Tesis Doctoral:

FINANCIAL MARKETS: A VIEW FROM STATISTICAL MECHANICS

LOS MERCADOS FINANCIEROS: UNA VISIÓN DESDE LA MECÁNICA ESTADÍSTICA

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Summary

This thesis aims to study the financial market using tools from Econophysics. The thesis begins by studying the importance of the market memory factor, measured through the Hurst exponent, in the 4-factor model of Fama and French, obtaining results that have supported the importance of this factor within the study of the financial market returns. Market research continues by studying the comovement in price and volatility among the US market stocks using comovement functions developed from colloidal particle functions. This methodology aims to identify the cause of consistent changes in volatility or price and to investigate the importance of the market in explaining comovement. One of the main results obtained was to verify how the equally weighted market can explain practically all the market comovement, which is in line with the CAPM model, where the market index is expected to largely explain the joint movement between two different stocks.

Resumen

La presente tesis tiene como objetivo el estudio del mercado financiero haciendo usos de herramientas provenientes de la econofísica. La tesis empieza estudiando la importancia del factor memoria de mercado, medido a través del Exponente de Hurst, en el modelo de cuatro factores de Fama y French, obteniéndose unos resultados que han respaldado la importancia de este factor dentro del estudio de los rendimientos del mercado financiero. Se continua la investigación sobre el mercado estudiando el comovimiento en precio y en volatilidad entre las acciones del mercado americano utilizando unas funciones de comovimiento desarrolladas a partir de funciones de partículas coloidales. Esta metodología tiene por objeto identificar la causa de los cambios coherentes en la volatilidad o el precio e indagar sobre la importancia del mercado en la explicación del comovimiento. Uno de los principales resultados obtenidos fue constatar como el mercado igualmente ponderado puede explicar prácticamente todo el comovimiento de mercado, lo que va en la linea del modelo CAPM, donde se espera que el índice de mercado explique en gran medida el movimiento conjunto entre dos valores diferentes.

Contents

Chapter 1

Introduction

1.1 Origin and evolution of Econophysics.

The origin of Econophysics is found in statistical mechanics, also called Statistical Physics. This new field of knowledge was created in the 19th century by physicists James Clerk Maxwell, Ludwig Boltzmann, and Josuah Willard Gibbs. Statistical mechanics has been defined as the branch of physics that combines the principles and procedures of statistics with the laws of classical and quantum mechanics, particularly in relation to the field of thermodynamics. The goal of this new branch is to predict and explain the measurable properties of macroscopic systems within those systems.

James Clerk Maxwell was the one who began to use the methods of statistical mechanics in the field of social sciences and, later, the development of this new application was promoted by Ludwig Boltzmann, who stated:

If we do not only think about inanimate objects, we have before us a new perspective. Let us consider the application of this method to the statistics of living beings, society, sociology and so on.

Since then, the interrelation between physics, mathematics and economics has been intensifying. In 1963 the physicist Tinbergen [28] won the Nobel Prize in Economics for his study of dynamic models in the analysis of economic processes.

In 1963 the mathematician Mandelbrot [43] carried out one of the most revolutionary works in the field of economics, by proposing the replacement of the Normal Distribution in the study of the prices of financial assets by the Levy distribution. From this work, Eugene Fama [21] opened the debate on the usefulness of the Normal Distribution in finance.

The repercussion of this new study approach was so huge that, based on the works of Mandelbrot [45] and Peters [52], a new proposal was developed to interpret the functioning of the financial market, the **Fractal Market Theory**, a theory that contrasts with the classic **Efficient Market Theory**.

After these great turns in financial research, new important changes took place in the 1970s due to the gradual increase in assets traded in financial markets. In 1973, currencies began to be traded 24 hours a day from Monday to Saturday. On the other hand, the volume traded grew year after year and, in 1995, the volume of transactions was already 80 times bigger than what was recorded in 1973.

Another breakthrough occurred in the 1980s thanks to the advent of electronic trading within the foreign exchange market. The development of new technologies enabled the improvement of the exchange of financial data, making it possible to obtain up-to-date market data with time intervals as short as seconds. The great development of financial data allowed not only to develop prediction models but also to create more accurate methods.

As a consequence of the above, in the 1990s physicists began to be interested in financial markets, since the large volume of data available allowed the application of statistical mechanics models in a totally different sphere.

The formal appearance of Econophysics takes place in 1996 with the work *Anomalous Fluctuations in the dynamics of complex systems: from DNA and physiology to Econophysics* by H.E. Stanley et al [62]. Econophysics is defined as the branch of interdisciplinary knowledge that applies the methods of statistical physics to solve problems in economics in general and finance in particular. Econophysics is not limited only to applying the laws of physics in the field of social sciences, but rather tries to apply the mathematical methods created by statistical physics to the study of the complex systems that form the behavior of the human being.

Throughout the 20th century and so far in the 21st century, Econophysics has been dealing with different aspects of the economy in general and financial markets in particular, but without a doubt the field of greatest interest is that of the *Financial Series*. Since the nineties, various authors such as Bollerslev et al.[8], Pagan [50] and Cont [19], have been establishing the following new empirical characteristics to describe them:

- **Absence of autocorrelation:** The linear correlation in the return series is practically insignificant, except for small scales of intraday data (less-than-20-minute intervals).
- **The returns appear to follow heavy-tailed distributions.**
- **Aggregation normality:** It is commonly accepted that as the time scale of the data considered increases, the distribution looks more and more like a normal one.
- **Presence of volatility clusters:** Different volatility measures show positive autocorrelation over several days, which highlights the fact that high volatility events tend to cluster over time.
- **Conditional heavy tails:** Even after smoothing out the effect of volatility clusters on the return series (for example, through GARCH-type models), the residual series still exhibit heavy tails.
- **Slow decay of the autocorrelation function of returns:** The autocorrelation function of returns in absolute value shows an exponential decay over time. This is closely related to what has been called memory in the series.
- **Correlation between volume and volatility:** Trading volume is strongly correlated with market volatility.

1.2 Memory in the series.

Without a doubt, one of the most important theories in finance is the *Efficient Market Theory*. The term efficiency refers to the fact that investors do not have the opportunity to achieve abnormal profits by operating in the market as compared with other investors, that is, it is not possible to beat the market unless a greater risk is assumed.

In 1970, Fama [22] established three efficiency hypotheses:

- **Weak-form efficiency:** the prices of financial assets reflect, at all times, the historical financial information, which implies one of the foundations of this theory, i.e., prices follow a random walk, with no memory over time.
- **Semi-strong efficiency:** the prices of financial assets reflect, at all times, the historical financial information and the public information, so that an advantage over the market cannot be obtained.

- **Strong-form efficiency:** the prices of financial assets reflect, at all times, the historical financial information, the public information and the private information on financial assets. In this scenario it is impossible for one investor to gain an advantage against other investor.

Since the establishment of these efficiency states, the financial literature has tried to contrast them through the use of different methodologies. Most researchers have ruled out the semi-strong and the strong-form efficiency, even in the most capitalized markets ([12], [33]), so the most contrasted hypothesis is, without a doubt, that of the weak-form efficiency.

One of the foundations of the Efficient Market hypothesis is to reject the existence of memory in financial series, since this would imply that prices do not follow a random walk. This hypothesis of no correlation in the financial series was already questioned by some authors ([39], [40]). Fama [23] carried out an analysis of the works that had found a correlation between past stock prices using short- and long-term horizons. Although in his work he does find signs of correlations, he ends by concluding that there is not enough statistical evidence to say that past prices can predict future ones, since the results may be due to the size of the sample and the seasonality of the series.

However, already in 1963 the mathematician Mandelbrot [43] questioned the random walk hypothesis by stating that price series show long-term memory, and it is at this point that physical models began to provide alternative theories to the efficient markets theory.

Thus, in 1991 Edgar E. Peters [52] proposed a new definition of market behavior based on chaos theory. I call this proposal the **fractal markets** hypothesis and it provides a new point of view to explain market liquidity. Thus, the market is stable when the investors who participate in it cover a wide variety of investment horizons, guaranteeing the liquidity of the traders. From this approach, the importance of the information would not depend on the type of information available but on the investment horizon of the investor, since there would be no liquidity in the market if the information had the same impact for all investors. Market collapses occur when there is an imbalance among buying and selling orders because a large number of agents place too many sell orders, which cannot be managed by market makers, causing prices to fall. On these occasions, investors question the validity of market information, causing investors with long-term horizons to withdraw from their investments or to switch to short-term investing.

The fractal market hypothesis considers that in periods of market stability all investment horizons are equally represented so that supply and demand are balanced without problems. The opposite happens in times of crisis. This behavior provides important information with which to study the behavior of the market over time. However, it is worth mentioning that today both theories cannot be considered opposed, but rather complementary.

As previously mentioned, the works of Cont [18, 19] show the correlation among the returns in absolute value of a financial asset, which causes an exponential fall in the autocorrelation function of the returns in absolute value to almost 0. This event is closely related to what is known as long memory and is in line with the work of Mandelbrot [43] where the idea that the returns on financial assets are distributed according to a fractional Brownian motion was defended, giving place to the existence of memory in the financial series.

In the field of econophysics, the presence of memory in the series is studied using what is known as the Hurst exponent. This methodology was developed by the English hydrologist H.E. Hurst in 1951 [34], based on Einstein's contributions regarding the Brownian motion of physical particles, to address the problem of control of water reserves near the River Nile dam. R/S analysis in economics was introduced by Mandelbrot [44] and Mandelbrot and Wallis [45] as a superior methodology to the classic ones used in economics: autocorrelation, variance analysis and spectral analysis.

In this line of study of market memory, the first study found in this thesis, *Extending the Fama and French model with a long term memory factor* (published in 2019 in the European Journal of Operational Research), has been developed.

The main points of interest in this study are briefly outlined below.

1.2.1 Hurst Exponent and the APT.

Since the beginning of portfolio theory with Markowitz [47], many researchers have studied the behavior of financial asset returns. The best known factorial model for the study of market returns is the Capital Asset Pricing Model (CAPM) created by Lintner [38], Mossin [49] and Black and Scholes [7]. The great contribution of the CAPM to finance was to relate the performance of a financial asset to its systematic risk, which is measured through the beta. This implies that part of the performance of an asset is a linear function of its market beta, and therefore, the beta can adequately describe its performance.

Despite the fact that there were many research works that supported the good functioning of the CAPM model ([6], [25]) the first criticisms did not take long to appear. Fama and French [24] carried out a study for the period 1941-1990 and reached the conclusion that the CAPM model was not capable of adequately describing the yields since there must be other additional factors that are essential for determining them.

