between the ‘recallers’ and ‘calculators’ strategy groups for multiplication are show in Table 2.

Table 2. *Differences between ‘recallers’ and ‘calculators’ strategy users on mathematical and working memory measures.*

Measures	Mean (Recallers)	Mean (Calculators)	t	p	d
Multiplication Production	50.67	91.18	2.582	<.05	.869
Mult’n Multiple Choice	86.25	121.7	3.553	<.01	.655
Intrusion Errors	0.33	1.24	2.602	<.05	.667
Total Errors	0.42	1.88	2.493	<.05	.616
Non-Word Repetition	20.25	18.06	2.309	<.05	.637
Visual Patterns	11.17	10.65	0.903	n/s	.229
Shapes Inhibition	18.75	18.65	0.237	n/s	.062

Discussion and conclusions (Experiment 1)

The weak, but significant correlation of the phonological working memory measure and performance on both measures of multiplication (but not addition) fact mastery suggests that multiplication facts may be encoded in terms of their phonemic features. This would be expected given that many children in the UK learn their multiplication tables by rote repetition of a verbal sequence. It could be that children who are less able to recall multiplication facts directly, which would lead to a faster solution, are constrained by their limited phonological working memory span. As solution speed in arithmetic is indicative of strategy (see Barrouillet and Lépine, 2005) there is a suggestion that phonological working memory could help children to make a move to a direct recall strategy. This hypothesis was supported by the finding that the children who were predominantly using a recall strategy had better phonological working memory than those children still using other, more calculation-based strategies.

The intrusion errors made in the shapes inhibition task correlated with the total number of multiplication errors made, but not significantly with the number of intrusion errors made (although the correlation was still high; 68% of all errors made were intrusion errors). It is also not at all clear that the shapes inhibition task is tapping inhibition exclusively. More research is needed, with a range of different inhibition tasks, before it is possible to say confidently that the ability to inhibit unwanted information is directly implicated in multiplication accuracy. Poor inhibitory skills may be an indirect cause of other errors, such as miscounting, made on multiplication fact questions. In fact, most of the errors that were not intrusion errors were operation errors, where the children added instead of multiplied (e.g. $4 \times 6 = 10$). As the children would have spent the majority of their mathematical careers adding rather than multiplying, it could be argued that failing to inhibit the pre-potent tendency to add rather than multiply constitutes an intrusion error of sorts. This would support the suggestion that multiplication facts are stored in associated networks and that inhibitory skills are important in eliminating children's multiplication errors in two ways: in preventing associative intrusion errors and in preventing operation errors.

Evidence suggests that the ability to master addition facts may not be related to phonological working memory, but may be connected to visuo-spatial skills (e.g. D'Amico & Guarnera, 2005). We did not find a significant correlation between performance on our addition task and a task measuring visual working memory. However, the children in our sample sco-

red at ceiling levels in terms of accuracy. There is a need for further research to establish whether children use visual-spatial working memory processes as they manipulate sets and establish an understanding of early addition. The precise role of visual-spatial working memory remains unclear; the children may be using VSWM as they carry out the calculation, or they may simply be better at addition because the successful visual manipulation of sets earlier in their educational career enabled them to build more robust concepts of addition.

The shapes inhibition task had a significant correlation with addition speed. As intrusion errors are unlikely to be a feature of addition. This raises the possibility that the shapes inhibition task is tapping more than simply the ability to inhibit unwanted information from working memory.

One possibility is that this task is providing a measure of general central executive functioning and that the central executive is very important in quantity manipulation operations like addition. This would be consistent with a number of studies showing that disruption to the central executive has a serious impact on addition performance (e.g. De Rammelaere, Stuyven, & Vandierendonck, 2001). This task could be tapping the ability of the children to hold information in memory while carrying out concurrent processing? The children would have to keep the red shape in mind, while carrying out a comparison of the two shapes. This ability to hold information in mind while carrying out concurrent processing (a function of the central executive) could be highly important in children's counting strategies for addition. Given the inconclusive nature of these findings, a second experiment was conducted to explore in more detail the link between central executive working memory and children's mathematical performance. This experiment followed the findings from Experiment 1. Given the apparently crucial role of the central executive in children's mathematical processing, and the complex nature of the central executive, this experiment sought to unravel the different functions ascribed to the central executive and to investigate their possible contribution to successful mathematical processing. This experiment used a correlational design similar to that used in Experiment 1.