In this context, many researchers began to look for factors that would help represent market returns. Banz [4] studied the size effect within the profitability study. Bhandari [5] studied the relationship between leverage and performance of a listed company. Stattman [63], Rosenberg et al. [57] and Chan et al. [16] studied the importance of the book-to-market ratio in calculating market returns.

Based on several of the works on factors, Fama and French [24] proposed their first factorial model, which was based on the incorporation of two new factors to the CAPM, size (capitalization) and value (book-to-market ratio). The authors have developed their model over the years incorporating other factors. Fama and French [26] incorporated the momentum factor proposed by Carhart [14] in order to solve the specification problems that arise due to economic cycles. Fama and French [27], under the premise that their three-factor model did not show all the reality of market returns, incorporated two new factors: profitability and investment.

In this line of work, the first article of this thesis was developed, whose purpose is to incorporate a factor that represents the long-term memory of the market within the model proposed by Fama and French [26]. In this way, the aforementioned Hurst exponent is used as the memory factor.

In fractal geometry, the Hurst exponent has been defined by H or by H_q and serves to quantify the relative tendency of a time series to strongly return to the mean or to cluster in one direction [36]. What is sought when applying the Hurst exponent is to see if the time series being studied is persistent or not, since if persistence is found, there could be some kind of dependency between the data. In finance, we would be checking if the data behave according to the ordinary Brownian movement or, on the contrary, there is memory within the data series.

After carrying out a study of the different methodologies that are usually used in market studies for the Hurst exponent, it was chosen to use the *FD algorithm* developed by Sánchez et al.[59]. This methodology has the advantage of being able to calculate a wider range of movements and these do not necessarily have to be Brownian.

Once we define the calculation of the memory factor, we incorporate it into the factorial model of Fama and French [26]:

$$r_{it} = R_{fi} + \beta_{mi}(R_{mt} - R_{ft}) + \beta_{si}SMB_t + \beta_{vi}HML_t + \beta_{Hi}HF_t + \beta_{mmi}WML_t + \epsilon_{it} \quad (1.1)$$

The sample used for the calculations is made up of 2,500 shares that represent the most liquid securities in the US market for the time period from 2012-2016.

To know which factors are the ones that provide the most information in the study of returns, several models were proposed based on formula 1.1. We began by creating a model with a single factor, choosing the one that best explains the performance of the share. Then, a second model with two factors, incorporating to the factor that we already had the second factor that best represents the share's returns, was proposed. We replicated the method by creating the three-, four-, and five-factor model.

To measure which model is the one that best represents the profitability of the shares, we use the Akaike criterion (AIC), which measures the quality of each model according to its goodness of fit and its complexity.

The objective of this study is not to analyze the performance of the classic factors, but to verify the evolution of the expected average returns of different portfolios based on the evolution of the memory factor in order to verify its effectiveness.

Table 1.1 displays the monthly average logarithmic returns for different portfolios. The portfolios were created using 10 quartiles for the H value. The values obtained show that as the H factor increases, the average expected returns of the portfolios also increase. With these results, it was concluded that the Memory factor is indeed significant in the study of stock returns, so we continued the investigation by incorporating it as a factor in the Fama and French model [26].

To represent the market factor, two forms of calculation have been taken: the first is using the S&P500 (SPX) index as a proxy for the market and the second is by weighting all the values in the

H Decile	Low	2	3	4	5
Monthly return	0.007	0.006	0.007	0.007	0.007
Annualized return	0.0898	0.0716	0.0885	0.0960	0.0934
H Decile	6	7	8	9	High
Monthly return	0.009	0.011	0.012	0.015	0.017
Annualized return	0.1240	0.1419	0.1616	0.2087	0.2367

Table 1.1: Evolution of expected average returns for a sample of 1,500 securities for the period 1980-2018, ordered by their H decile value.

sample (EW) equally. Results change substantially depending on how the market is calculated. For example, when the tests were carried out creating portfolios composed of 30 shares, the market represented by the SPX proxy did not provide any type of information to the model, being size (SMB) and memory (H) the factors that best explain the profitability of the portfolios, as shown in Table 1.2.

	2012	2013	2014	2015	2016
HML %	24	68	100	99	94
SMB %	100	100	100	100	100
H %	100	100	100	100	100
MOM %	49	100	35	4	83
SPX %	0	0	0	0	0

Table 1.2: Percentage of presence in the models of each factor for 10,000 random portfolios of 30 securities, with the SPX as market factor.

Instead, when the EW method is used to calculate the market, this factor becomes the most important one to represent the performance of the stocks followed by the HML, SMB, H and MOM factors. In this case, momentum (MOM) is the least significant factor:

	2012	2013	2014	2015	2016
HML %	49	52	62	66	64
SMB %	40	45	56	57	56
H %	47	43	42	37	47
MOM %	27	29	27	23	32
EW %	100	100	100	100	100

Table 1.3: Percentage of presence in the models of each factor for 10,000 random portfolios of 30 securities, with EW as market factor.

As can be seen in the simulations, the market memory factor achieves very good results as an explanatory factor for portfolio returns. It is interesting to see the changes in the results according to the method of calculating the market, especially for the EW method, which has had exceptional results in explaining profitability, so it was decided to continue using this method of market representation for the following research that has been developed for the thesis.

1.3 Market comovement.

As seen in section 1.2.1, one of the objectives of the CAPM is to explain the correlation among different assets through their relationship with the market index, so it is expected that the market index can explain the comovement between two different assets, leaving the residual ϵ_{it} to differentiate the asset price trend.

Asset comovement has captured the attention of researchers for decades due to its importance for asset selection, portfolio diversification or risk management. The causes of comovement have been studied from different points of view. Roll [56] found that the level of joint stock movement depends on the relative amounts of firm-and market-level information capitalized into stock prices. Domowitz et al. [20] showed that the comovement of liquidity is determined by the flow of orders and that the comovement of profitability is caused by the type of order (market and limit orders). Byrne et al. [11] show that comovement is responsible for two-thirds of the variability in global bond yields. These authors concluded that global inflation explains most of the global stock return comovements.

Morck et al. [48] showed that returns on equity are more synchronous in emerging markets. The authors also show evidence that comovement is not a consequence of the structural characteristics of economies, like country and market size. Jach [35] studied the comovement variability over time, bivariate and multivariate among the profitability of various international stock markets. This work concluded that development and region are not always decisive factors. Parsley and Popper [51] found that the comovement of profitability does not depend on the wealth of the country, but it is more affected by variables that reflect different institutional aspects, including the stability of international macroeconomic policy.

Other authors have focused their attention on the comovement among different assets or countries. Bonfiglioli and Favero [9] found no evidence of long-term interdependence between the US and German stock markets. Rua and Nunes [58] analyzed the comovement among the major developed countries over 40 years. These authors conclude that it is stronger among countries at lower frequencies and it is also different in each one of them. They also state that the degree of comovement varies over time.

Akoum et al. [1] examined the joint movements of stock markets and oil prices in the GCC region. These authors showed evidence of a strong dependency from 2007 in the long term. However, in the short term, there is no clear evidence of dependency. Reboredo [55] did not find any extreme joint movement between oil prices and exchange rates in the periods before and after the financial crisis. Magdaleno and Pinho [42] reported a strong and significant relationship between index prices. The authors show that movements in the US and UK stock markets are not quickly transmitted to other markets. They also found that, both economically and geographically, economies show higher levels of comovements, except for Japan. Loh [41] studied the joint movement of 13 Asian-Pacific, European and American stock markets. This author found a significant joint movement among most of them in the long term. During the financial crisis, this author reports evidence of an important variation in comovement over time.

Some authors have also analyzed relevant factors in comovement. Baca et al. [3] showed evidence of the importance of global industry factors in explaining variation in international profitability. In a similar vein, Cavaglia et al. [15] showed that the country effect is more relevant than industry factors in the late 1990s and Griffin and Karolyi [32] found that global industry factors only explain the four percent of the variation of the local stock exchanges. However, L'Her et al. [37] reported evidence to the contrary, finding that global industry effects outweighed country effects in importance between 1999-2000. Brooks and Del Negro [10] analyzed the link between international stock market co-movement and firm-level variables that measure international diversification, finding a significant relationship between beta stock returns and these variables. Antonakakis and Chatziantoniou [2] showed that there is a clear negative correlation between political uncertainty and stock market returns, except during the financial crisis.

Regarding the measurement of comovement, different approaches have been proposed by the scientific community. Among them we can mention the cross-correlation analysis (Akoum and others [1]), spatial techniques (Fernández-Aviles and others [29]), regression coefficient (Brooks and De Negro [10]), quantiles (Cappiello and others [13]), queue dependency coefficient (Garcia and Tsafack [31]), copula (Rebodero [55]), time series analysis (Antonakakis et al. [2]), deficit multidimensional scaling approach (Fernández-Aviles and others [30]), or the approach based on the Hurst exponent (Ramos Requena et al. [54]).

In this line of research, the following works have been carried out:

1.3.1 Price comovement.

In the second article of this thesis, *A new look on financial markets on co-movement through cooperative dynamics in many-body physics* a totally new approach is proposed to study the comovement of the entire market according to some functions based on particle systems, based on previous works by Clara et al. [17], Sánchez et al. [60] and Puertas et al. [53].

In order to study comovement, the analysis of particle systems was borrowed and adapted to financial markets. From a physical point of view, a portfolio can be seen as a system of particles, in which the index represents a specific point in the system that characterizes the entire system, just like a center of mass. The main contribution to the movement of the assets is described by Brownian motion, and the interactions among the assets are unknown, if they exist at all, and induce collective motion.

In this second work, three proposals were elaborated to represent the comovement of shares based on a function of colloidal particle systems. The main points of the work carried out are summarized below.

Methodology

One of the functions that were created for the study of comovement from colloidal systems and that yielded very good results is the C_{3t} function:

$$C_{3t}(\tau) = \frac{\sum_{i,j} \delta x_i(t,\tau)\delta x_j(t,\tau)}{\sum_{i,j} |\delta x_i(t,\tau)\delta x_j(t,\tau)|} \tag{1.2}$$

$$C_3(\tau) =< C_{3t}(\tau) > \tag{1.3}$$

Where $x_i(t)$ is the logarithm of the price of asset i at time t, and $\delta x_i(t,\tau) = x_i(t+\tau) - x_i(t)$ is the logarithmic return of asset i in the period from t to $t+\tau$. The sums are made for each pair of assets i, j, excluding equal pairs $(i = j)$.