Method (Experiment 2)

Participants

The sample consisted of 25 children, from 3 state primary schools in the South West of England. All the children were in Year 5 (mean age = 116 months, S.D = 5.2 months, range = 112-123 months) at the time of testing. There were 15 males and 10 females.

Instruments

Mathematical Tasks

The mathematical measures used were identical to those described above for Experiment 1. In addition, the children were given a task to measure performance on more complicated addition questions that involved carrying. These were presented using PowerPoint on a lap-top computer in the same way as the other addition production measure. The children gave their answers verbally. Their completion times and errors were recorded.

Central Executive Tasks

The working memory tasks used were designed to tap different aspects of the central executive. The tasks assessed the children's ability to recall phonological and visual information in the face of concurrent processing. The other tasks assessed the children's ability to inhibit unwanted information from working memory and to update the contents of working memory by discarding unwanted information to retain only important information. Baddeley (1986) ascribes all these functions to the central executive. What is not totally clear is whether the central executive is fractionated, or whether there is any variability in these different functions.

Animals Task

This is a test of processing and inhibition. This task is adapted from one used by Pasolunghi and Siegel (2001). It involved the children listening to strings of words and tapping the table every time an animal was mentioned. The children then had to recall the final word in each string. There were four words in each string and working memory load was manipula-

ted by altering the number of strings in a trial. The test was stopped after two consecutive failures to recall all the final words in a trial. Correct responses and inhibition errors (i.e. recalling animal names that were not the last word in the list) were noted.

Updating

The children were presented with strings of words and asked to recall the 2, 3 or 4 smallest objects in the list. The difficulty of the task was increased by increasing the length of the list and the number of words that had to be recalled from the list.

Backward Digit Recall

This task was taken from the Working Memory Test Battery for Children (WMTB-C, Pickering & Gathercole, 2001). In this task children were presented with strings of digits and asked to recall them in reverse order e.g. for the string 4, 7, 2, 9, the correct response would be 9, 2, 7, 4. The working memory load was increased by increasing the length of the string. The task was administered in blocks of six trials. The length of the strings was increased as a child moved from one block to the next. The test was stopped if a child made a mistake on any three trials in a single block. Once a child had completed four trials successfully in any one block, he/she moved to the next block.

Concurrent visual processing and storage (Mr X)

This task was taken from the Automated Working Memory Assessment (AWMA, Alloway, Gathercole, & Pickering, 2004). This is a test of concurrent visual processing and storage. This task involved the retention of visual-spatial information during concurrent visual-spatial processing. The participants were shown two people (Mr Xs) both holding a ball. For each trial, the child had to say whether the two figures were holding the ball in the same or different hands. They then had to recall the location of the ball being held by the figure on the right. The figures were presented in blocks of six trials. Working memory load was manipulated by increasing the number of pairs of figures shown in a single trial. The rules for progression from block to block and for the termination of the task were the same as those for the backward digit recall task described above.

The testing took place in two separate sessions separated by a week. In the first of the two sessions, the children were given the addition tasks, the animals task and Mr X. In the second session the children were given the multiplication tasks, backward digit recall and the updating task.

Results (Experiment 2)

Independent samples t-tests revealed no significant differences between the boys and the girls on any of the mathematical or working memory measures. Correlations between the working memory and the mathematical measures are shown in Table 3 below.

Table 3. *Correlations between working memory and mathematical measures.*

Measures	BDR	Animal Recall	Animal Inhibition	Mr X Recall	Updating
Addition Mastery	-.603(**)	-.655(**)	.687(**)	-.593(**)	-.452
Addition M-Choice	-.472(*)	-.524(**)	.612(**)	-.615(**)	-.487(*)
Complex Addition	-.562(**)	-.559(*)	.704(**)	-.551(*)	-.463(*)
Mult'n Mastery	-.427	-.568(*)	.275	-.544(*)	-.453(*)
Mult'n M-Choice	-.309	-.381	.366	-.531(*)	-.385
Total Mult'n Errors	-.543(**)	-.478(*)	.546(**)	-.469(*)	-.565(**)
Mult'n Inhib'n Errors	-.491(*)	-.497(*)	.509(**)	-.493(*)	-.551(**)

$p < .05$; ** $p < .01$.

Note: Correlations are negative (except for those for the animal inhibition task) as higher scores for the maths tasks (completion times and error rates) indicate poorer performance, whereas higher scores for the working memory tasks, except the animal inhibition task, which indicates better performance.