The possible results of this function range from -1 to 1. If the functions are close to 0, it means that there is no comovement; if they are greater than 0, it means that the values move in the same direction, and when they are less than 0, the values move in the opposite direction.

In order to achieve a deeper study of the market, the thesis proposes a second way of calculating comovement by subtracting the market average from function 1.2. Market average is subtracted as follows:

$$\delta x_i(t,\tau) = x_i(t,\tau) - x_i(t) - (m(t+\tau) - m(t)) \tag{1.4}$$

Where, x_i is the logarithm of the price of asset i and $m =< x_i >$ is the market (the average of the logarithms of the prices of all assets).

Results.

In this article, more than 3,000 securities have been considered (attending to liquidity, capitalization and free float criteria) in the US market from 2003 to March 2020, to carry out different simulations and study the behaviour of price comovement according to different ways of representing the market:

- EW: all shares have the same weight.
- CAP: the stocks in the sample are weighted according to their capitalization.
- SPX: corresponds to the SP 500 index.
- IWM: corresponds to the Russell 2000 index.

After analyzing the different results with the different market calculation methods, it was found that the EW method is the one that best explains the market comovement, obtaining values very close to zero when the market average is subtracted, as shown in the table 1.4.

These results of the market comovement so close to zero show that the market itself provides the vast majority of the information on the behaviour of the comovement, a thought that is in line with the theories of Beta.

Beta is used in the literature to model the log return of a stock relative to the log return of the market. The simplest model is the Sharpe model [61], but other models, such as the Fama-French factor model [24], also consider the beta as part of the model.

To verify the importance of the beta, it was decided to replicate the simulations with all the ways of representing the market, but incorporating the beta to the comovement model. To incorporate the beta to the model, the following was calculated:

$$\delta x_t(t,\tau) = \frac{x_i(t+\tau) - x_i(t)}{\beta_i} - (m(t+\tau) - m(t)) \tag{1.5}$$

Being m the different ways of calculating the market (EW, CAP, SPX, IWM). Assuming the beta model, the comovement should be less than the comovement calculated without considering the beta.

Table 1.5 shows the results of the comovement with the market beta and also representing the market according to EW. Comparing the results, it can be said that this version does not significantly improve the results obtained previously and, therefore, the previous model without the market beta is preferred since the model is simpler.

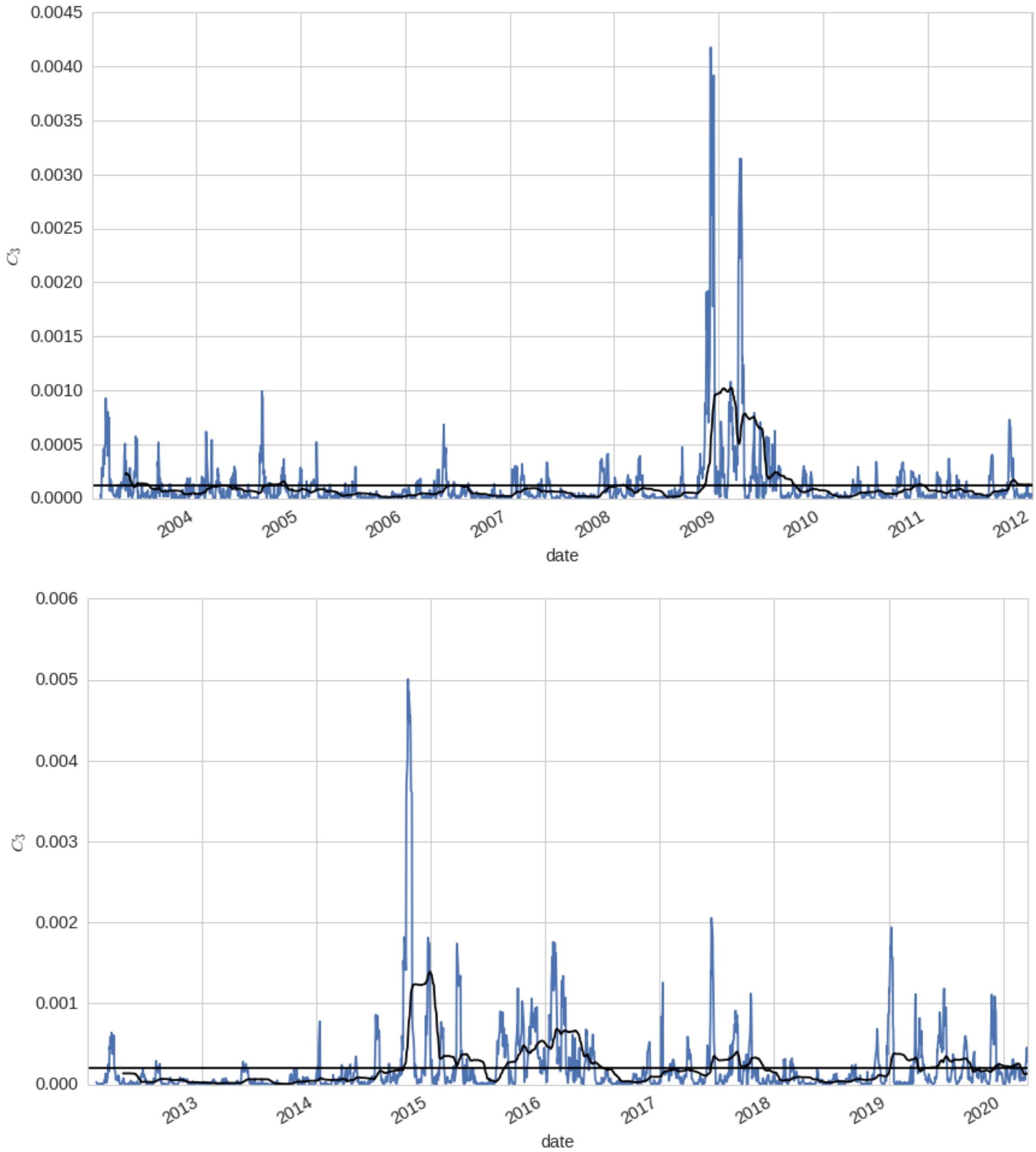
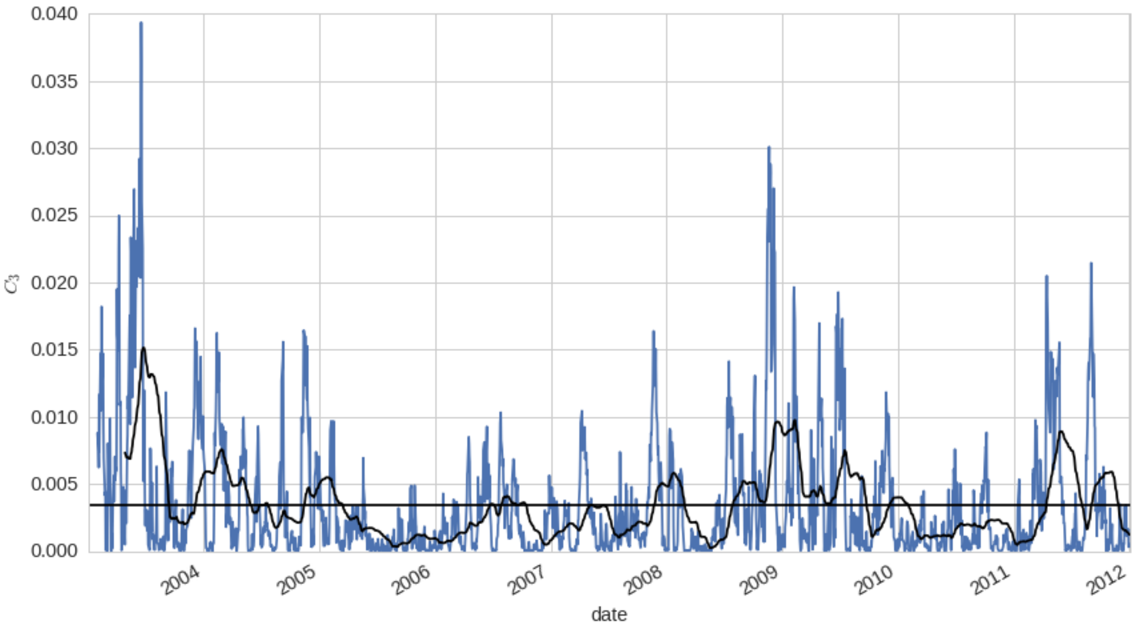


Table 1.4: Comovement ($C_{3t}(20)$) of the entire market during 2003-2011 (top graph) and 2012-2020 (bottom graph) with the market removed. The blue line is the daily comovement, while the black line is a moving average of the blue line with a time window of 60 trading days. Market representation is Equal Weighted (EW). The horizontal line represents the average comovement ($C_{3t}(20)$) throughout the entire period.



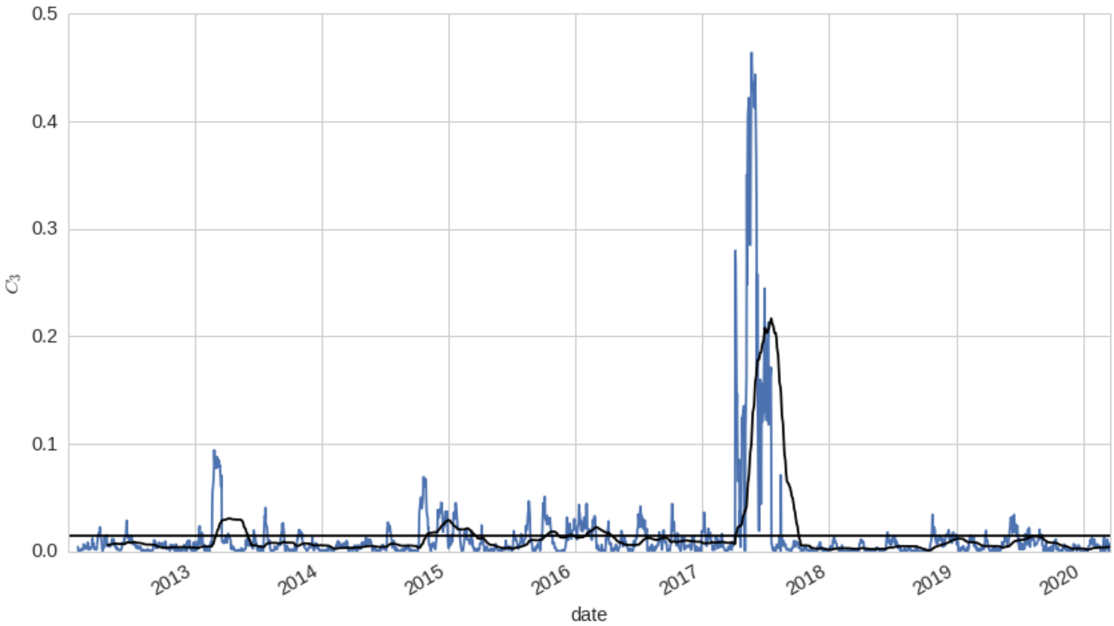


Table 1.5: Comovement ($C_{3t}(20)$) of the entire market during 2003-2011 (top graph) and 2012-2020 (bottom graph) with the market eliminated when considering the beta of each stock. The blue line is the daily comovement, while the black line is a moving average of the blue line with a time window of 60 trading days. The horizontal line represents the average comovement ($C_3(20)$) throughout the entire period.