Pearson correlation analysis revealed some significant correlations between the working memory scores and the mathematical scores. These are summarised below:

- Both the central executive measures that rely on inhibition (Animal Inhibition and Updating) correlated very strongly with the number of multiplication errors.
- The measure of multiplication speed appears to have a much weaker correlation with the central executive measures than does the equivalent addition measure.

- The two central executive measures that require storage in the face of concurrent processing of other information (Animal Recall and Mr X) showed surprisingly similar patterns of results. The difference between the two tasks is that one uses phonological stimuli (Animal Recall) while the other uses visual-spatial stimuli (Mr X). It was expected that the phonological central executive task might correlate more strongly with the multiplication performance and the visual-spatial task more strongly with addition performance. This pattern was not seen. In fact, both measures correlated more strongly with the addition tasks than the multiplication tasks.

As in Experiment 1, the children were observed as they carried out the addition and multiplication tasks and were then questioned about the strategies they used to reach an answer. Following this discussion, they were assigned to one of two strategy groups based on their predominant strategy ('Counters' and 'Mixed' for addition; 'Recallers' and 'Calculators' for multiplication).

Given that strategy use is highly predictive of success in the mathematical tasks, the following analysis sought to determine whether strategy use might be predictive of working memory performance. Independent samples t-tests for the two multiplication strategy groups ('Recallers' and 'Calculators') showed no significant differences between the groups on any of the working memory tasks. However, the situation was very different for the addition groups ('Counters' and 'Mixed'). The results are shown in Table 4 below.

Table 4. *Differences between the addition strategy groups on working memory performance.*

Measures	Mean (Mixed)	Mean (Calculators)	t	p	d
BDR (Direct Trials)	17.94	14.89	1.622	n/s	.469
Animal Recall (Words Correct)	18.81	13.67	2.359	<.05	.692
Animal Inhibition (Errors)	.81	1.67	2.076	<.05	.565
Mr X Recall (Correct Trials)	15.69	10.43	2.449	<.05	.781
Updating (Words Correct)	24.87	15.67	2.834	<.01	.801

The existence of a statistically significant difference between the two strategy groups for addition suggests that central executive working memory may be predictive of strategy

use. Children whose central executive functioning is good are more likely to be using more sophisticated addition strategies. The absence of this difference when the different multiplication strategy groups were compared could have two possible explanations. Either central executive working memory does not play a role in the development of multiplication strategy (possibly some other factors such as verbal working memory; see Experiment 1), confidence, the favouring of speed rather than accuracy as a mark of success). Alternatively, central executive working memory plays less of a role in multiplication than it does in addition. This explanation would support the findings from Experiment 1.

However, Experiment 1 and the correlational data from Experiment 2 suggest some correlation between multiplication errors, and central executive working memory. In order to investigate this further, the strength of the correlations between the central executive measures and the multiplication measures (accuracy and speed) for the two strategy groups were compared. The results are shown in Table 5 below:

Table 5. Correlations between multiplication and central executive measures for the two multiplication strategy groups.

Measures	Multiplication Times		Multiplication Accuracy	
	Recallers	Calculators	Recallers	Calculators
BDR (Direct Trials)	.480	.136	.665	.556
Animal Recall (Words Correct)	.351	.359	.402	.451
Animal Inhibition (Errors)	.277	.160	.110	.586
Mr X Recall (Correct Trials)	.190	.686	.074	.475
Updating (Words Correct)	.252	.565	.123	.591

The results show that, for the accuracy measure, the correlations were stronger for the 'Calculators' than for the 'Recallers', with the exception of the BDR score. The differences were particularly pronounced for the tasks requiring some kind of inhibition (Animals Inhibition and Updating).

Discussion and conclusions (Experiment 2)

The analysis of the correlations between the mathematical tasks and the working memory tasks showed some striking and somewhat unexpected patterns. Both the central executive tasks that involved inhibiting information (Animal Inhibition and Updating) correlated most strongly with the measures of multiplication errors. This is in line with the findings from Experiment 1, which suggested that the ability to inhibit unwanted or superfluous information from working memory was predictive of multiplication accuracy, but not speed. These findings support the contention made by Campbell (1987) that multiplication facts are stored in associated networks, which are activated on presentation of the multiplicands. The ability to inhibit the activated, but incorrect, associations mediates accurate recall of the multiplication fact. This supports that idea that the inhibitory component of central executive functioning is primarily important in multiplication accuracy.