1.3.2 Volatility comovement.

In the third publication of the thesis, *Volatility Co-Movement in Stock Markets*, the comovement was recalculated using the same methodology proposed in the previous article but using volatility as the study variable. As was the case before, it was found that subtracting the market the comovement takes values very close to zero. In this work we wanted to focus above all on the behaviour of the volatility variable due to its great importance in the field of finance. The main results obtained in this latest investigation are shown below.

Results.

On this occasion, 3,577 US stocks were used as a sample for the period between 2008 and 2020, sampled daily. The volatility of stock i at time t, $v_i(t)$, is calculated as the standard deviation of the log returns for a year (250 trading days) ending in t.

When representing the volatility variable in figure 1.1, it was observed that the range of volatilities is very wide, with most of the stocks being between 0 and 1. Since the range of volatilities is so wide, it was thought that there could be some dependence between the stocks classified according to their volatility. In addition, volatility is known to be a variable that changes over time, also changing the dependency between stocks. To study this behaviour, the securities were classified according to their volatility and we carried out an analysis.

Table 1.6 shows the comovement of volatility when we compare stocks with similar volatility in 0.1-width intervals and over time. In panel (a) the results are shown without subtracting the market, and in panel (b) they are shown subtracting the market.

In panel (a) the comovement of volatility decreases as volatility increases, so when we compare similar volatilities, the lowest volatilities reflect the highest comovement. In panel (b) the comovement results are very close to zero, although for similar volatilities, the results show an above-average comovement.

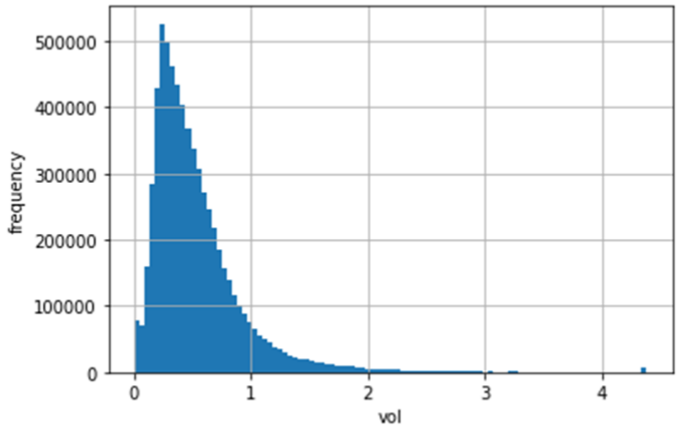


Figure 1.1: Histogram of the volatility variable

To study the behaviour of the comovement of volatility when different volatilities are compared, heatmaps 1.7 are presented. In these maps, the intensity of the comovement is graduated with colours that go from intense red (high comovement in the same direction) to intense blue (high comovement in the opposite direction), leaving white as the colour that expresses the non-existence of comovement.

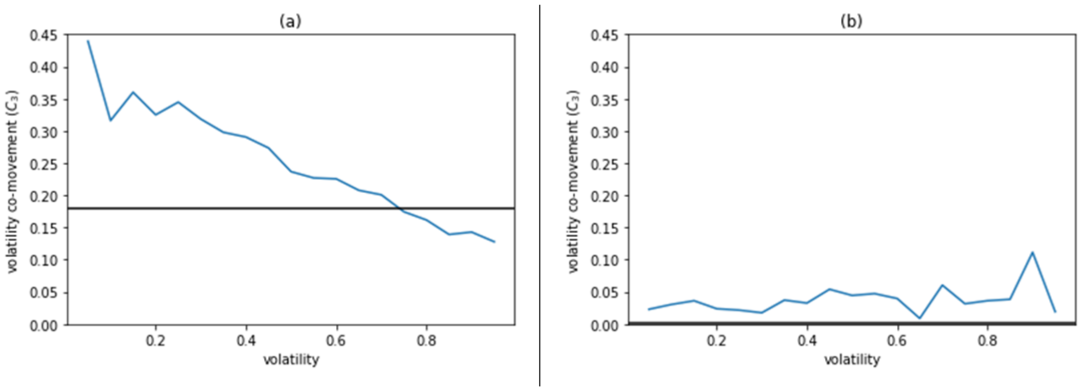


Table 1.6: Monthly volatility comovement ($C_3(20)$) for securities with similar volatility for the period 2008-2020; (a) the market is not subtracted; (b) the market is subtracted.

Panel (a) shows the results without subtracting the market. All the boxes are red in the chart, indicating a direct comovement in volatility among stocks with different degrees of volatility. The existence of a diagonal is appreciated, where the highest values of comovement are concentrated, being the extremes of the figure, where the highest volatilities are compared with the smallest, where the weakest comovements are concentrated.

Panel (b) shows the results subtracting the market. As expected, the results show a very weak comovement for all combinations among volatilities. This result confirms that most of the cooperative movement is indeed market-driven, calculated as the simple average of all stocks but not everything. In fact, an increase in the comovement around the diagonal is observed for large volatility stocks.

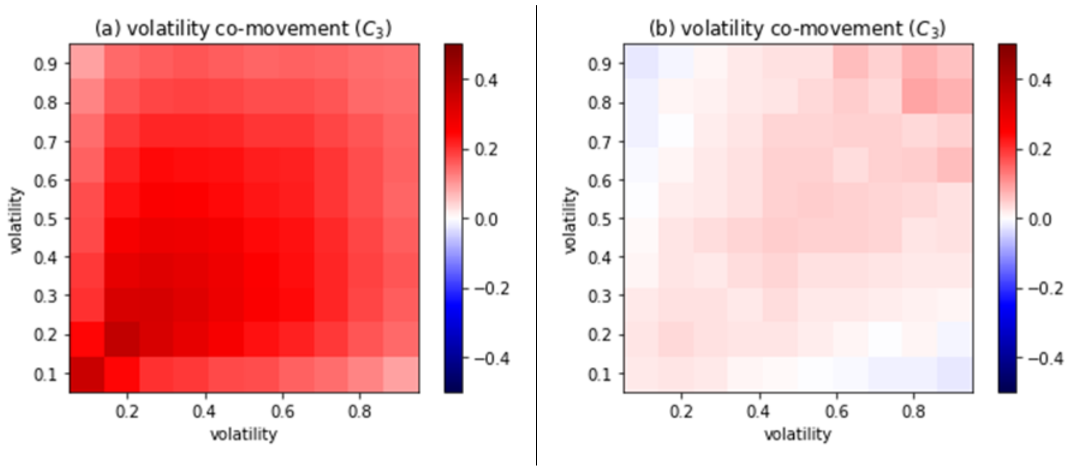


Table 1.7: Monthly comovement of volatility between stocks of different (and similar) volatilities. (a) Without subtracting the market; (b) subtracting the market.

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Chapter 2

Publications

Extending the Fama and French model with a long term memory factor

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Extending the Fama and French model with a long term memory factor

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ABSTRACT

In this paper, a new long-term memory factor for extending the well-known Fama and French model is proposed and discussed thoroughly. The new long-term memory factor is based on the Hurst exponent and is calculated using the fractal dimension (FD) algorithm. The relevance of the new factor is illustrated using a sample of 1500 largest U.S. companies from different sectors.

1. The foundations of capital asset pricing model

The behavior of stock market prices and returns is one of the fundamental topics in finance theory. Research efforts are devoted to identifying a proper distribution function, which fits market returns (interesting contributions were made by Boyarchenko & Levendorskii, 2000; Hirschberg, Mazumdar, Slottje, & Zhang, 1992; Kim, Rachev, Bianchi, & Fabozzi, 2008; Kim, Rachev, Chung, & Bianchi, 2009; Kim et al., 2009; Kozubowski & Panorska, 1999; Kozubowski & Podgorski, 2001; Mandelbrot, 1963). The research also concentrates on different models determining the important pricing aspects and dependencies such as the relationship between risk and return or what are the explanatory variables of stocks returns.

In this last area one of the most important initial contribution was done by Sharpe (1964), who proposed the single index model also known as the diagonal model. The initial motivation behind the model was the aim to solve the covariance matrix estimation problem associated with the Markowitz model. Later papers of Lintner (1965), Mossin (1966) and Black (1972) extended and finalized the model. The classical capital asset pricing model (CAPM) was born.

The first CAPM empirical tests were done by Black, Jensen, and Scholes (1972) and Fama and MacBeth (1973), where the authors confirmed that the model is valid and explains returns of companies before 1969. Since then, beta is a parameter frequently used by investors to characterize risk of stocks. However, the first empirical problems quickly appeared and suggested that the model is incomplete. Reinganum (1981) and Lakonishok and Shapiro (1986) found the first CAPM anomalies for market returns in the period from 1964 to 1990. Later Fama and French (1993) found anomalies for the period from 1941 to 1990. The reason why the model is not fully valid seems to be the existence of additional factors, which are relevant for asset pricing. Litzenberger and Ramaswamy (1979) showed a positive relationship between dividend yield and return for common stocks. Basu (1977, 1983) found that price earning ratios and risk adjusted returns are also related. Later, Banz (1981) showed the importance of the total market value for asset pricing. The author found that average returns of small NYSE firms were high compared to estimated betas while for large caps these were too small over the forty-year period considered. Banz called this market anomaly the size effect. Other interesting CAPM anomaly was showed by Bhandari (1988), where the author claimed using market beta as well as size factor that expected common stock returns are positively related to the debt/equity ratio (non common equity liabilities).

Stattman (1980), Rosenberg, Reid, and Lanstein (1985), and Chan, Hamao, and Lakonishok (1991) proved the relevance of book to market ratio to determine the expected return of a stock. After all these empirical contradictions to the CAPM theory, Fama and French have proposed a new three-factor model extending the

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classical CAPM. In this paper, we propose to include a memory factor measured through its Hurst exponent as an additional explanatory variable for stocks expected returns.

2. The Fama and French factor models

According to the traditional CAPM a stock (or portfolio) returns can be described as a lineal function of its market beta:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it} \tag{1}$$

where R_i is the return on security i for period t, R_{mt} is the market return (usually proxied through its main equity index or an all-shares value-weighted portfolio), and ϵ_{it} is a zero-mean residual.

Adding the riskfree return R_f to (1), the equation can be rewritten as:

$$R_{it} - R_{ft} = \alpha_i + \beta_i (R_{mt} - R_{ft}) + \epsilon_{it} \tag{2}$$

Based on the empirical evidence on U.S. stocks and peculiarities associated with the CAPM thereon, Fama and French (1993, 1995) proposed an extension of (1) by introducing two new factors, capturing patterns associated with the size and value versus growth stocks. The three-factor empirical asset pricing model is defined then as:

$$R_{it} - R_{ft} = \alpha_i + \beta_i (R_{mt} - R_{ft}) + s_i SMB_t + h_i HML_t + \epsilon_{it} \tag{3}$$

where SMB_t is the returns on a diversified portfolio of small stocks minus the returns on a diversified portfolio of big stocks, and HML_t is the difference between the returns on diversified portfolios of high book-to-market and low book-to-market stocks.

Many studies[1] showed that (3) is an incomplete model for expected returns because its three factors miss much of the variation in average returns. The empirical evidence shows that extreme growth stocks as well as microcap extreme growth stocks represent a problem for the original three-factor model. For this reason Fama and French (2015) extended the original model with two new factors. Motivated by the dividend discount valuation model, Fama and French consider the expected earnings as a critic factor for expected returns and use (following Novy-Marx, 2013) profitability as a proxy value for expected earnings. A further factor included in the new five-factor model is the company investment based on Miller and Modigliani (1961) and on a previous work of Aharoni, Grundy, and Zeng (2013). The Fama and French five-factor model is defined then as:

$$\begin{aligned} R_{it} - R_{ft} &= \alpha_i + \beta_i (R_{mt} - R_{ft}) + s_i SMB_t + h_i HML_t \\ &\quad + r_i RMW_t + c_i CMA_t + \epsilon_{it} \end{aligned} \tag{4}$$

where RMW_t is the difference between the returns on diversified portfolios of stocks with robust and weak profitability correspondingly, and CMA_t is the difference between the returns on diversified portfolios of the stocks of companies with low and with high investment practices.

3. Long memory in financial markets.

The study of long memory in financial markets has moved into spotlight after Mandelbrot (1963) conjectured that the stock market prices may follow a fractional Brownian motion and the latter represents is a long-memory generalization of a standard Brownian motion process. After this contribution, the long memory topic has caught attention of many researchers.[2]

In order to see whether a time series possesses a long memory or not, the hydrologist H.E. Hurst introduced is 1951 the *Hurst exponent*, which is based on the idea that natural phenomena possess a long memory feature. In fractal geometry, the Hurst exponent (usually denoted by H or H_q) provides a measure for long term memory and fractality of a time series. H allows us to roughly classify series into three categories: $H = 0.5$ indicates that we have a random series, an H value between 0 and 0.5 indicates an anti-persistent series, and finally H greater than 0.5 indicates that we have a persistent series, meaning that the data have a long memory. An anti-persistent series is mean-reverting, which means that an up value is more likely followed by a down value and vice versa.

The most popular method to estimate Hurst exponent is R/S analysis (Hurst, 1951; Mandelbrot & Wallis, 1969). However, taking into account that several authors (Couillard & Davison, 2005; Lo, 1989; Weron, 2002 and recently Sanchez Granero et al., 2008) showed that R/S analysis is biased when the series are too short, the attempts to improve H estimation has been a popular topic over the recent decades and several methods have been developed and tested by researchers. Alternative approaches are Multifractal Detrended Fluctuation Analysis (MF-DFA), the Lyapunov exponent, Hudak's Semiparametric Method (GPH), Quasi Maximum Likelihood analysis (QML), Centered Moving Average (CMA), Generalized Hurst Exponent (GHE), the Periodogram Method, Wavelets Methodology, Geometric Method-based procedures (GM) and Fractal Dimension Algorithms (FD).

3.1. Background: momentum factor

The momentum factor was introduced by Carhart (1997) to amend the three-factor model and the enhanced model was applied to analyze the classical problem of mutual fund return persistence.[3] The four-factor model is defined as:

$$R_{it} = \alpha_i + \beta_{mi} R_{mt} + \beta_{si} SMB_t + \beta_{vi} HML_t + \beta_{mmi} MOM_t + \epsilon_{it} \tag{5}$$

where MOM_t is a momentum return at time t measured as difference between the month t returns of diversified portfolios of the winners and losers of the past year.

Since then regressions (3) and (5) have been widely used to explain stocks return. Avramov and Chordia (2006) showed that the momentum factor is not consistent in capturing the impact of past returns on the cross-section of returns when applied to a sample of NYSE-AMEX and NASDAQ stocks. The authors found that momentum is able to capture the impact of NASDAQ turnover on returns, though. The authors also suggested that the benefits of the momentum factor are rather associated with the business cycle. A further conclusion was that the momentum factor is lacking a tangible economic rationale. Other authors, such as De Bondt and Thaler (1985) or Jegadeesh and Titman (1993), argued that stocks with low past returns tend to have higher future returns or that a tendency in returns is likely to continue in the future. In any case, the use of momentum has been very popular in recent years. Some interesting contributions are for example Billio, Calés, and Guégan (2011), in which authors present a theoretical framework for modeling the evolution of a dynamic portfolio considering the momentum effect; Post (2008), where the author has investigated the effect of short-selling restrictions on momentum strategies or Pätäri, Karell, Luukka, and Yeomans (2018) where the efficiency of

[1] See, for example, Novy-Marx (2013), Titman, Wei, and Xie (2004).

[2] See earlier works by Diebold and Rudebusch (1989), Shea (1991), Backus and Zin (1993), Hassler (1994) and Hassler and Wolters (1995) or Peters (1996) as well as recent contributions done by Jiang, Xie, and Zhou (2014) or Buonocore, Aste, and Di Matteo (2017) and by the authors Sanchez Granero, Trinidad Segovia, and Garcia Pérez (2008), Trinidad Segovia, Fernández-Martínez, and Sánchez-Granero (2012), Sanchez Granero, Fernández Martínez, and Trinidad Segovia (2012), and Sanchez Granero, Trinidad Segovia, Garcia, and Fernandez Martinez (2015).

[3] Momentum based strategies were first used by Jegadeesh and Titman (1993), and later applied by Grinblatt, Titman, and Wemers (1995) to determine whether mutual fund managers could outperform exploiting such strategies.

four multicriteria decision-making (MCDM) methods in identifying the future best-performing stocks has been evaluated, using different OR techniques to combine value and momentum indicators into a single efficiency score.

In 2012 Fama and French (2012) proposed a revised model (5), which was applied to a sample of international companies. The main contribution is that the sample covers all size groups and the authors examined how well (3) and (5) capture average returns for portfolios formed based on size and value, or size and momentum. Further, the GRS test rejects the hypothesis that the true intercepts are zero for both, three- and four-factor models, however, when microcaps are excluded, the intercepts for the global four-factor model suggest that it is acceptable in explaining average returns of global portfolios. The authors conclusion is that (5) is valid in explaining the returns on global portfolios as long as the portfolio does not have a strong tilt toward microcaps or toward the stocks of a particular region. It seems that the four-factor asset pricing model does not perform properly when it is applied to local size-momentum portfolios. The authors also suggested that the problems of the model are concentrated around portfolios with extreme tendency toward winners or losers.

In this paper we propose the use of the Hurst exponent as an additional factor in the four-factor model. We will denote as H our new factor by setting a portfolio with the difference of the average returns of stocks with higher semi-annual H and the average returns of stocks with lower semi-annual H. The Hurst exponent will be calculated through *FD4* algorithm.

3.2. Estimating Hurst exponent using fractal dimension algorithms

The family of fractal dimension algorithms (FD algorithms henceforth) was introduced in Fernández-Martínez and Sanchez-Granero (2012), based on the concept of fractal dimension of a curve previously discussed in Fernández-Martínez and Sanchez-Granero (2012). The authors considered fractal dimension as a generalization of the Hurst exponent, and the main advantage of these algorithms is that they can be used for a wider range of underlying distributions than Brownian motions. Four versions of the FD algorithms were defined: the first three of them were introduced in Fernández-Martínez and Sanchez-Granero (2012) and the last one in Fernández-Martínez, Sánchez-Granero, Trinidad Segovia, and Román-Sánchez (2014), where the first three are obtained from a multifractal approach. In general, the FD algorithms are calculated as follows.

Let X be a random variable for which its absolute qth-moment is defined as $m_q(X) = E[X^q]$ for each $q > 0$, provided that such expected value exists. Thus, let $\mathbf{X}(t, \omega)$ be a random process with stationary increments and let us suppose that there exists a parameter $H > 0$ such that $M(T, \omega) \sim T^H M(1, \omega)$ holds (for example, if $\mathbf{X}$ is an H-self similar process), where $M(T, \omega)$ is the T-period cumulative range of the random process $\mathbf{X}(t, \omega)$ and $\sim$ means that they have the same finite joint distribution functions. Hence, if we take q-powers in the previous equation then we have that

$$M(T, \omega)^q \sim T^{qH}\ M(1, \omega)^q \tag{6}$$

for any $q > 0$. Let us consider X_n to be the $\frac{1}{2^n}$-period cumulative range of the random process $\mathbf{X}$, namely $X_n = M(1/2^n, \omega)$ for all $n \in \mathbb{N}$, then we have that $X_n^q \sim (\frac{1}{2^n})^{qH}\ X_0^q$ and $X_n^q \sim 2^{qH}\ X_{n+1}^q$ for all $n \in \mathbb{N}$ and all $q > 0$.

Hence, the mean of these distributions must be equal, namely:

$$m_q(X_n) = \frac{1}{2^{nqH}}\ m_q(X_0). \tag{7}$$

and

$$m_q(X_n) = 2^{qH}\ m_q(X_{n+1}), \tag{8}$$

Table 1
Evolution of expected average returns for a sample of 1500 stocks for the period 1980–2018, ordered by its H value decile.