The second striking finding from the correlational analysis is that the correlations between the measures of addition performance and the central executive measures were much stronger than the correlations between the multiplication measures and the central executive measures. This suggests that, by Year 5 at least, children doing multiplication are drawing less heavily on central executive capacity than those same children doing addition. There is some evidence (Lee & Kang, 2002) that phonological working memory is more important in multiplication, whereas other (central executive) working memory processes are more involved in quantity manipulation operations such as addition. Again, this is consistent with the findings from Experiment 1. Experiment 2 did not take a measure of phonological working memory, as it was looking specifically at the central executive. This finding would support the idea that, by the time they reach Year 5, children are beginning to ‘master’ multiplication facts, which are stored as a rote verbal code and are recalled in a more automatic way without the need for the involvement of the central executive. This would support the suggestion made by Holmes and Adams (2006), which speculates that the phonological loop and not the central executive may be implicated in the retrieval of information from long-term memory.

In order to investigate this idea further, a correlational analysis was done looking at the relationship between multiplication accuracy and the central executive measures for the ‘Recall’ strategy group and the ‘Calculators’ strategy group. If the above contention is correct, then the correlations between these measures should be stronger for the ‘Calculators’

group, (who are presumably using some form of quantity manipulation to arrive at their answers) than for the 'Recall' group, who are using a direct recall from long-term memory.

This pattern of results was seen (with the sole exception of the BDR performance and accuracy) and is shown in Table 5. This suggests that using a direct recall strategy puts less demand on the central executive of working memory, even in terms of the inhibitory processes that seem to be important in being accurate. Possibly, inhibiting an unwanted association is a feature of the recall strategy, but also of counting-based strategies. For example, when counting in steps of 3 to work out a sum such as 6×3 , children would still face the prospect of intrusion from other multiples of 3 in the count. They would also need to keep a running total of their count, while tracking the number of steps of 3 they had counted. This would put considerable demands on central executive working memory resources and could explain why a direct recall strategy makes less demands of central executive working memory.

One of the research questions that underpinned this study concerned the different aspects of central executive working memory and their involvement in different mathematical operations. The study incorporated two central executive measures that focused on storage in the face of processing of other stimuli (Animal Recall and Mr X); one of which uses phonological stimuli and the other visual-spatial stimuli. These two tasks showed very similar patterns for the different mathematical operations. Both conformed to the pattern of a weaker correlation for the multiplication tasks. There was very little difference between them on any of the mathematical tasks. The findings of this study therefore do not support the idea of a difference in the importance of visual-spatial or phonological central executive working memory in mathematical performance.

The most likely explanation for this finding is that these two tasks are tapping some domain-general central executive capacity, as the storage component of these tasks is likely to be carried out by the two 'slave' systems. If these tasks are tapping the ability to hold information in the face of concurrent processing demands (Baddeley, 1996 identified this as a central executive function separate from any storage role), rather than the system's storage capacity, then no differences based on the nature of the stimuli would be expected.

The findings from this study reinforce those from Experiment 1 that differences in strategy use are highly important in determining mathematical performance in terms of both

accuracy and completion times (multiplication) and completion times (addition). The lack of an interaction with the addition errors needs to be treated with caution as very few addition errors were made at all.

The relationship between strategy use and working memory was interesting and unexpected. Is it possible to say from this that working memory plays a role in the development of addition strategies? The data suggest that this may be the case, but are not conclusive as the direction of any causal link cannot be determined from these data. While it is perfectly possible that superior central executive working memory helps children to move to more sophisticated addition strategies, it is equally plausible that using more sophisticated addition strategies causes an improvement in working memory performance. The fact that there were no significant central executive working memory differences between the two multiplication groups strengthens the idea, from the correlational analysis, that the central executive is much less involved in multiplication. Data from Experiment 1 did show a significant difference between the two multiplication strategy groups on a measure of phonological working memory, strengthening the suggestion that it is this component of the working memory model that is primarily involved in the move to a direct recall strategy. This would support Holmes and Adams' (2006) suggestion that the phonological loop is important in the recall of information from long-term memory; something that would be vital in developing an effective direct recall strategy.

Summary of Findings.

- The central executive component of working memory is highly involved in children's addition, even simple addition, but less so in multiplication.
- Phonological working memory appears to play an important part in children's multiplication, possibly by enabling children with good phonological working memory to progress more quickly to a direct recall strategy.
- The role of the central executive in multiplication appears to be in inhibiting activated, but incorrect answers (usually multiples of one of the multiplicands), or in inhibiting the pre-potent tendency to add.
- This inhibitory component of the central executive appears to be an important mediator of multiplication accuracy in non-recall strategies also.

Central executive working memory is highly predictive of addition accuracy.