H Decile	Low	2	3	4	5
Monthly return	0.007	0.006	0.007	0.007	0.007
Annualized return	0.0898	0.0716	0.0885	0.0960	0.0934
H Decile	6	7	8	9	High
Monthly return	0.009	0.011	0.012	0.015	0.017
Annualized return	0.1240	0.1419	0.1616	0.2087	0.2367

By taking 2-base logarithms on Eq. (8), we obtain:

$$\log_2\left(\frac{m_q(X_n)}{m_q(X_{n+1})}\right) = qH, \tag{9}$$

The Hurst exponent (or self-similarity index) H of the random process $\mathbf{X}$ could be estimated using (9) as:

$$H = \frac{1}{q}\ \log_2\left(\frac{m_q(X_n)}{m_q(X_{n+1})}\right) \tag{10}$$

This approach is called *FD*. We refer the reader to Fernández-Martínez et al. (2014) for further discussion and different ways to calculate H based on this approach. In this paper, we will use the *FD4* algorithm. That is, we will use Eq. (10) with $q = 0.01$ and n such that X_{n+1} and X_n are the cumulative range of one and two days, respectively.

4. Empirical application

In Fama and French (2012), the authors analyzed the behavior of different portfolios by regions studying the changes in portfolio returns depending on the pair of factors. They set up 25 portfolios formed by size and book to market factors and other 25 with size and momentum factors. The authors found that average expected returns increased when the value of book to market and momentum factors also increased. Further, they observed that a size effect is caused mainly by the micro caps.

We are not going to analyze the performance of the classical factors on our sample as we consider that previous studies such as Fama and French (2012) are representative. However, we have tested the evolution of the average expected returns of different portfolios according to the evolution of the H factor. Table 1 shows the average monthly (and annualized) log-returns. The portfolios have been created using 10 quantiles for the value of H ($1 :< 0.5218; 2 : 0.5218–0.5429; 3 : 0.5429–0.5584; 4 : 0.5584–0.5724; 5 : 0.5724–0.5867; 6 : 0.5867–0.6031; 7 : 0.6031–0.6255; 8 : 0.6255 – 0.6691; 9 : 0.6691–1.9103; 10 :> 1.9103$). Table 1 outlines that similarly to the case of book to market and momentum factors, the expected average returns of different portfolios increase when the corresponding H factor increases.

We conclude that H seems to be a significant factor in stock return. A similar analysis but using other algorithms to calculate H is performed in Fernández-Martínez, Sanchez-Granero, Muñoz Torrecillas, and Mckelvey (2017).

Now we analyze the performance of the H factor in a Fama and French five-factor model. We propose the following regression with five factors:

$$R_{it} = \alpha_i + \beta_{mi}R_{mt} + \beta_{si}SMB_t + \beta_{vi}HML_t + \beta_{mmi}MOM_t + \beta_{Hi}H_t + \epsilon_{it} \tag{11}$$

where H_t is the return of a portfolio composed by the stocks with the greatest semi-annual H value minus the stocks with the lowest semi-annual H value, with H calculated for each individual stock as described in the previous section using the *FD4* algorithm.

Table 2
Percent of presence in the models of each factor, with *SPX* as market factor.

	2012	2013	2014	2015	2016
HML (%)	44	43	79	77	69
SMB (%)	91	69	62	87	92
H (%)	83	86	83	88	74
MOM (%)	31	60	22	17	39
SPX (%)	2	3	3	4	1

Table 3
Percent of presence in the models of each factor, with *EW* as market factor.

	2012	2013	2014	2015	2016
HML (%)	47	43	52	55	60
SMB (%)	47	48	58	59	55
H (%)	52	41	48	40	47
MOM (%)	28	26	22	22	29
EW (%)	89	92	90	95	88

Let us study which factor in the five factor model is more significant to explain stock returns. To do this, first of all, we apply a one factor model choosing F_1 as the single factor that provides a better explanation for the data. After that, we apply a two factor model, one factor is F_1 and an additional factor, F_2, chosen from the rest of factors in such a way that a two-factor model with F_1 and F_2 provides a best explanation of the data. We repeat the procedure with 3, 4 and 5 factors.

We assume that the five-factor model will perform better that any model with less factors, but if the improvement is not statistically significant the model with less factors will be preferred. For that we use the Akaike (1974) test (AIC) defined as:

$$AIC = 2k - \ln(L) \tag{12}$$

where k is the number of factors and L is the maximum value of the verisimilitude function of the estimated parameters.

For a given group of models, it will be chosen the one with the minimum AIC. We propose 5 models: F_1, $F_1 + F_2$, $F_1 + F_2 + F_3$, $F_1 + F_2 + F_3 + F_4$ and $F_1 + F_2 + F_3 + F_4 + F_5$. The first model is the one which better fits if only one factor is used, the second model is the better fit if two factors are used and so on.

We use a sample of 2500 stocks with the highest liquidity in the U.S. market (the 200-day median daily dollar volume must be greater than \$2.5 million and the market capitalization must be greater than \$350 million). For each stock, we will choose a model with one, two, three, four or five factors, according to the Akaike test.

We proceed with our study of comparing first the percentage of appearances of the *SMB*, *HML*, *MOM*, *H* and market factors in those models. Our objective is to check how many of those models use each of the factors. For the market factor, one option is to use a broad index as a proxy of the market. In this case, we will use the S&P500 index (henceforth denoted by *SPX*). As an alternative for the market factor, we will use an equally weighted portfolio with all the stocks considered (denoted by *EW*).

If we use *SPX* as the market factor, as Table 2 outlines, factors *H*, *HML* and *SMB* are the ones with the most presence in the models in our sample, with *SMB* and *H* being the two more significant factors. It should be noted with respect to *H* that, excepting 2016, the factor is present above 80 percent every year. It is also interesting to note that *SPX* does not improve the model in hardly any case.

Surprisingly, if we use *EW* as the market factor, the results are very different. As Table 2 shows, in this case the market factor appears in over 90% of the models, while *HML*, *SMB* and *H* appears in the model roughly half of the time. The momentum factor *MOM* is the least significant factor in this case.

The results obtained for the EW factor are in line with the ones showed by Racicot and Rentz (2015, 2016), Racicot, Rentz, and Theoret (2018) and Racicot (2015) where some variants of the Fama and French model are analyzed using a new generalized method of moments estimator (GMM_d) that relies on robust instruments to estimate panel data regression models containing errors in variables. In those papers the authors conclude that all factors, except the market one, are not significant. However, if a standard econometric (OLS) method is used, the five-factor model seems quite effective in explaining the returns. In this paper, we propose a different approach to study the relevance of the *H* factor, but, as discussed, there are different ways to measure the significance of a factor which can lead to different conclusions.

Table 4
Percent of presence in the models of factors *H* and *MOM*, with *SPX* as market factor.

	No one (%)	*H* but no *MOM* (%)	*MOM* but no *H* (%)	*H* and *MOM* (%)
2012	12	57	5	27
2013	8	32	5	55
2014	14	64	3	19
2015	8	74	3	14
2016	20	41	7	33

Table 5
Percent of presence in the models of factors *H* and *MOM*, with *EW* as market factor.

	No one (%)	*H* but no *MOM* (%)	*MOM* but no *H* (%)	*H* and *MOM* (%)
2012	35	37	12	15
2013	45	29	14	12
2014	42	36	11	11
2015	46	32	14	8
2016	40	32	13	15

Actually, one may think that *H* is a kind of momentum factor. As Tables 2 and 3 show, the *H* factor is more explicative than the *MOM* factor.

According to this observation, Tables 4 and 5 summarize the results where we compare the presence of the *MOM* and *H* factors in order to see whether these factors are substitutive. We also want to test whether *H* performs better than *MOM*.

From results presented in Tables 4 and 5, we can clearly see that both factors are not substituting each other. There is a low to moderate amount of stocks for which we would choose a model with both factors, and there exists also a large amount of stocks for which the use of the *H* factor but not the *MOM* factor is important. On the opposite, the number of stocks for which the model would use the *MOM* factor but not the *H* one has order of 5–10% only. Therefore, we can conclude that *H* provides more information than *MOM* to explaining stock returns and that both factors are substantially different.

4.1. Portfolio factor analysis.

In order to evaluate the robustness of the previous results, we apply a similar analysis to random (equally weighted) portfolios composed of 10 and 30 stocks. So we use again Eq. (11) but now R_{it} is the return of such a random portfolio. Then we use the Akaike test, similarly to the previous analysis for individual stocks, to study which ones of the five factors are more relevant and which ones can be removed from the model. Ten thousand random portfolios of 10 (or 30) stocks are used in this study. As illustrated in Tables 6 and 7, if *SPX* is used as the market factor, the results are

Table 6
Percent of presence in the models of each factor for 10,000 random portfolios of 30 stocks, with *SPX* as market factor.

	2012	2013	2014	2015	2016
HML (%)	24	68	100	99	94
SMB (%)	100	100	100	100	100
H (%)	100	100	100	100	100
MOM (%)	49	100	35	4	83
SPX (%)	0	0	0	0	0

Table 7
Percent of presence in the models of each factor for 10,000 random portfolios of 10 stocks, with *SPX* as market factor.

	2012	2013	2014	2015	2016
HML (%)	37	57	93	91	83
SMB (%)	100	100	92	100	100
H (%)	100	100	100	100	100
MOM (%)	38	94	29	9	66
SPX (%)	0	0	0	0	0

Table 8
Percent of presence in the models of each factor for 10,000 random portfolios of 30 stocks, with *EW* as market factor.

	2012	2013	2014	2015	2016
HML (%)	49	52	62	66	64
SMB (%)	40	45	56	57	56
H (%)	47	43	42	37	47
MOM (%)	27	29	27	23	32
EW (%)	100	100	100	100	100

Table 9
Percent of presence in the models of each factor for 10,000 random portfolios of 10 stocks, with *EW* as market factor.

	2012	2013	2014	2015	2016
HML (%)	49	49	64	66	63
SMB (%)	41	47	56	58	57
H (%)	48	43	43	38	48
MOM (%)	28	30	27	23	32
EW (%)	100	100	100	100	100

clear: the more relevant factors to explain the return of a portfolio are *SMB* and *H*. Also, factors *HML* and *MOM* often improve the model, while the market factor does not provide an improvement of the model in any case, so any explanatory power of the market factor is better provided by the other four factors.

If *EW* is used as the market factor, the results again are very different, confirming the results obtained in the previous section (Tables 8 and 9): *EW* is the more significant factor (it appears in all models), while factors *HML*, *SMB* and *H* appear in roughly half of the models. The momentum factor is the least significant again.

5. Conclusions

In this paper a new memory factor *H*, which extends the classical Fama–French factor model, is introduced. The factor utilizes the concept of long memory, which has been a popular topic in finance over the last decade, see, for instance, Lopez Garcia and Ramos Requera (2019). More precisely, the *H* factor is quantified through the Hurst exponent and we propose its estimation via the *FD*4 algorithm. In general, a factor based on long memory has a financial rationale, since there are several causes which may be sources of long memory (e.g. economic cycles, global tendencies, herding behavior or fashion stocks or industries).

As an empirical illustration, we show that *H* factor is one of the significant factors in the extended model. In fact, its significance level is similar to the capitalization (*SMB*) and the book-to-market (*HML*) factors and even greater than the momentum (*MOM*) factor. The relevance of the *H* factor is even greater if portfolios of 10 or 30 stocks are considered instead of individual stocks. We also observe that the way we calculate the market factor is very relevant for the analysis: while the market factor with an equally weighted portfolio is very significant in the model, the market factor calculated as a capitalization weighted portfolio (the S&P500 index in this case) is very close to be irrelevant in the model. Finally, though *H* and *MOM* factors may be related (as the momentum and long memory may seem to be related phenomena), we show that they are not the same factor and also that *H* is more significant than *MOM*.

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A new look on financial markets co-movement through cooperative dynamics in many-body physics

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A New Look on Financial Markets Co-Movement through Cooperative Dynamics in Many-Body Physics

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Abstract: One of the main contributions of the Capital Assets Pricing Model (CAPM) to portfolio theory was to explain the correlation between assets through its relationship with the market index. According to this approach, the market index is expected to explain the co-movement between two different stocks to a great extent. In this paper, we try to verify this hypothesis using a sample of 3.000 stocks of the USA market (attending to liquidity, capitalization, and free float criteria) by using some functions inspired by cooperative dynamics in physical particle systems. We will show that all of the co-movement among the stocks is completely explained by the market, even without considering the market beta of the stocks.

Keywords: econophysics; collective motion; finance; stock market; capital assets pricing model

1. Introduction

The Capital Asset Pricing Model (CAPM) by Sharpe [1] Lintner [2] and Black [3,4] is one of the most relevant models within financial economics. The contribution of the CAPM to finance was associating the return on a financial asset with its systematic risk, which is measured by the market parameter Beta.

The CAPM approach to the study of the return on financial assets is as follows:

$$r_i - R_f = \alpha_i + \beta_i (R_m - R_f) + \epsilon_{it} \tag{1}$$

where,

r_i is the return on the asset i,
α_i is the part of the return on the asset i that is due to aspects of the asset itself, such as profit, business, etc.
R_f is the return of the risk-free asset,
R_m is the market return (usually proxied through its main equity index or an all-shares value-weighted portfolio),
$(R_m - R_f)$ is known as the market risk premium, and
ϵ_{it} is a zero-mean residual.

In a sense, the CAPM is used to explain the differences in risk premiums for different assets in the market, which are caused by differences in the risk of asset returns. According to the CAPM, β_i is the factor that measures the sensitivity of the return on asset i to market movements. Accordingly, if the risk premium of a particular asset needs to be predicted, it is only necessary to select a risk-free rate and the beta of this asset. This model has been widely studied not only in the financial literature, but also by practitioners. However, there is a permanent debate about the usefulness of the market model, since, over time, empirical work has emerged with contrary results about whether market beta is indeed a useful factor in the study of a portfolio's performance. Blume [5], Black et al. [6], and Fama and Macbeth [7] performed the first tests over the CAPM and stated that indeed it provides a good prediction model. Conflicting results were obtained by King [8], who found the existence of variance due to industry effects and that the market effect accounts for only about 50 per-cent of the variance. Meyer [9] also found numerous unexplained components that represent some persistent significant source of interdependence among stock prices. Banz [10] found that the size effect of firms explained better than market *beta* the variation in average returns at a portfolio. Roll [11] found that market *beta* explains monthly stock price changes only in an average of 30%. However, testing other factors, such as industry or size, this author concluded that explanatory power by systematic economic factors is not different across industries. Bhandari [12] also found a positive relationship between leverage and average return. This author concluded that *beta* leaves out important information in order to explain more accurately the reality. Based on previous results of Stattman [13] and Rosemberg et al. [14], Fama and French [15] proposed an alternative model including three factors: the market *beta*, the size effect and the book to market ratio. According to their results, the market *beta* barely had the capacity to explain the variations in the average yields of the selected stocks, while size and value factors showed interesting results. The new factorial model proposed by Fama and French made clear the insufficiency of the market *beta* to cover all of the information needed to explain the variations in returns. Over time, advocates of the market *beta* have been studying the reasons why the CAPM no longer performs well in empirical studies. A good example is found in Isakov [16]. In this paper, the authors stated that *beta* has no chance of being a useful variable in recent empirical studies for two reasons. The first reason is that the model is expressed in terms of expected yields, but the tests can only be performed on yields that have been realized. The second reason is that excess realized market returns do not behave as expected, i.e., they are too volatile and often negative. There are similar works with different methodologies proposed, where it is shown that the market beta is still valid under some circumstances (see, for example, Pettengill, Sundaram, and Mathur [17], Chan and Lakonishok [18], Grundy and Malkiel [19], or Lopez et al. [20]).

For our purposes, one of the main goals of the CAPM is to explain the correlation of different assets through its relationship with the market index given by Equation (1). This represented an important advance in portfolio selection, because the market index is expected to explain all of the co-movement between two different assets, leaving only the residual ϵ_{it} to differ the trend of asset prices.

Assets co-movement has captured the attention of researchers due to its importance for asset allocation, portfolio diversification, or risk management. Causes of co-movement have been studied from different points of view. Roll [11] found that the level of stocks co-movement depends on the relative amounts of firm-level and market-level information capitalized into stock prices. Domowitz et al. [21] proved that liquidity co-movement is determined by order flow and return co-movement is caused by order type (market and limit orders). Byrne et al. [22] found that co-movement is responsible for two-thirds of variability in global bond yields. These authors concluded that global inflation explains most of the global yield co-movement.

Morck et al. [23] found that stock returns are more synchronous in emerging markets. The authors also show evidence that co-movement is not a consequence of structural characteristics of economies, such as fundamentals volatility or country and market size. Jach [24] quantified time-varying, bivariate, and multivariate co-movement between international stock market return. This work concluded that development and region are not always decisive factors. Parsley and Popper [25] observed that return co-movement does not depend on the richness of the country, but it is more affected by variables that reflect different institutional aspects, including international macroeconomic policy stability.

Other authors have focused their attention on the co-movement between different assets or countries. Bonfiglioli and Favero [26] did not find evidence of long-term interdependence between US and German stock markets. Rua and Nunes [27] analyzed the co-movement across major developed countries among 40 years. These authors found that the co-movement is strong across countries in lower frequencies and it is different in each one. They also remarked that the degree of co-movement varies over time. Akoum et al. [28] examined the co-movements of stock markets in the GCC region and crude oil prices. These authors showed evidence of a strong dependence after 2007 in the long term. However, in the short term, there is no clear evidence of dependence. Reboredo [29] found no extreme co-movement between oil prices and exchange rates in the periods before and after the financial crisis. Magdaleno and Pinho [30] reported a strong and significant relationship between index prices. The authors show that innovations in the US and UK stock markets are not rapidly transmitted to other markets. They also found that, economically, as well as geographically, economies show higher levels of co-movements, except Japan. Loh [31] studied the co-movement of 13 Asia-Pacific stock markets and the European and US ones. These authors found a significant co-movement between most of them in the long run. During the financial crisis, this author reports evidence of important variation in co-movement in time.

Factors revelant in co-movement have also been analyzed by some authors. Baca et al. [32] showed evidence of the importance of global industry factors in explaining international return variation. In a similar line, Cavaglia et al. [33] showed that country effect is more relevant than industry factors in the late-1990s and Griffin and Karolyi [34] found that global industry factors only explain four percent of the variation in local stock markets. However, L'Her et al. [35] reported evidence in an opposite sense, finding that global industry effects surpassed country effects in importance in 1999–2000. Brooks and Del Negro [36] analyzed the link between international stock market co-movement and firm level variables that measure international diversification, finding a significant relationship between stock returns *betas* and those variables. Antonakakis and Chatziantoniou [37] showed that there is a clear negative correlation between the policy uncertainty and stock market returns, except during the financial crisis.

Different approaches have been used to measure co-movement, such us cross-correlation analysis (Akoum et al. [28]), spatial techniques (Fernández-Aviles et al. [38]), regression coefficient (Brooks and De Negro [36]), quantiles (Cappiello et al. [39]), tail-dependence coefficient (Garcia and Tsafack [40]), copula approach (Rebodero [29]), time series analysis (Antonakakis et al. [37]), shortfall-multidimensional scaling approach (Fernández-Aviles et al. [41]), or a recent approach based in Hurst Exponent (Ramos Requena et al. [42]). In this paper, we propose a new approach to study the co-movement of the whole market based on some functions borrowed from or based on physical particle systems, following our previous works on this topic (Clara Rahola et al. [43], Sánchez Granero et al. [44], Puertas et al. [45]).

From a physical point of view, a portfolio can be seen as a many-body system, with the index representing a particular point of the system characterizing the whole system, such as the center of mass. The main contribution to the asset motion is described by Brownian motion, and the interactions among the assets are unknown, if they exist, and they induce collective motion. However, as given by classical mechanics of many-body systems, internal forces only affect the relative motion of a body with respect to the center of mass. Thus, a proper account of cooperative motion, within a physical approach can only be made if the center of mass (or index) is subtracted.

We study co-movement in the whole market in this paper by considering some functions inspired by co-movement in physical particle systems. It is showed that the market (represented by an equal weighted portfolio of all stocks) explains all of the co-movement in the whole market. Alternative ways to represent the market are considered, for example, portfolios weighted by capitalization or known indexes, in order to check whether these representations are also able to explain all the co-movement in the market. Some other alternatives that take into account the market beta are also considered.