Implications for the classroom

Results from this study would suggest that there is a role for phonological working memory in learning multiplication tables. This does not mean that understanding of multiplication should be sacrificed for faster recall of multiplication facts through endless rote repetition, simply that phonological working memory may be a factor in a child's ability to master multiplication facts. If teachers are more aware of the children in their care who have weaker phonological working memory skills, they are in a better position to provide ways of learning multiplication tables that do not rely heavily on the retention and recall of a rote verbal sequence. The use of more visual and manipulative ways of committing multiplication facts to memory would certainly help those children and may be of benefit to a number of children without specific problems with phonological working memory. Teachers who work with children whose phonological working memory is weak, but whose visual working memory is better could help the child to find strategies that connect the stronger visual representations of multiplication facts to the weaker phonological ones e.g. by encouraging the child to say the multiplication facts while writing them down or arranging visual arrays.

The evidence from this study suggests that the central executive is important in the development of more sophisticated addition strategies, specifically the move away from counting strategies to those using known facts. The process by which this might happen is not clear. It could be that children with weaker working memory are unable to store representations of the addends mentally while they carry out the calculations and so retain finger counting strategies for longer than those children who are able to represent the numbers mentally in working memory. It is possible that the use of more sophisticated addition strategies such as adding ten and adjusting requires more central executive processing in terms of tracking through a calculation and performing concurrent processing and storage than simply counting on nine. Children who lack the central executive resources to keep track of the multi-step process of adding ten and adjusting may prefer to retain the 'safer', although more error prone and slower, strategy of counting on nine.

Teachers seeking to help children move towards more sophisticated addition strategies could help by encouraging the children to record the steps in the strategy. This might help to

reduce the working memory load imposed by such strategies. Fairly simple recording might help children to keep track of what they had done with a particular calculation and what the interim result was. This would reduce the demands for concurrent processing and storage, which children with poor central executive working memory might find limiting. A compensation strategy for the calculation $35 + 9$ could be recorded as follows: $35 \rightarrow 45 \rightarrow 44$.

Once a child begins to have some success with this strategy, its benefits may become more apparent and the child may be motivated to move readily to more sophisticated addition strategies.

There was some indication from the findings that the concurrent storage and manipulation of visual information was closely related to addition performance. Teachers working with younger children commonly give them concrete objects to work with to establish an understanding of additive processes. Teaching and encouraging children to visualise sets of objects may promote early success with addition facts and therefore lead to better ‘mastery’ of these facts. Research by Hughes (1986) has suggested that children were almost as successful imagining and mentally manipulating sets of objects as were children who used physical objects to calculate addition sums. This would lend support to the notion that visual-spatial working memory may provide a mental workspace in which children begin to move from the concrete to the abstract (Holmes & Adams, 2006).

Early screening of children’s working memory skills could be used to identify children at risk of academic problems and so allow early intervention. It is possible that working memory skills could mediate the acquisition of both addition facts (through visualisation of sets of objects) and multiplication facts (through the phonological encoding of the operands and the resulting product). Increasing teachers’ knowledge of the working memory strengths and weaknesses of the children in their care could inform their teaching strategy and input for individual children. The application of this knowledge early on could help to target specific working memory practice early and so moderate some of the potential effects of working memory deficits.

Knowing the working memory profiles of the children in the class could help teachers to structure activities so as to reduce the working memory load on the child. This knowledge of the child’s working memory profile may also be helpful in understanding why a child is

experiencing particular difficulties. A child, who is having trouble learning multiplication facts, may have a sound understanding of the concept of multiplication. Giving such a child some support to reduce working memory load (for example a multiplication square when doing more complex multiplication questions) might allow the child to access activities that the less well-informed teacher may have assumed were beyond the child's capabilities.

Older children are often very good at coming up with their own strategies to help overcome working memory problems, provided that those problems have been identified and the strategies therefore 'legitimised' in the eyes of the child, the teacher and the other children in the class. Encouraging children whose working memory is being stretched beyond its limits, to ask for help, to seek resources to lighten the working memory load, to record information in different ways or to ask for clarification or simplification of the task and making those requests legitimate can go a long way to helping such children work with their working memory deficiencies.

Finally, the work done here presents us with the tantalising possibility that the correlation between more sophisticated strategy use and working memory performance may not be uni-directional. If this correlation works both ways i.e. if using more sophisticated strategies improves working memory performance, we are left with the very real possibility that teachers can bolster the working memory of their pupils, with the resulting gains in learning that such an intervention might bring.

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