2. Methodology

Cooperative dynamics have been observed in many physical particle systems. Furthermore, due to the short-time Brownian motion of assets, here we focus on particles with Brownian dynamics, namely colloidal systems. In colloids, as well as in atomic or molecular fluids, upon lowering the temperature the dynamics of the system slows down until the glass transition is reached, and the dynamics freezes, ideally. Accompanying this slowing down, the dynamics becomes more cooperative with more mobile particles clustering together in string-like structures (Donati et al. [46], Cates and Evans [47], Weeks et al. [48]). To study this phenomenology, several so-called four points correlation functions have been proposed, relating the dynamics of two particles at two different times (Glotzer et al. [49], Berthier et al. [50]). The most direct one is the evolution of the distance between a given pair of particles (Muranaka et al. [51], Zahn et al. [52]). Inspired by these functions, here we propose three simple observables that can be used to monitor the cooperative dynamics in financial systems:

- C_1. Consider the functions

$$C_{1t}(\tau) = \frac{\sum_{i,j}(\delta x_i(t,\tau) - \delta x_j(t,\tau))^2}{\sum_{i,j}(\delta x_i(t,\tau)^2 + \delta x_j(t,\tau)^2)}$$

$$C_1(\tau) = < C_{1t}(\tau) >$$

  where $x_i(t)$ is the log-price of asset i at time t, $\delta x_i(t,\tau) = x_i(t+\tau) - x_i(t)$ and the sum is for each pair of assets i, j, excluding the pairs i, i. C_1 is the average of $C_{1t}(\tau)$ over time origins t.

  The function C_{1t} is a measure of co-movement, from time t to time $t+\tau$, along the time t and the function C_1 is a measure of co-movement for the full period considered.

  The interpretation of these functions is as follows: if the resulting function is close to 1, it means that there is no co-movement in the whole market; if it yields values lower than 1, then we can say that the stocks move together; and if, on the contrary, the values are greater than 1, the stocks tend to move in the opposite direction.

- C_2. Consider the functions

$$C_{2t}(\tau) = \frac{\sum_{i,j}\delta x_i(t,\tau)\delta x_j(t,\tau)}{\sum_{i,j}(\delta x_i(t,\tau)^2 + \delta x_j(t,\tau)^2)}$$

$$C_2(\tau) = < C_{2t}(\tau) >$$

  where the same notation is used.

  In this case, if the functions are close to 0 it means that there is no co-movement; if they are greater than 0, it means that the stocks move in the same direction and, when they are less than 0, the stocks move in the opposite direction.

- C_3. Consider the functions

$$C_{3t}(\tau) = \frac{\sum_{i,j}\delta x_i(t,\tau)\delta x_j(t,\tau)}{\sum_{i,j}|\delta x_i(t,\tau)\delta x_j(t,\tau)|}$$

$$C_3(\tau) = < C_{3t}(\tau) >$$

where the same notation is used.

These functions are interpreted in the same way that C_2. Note that C_{3t} and C_3 take values between -1 and 1.

Note that functions C_1 and C_2 are analogous to functions $\gamma(\tau)/ < (\delta x)^2 >$ and $C(\tau)$ in Puertas et al. [45], respectively, while function C_3 is a variation of C_2. The three functions are intended to measure the degree of co-movement among all the stocks in the market, though they are slightly different.

3. Results

In this work, more than three thousand stocks (attending to liquidity, capitalization and free float criteria) in the USA market are considered from 2003 to March 2020. We will first show the results of the three functions proposed above, and the importance of subtracting the market to the evolution of the asset prices. Next, we will show how these parameters evolve in time and can be used to identify high-volatility periods. In the final subsection, other forms to represent the market are studied.

3.1. Co-Movement for Different Lag Times with and without Considering the Market

In this section, we want to study the behavior of the USA market through functions that show the co-movement of the shares. To calculate the shares, we will use, as data, the logarithm of the share price in two specific years, 2008 and 2018, in order to be able to check how the co-movement is affected in crisis stages and other more stable ones.

We will calculate the functions of the co-movement in two different ways: the first as described in the previous section, and the second by subtracting the market average, which is, $\delta x_i(t,\tau) = x_i(t+\tau) - x_i(t) - (m(t+\tau) - m(t))$, where x_i is the log price of asset i and $m =< x_i >$ is the market (the mean of the log-price of all assets). The purpose is to check how the market average affects the stock co-movement calculations. Note that $m(t+\tau) - m(t) =< x_i(t+\tau) - x_i(t) >$, which is, the market log-return is the equally weighted mean of the log-return of all the stocks.

The subtraction of the market is equivalent to removing the motion of the center of mass in physical systems. There, the dynamics of a system of particles can be separated in the motion of the center of mass, and the dynamics respect to it. Whereas external forces affect the center of mass, the internal dynamics is controlled by the internal forces.

We will use the three functions described in the previous section to measure the degree of co-movement.

In Figures 1–3, we show the results that were obtained with C_1, C_2, and C_3, respectively, for the years 2008 and 2018, depending on whether or not the market average is taken into account.

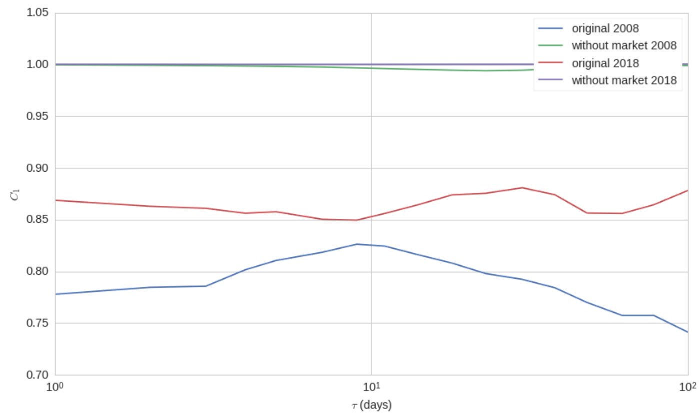


Figure 1. Co-movement with C_1 function. A value of 1 means that there is no co-movement, while values of less than 1 mean that the stocks tend to move in the same direction.

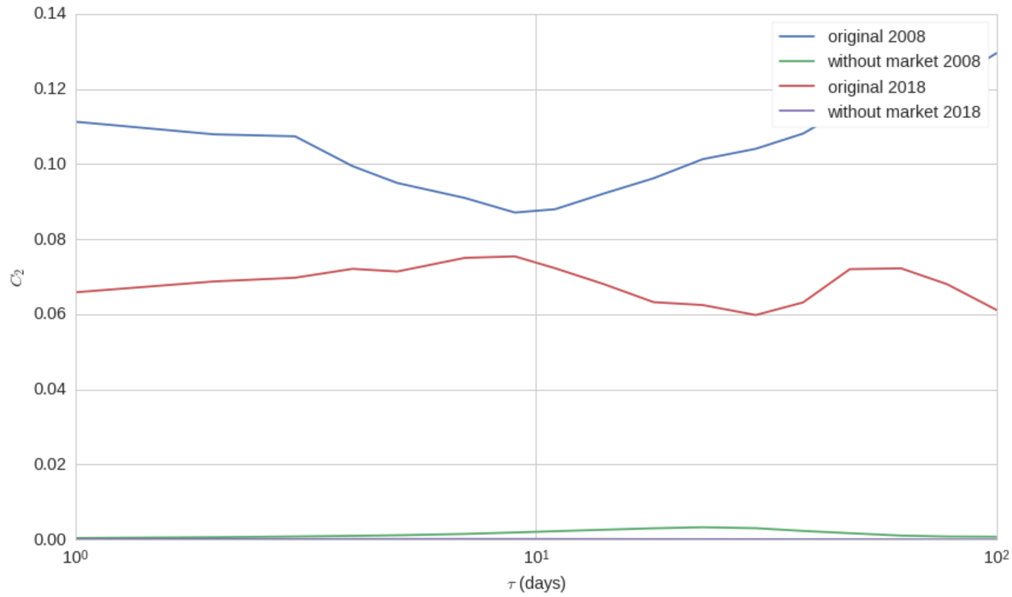


Figure 2. Co-movement with C_2 function. A value of 0 means that there is no co-movement, while values greater than 0 mean that the stocks tend to move in the same direction.

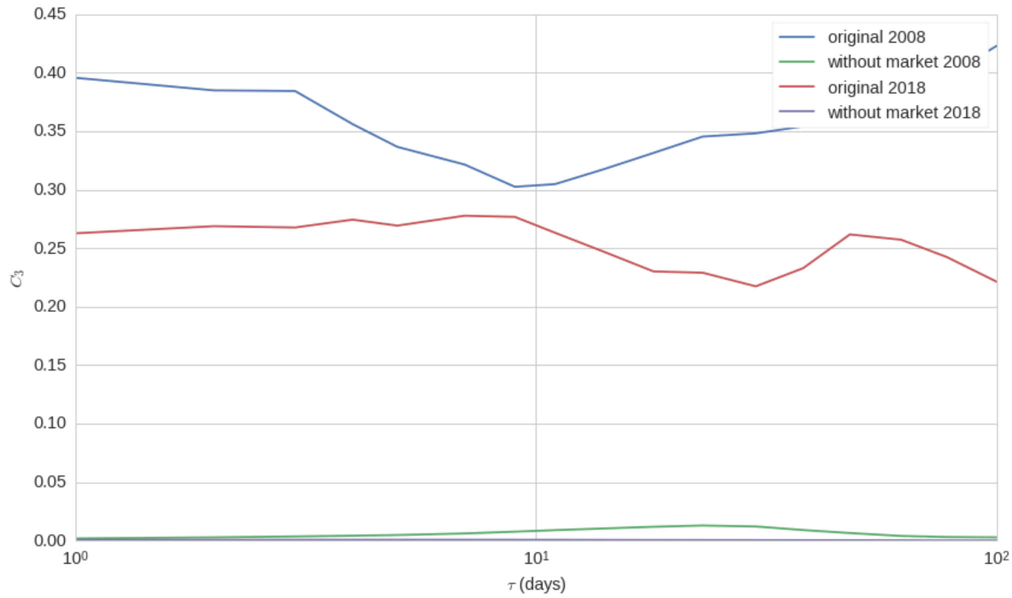


Figure 3. Co-movement with C_3 function. A value of 0 means that there is no co-movement, while values that are greater than 0 mean that the stocks tend to move in the same direction.

In both years and for the three functions, it is clear that there is some co-movement in the whole market (in the three cases, the co-movement in the year 2008 is a bit greater than in the year 2018), but all of the co-movement is explained by the market, since the co-movement disappears when we subtract the market.

Because the three functions essentially provide the same information, we use in the following $C_{3t}(\tau)$ and $C_3(\tau)$.

3.2. Co-Movement along the Time

To study the evolution of the co-movement with a wide time horizon, we have used the function $C_{3t}(20)$, which is, the degree of co-movement after 20 working days.

In Figures 4 and 5, we show the function $C_{3t}(20)$ for the period (2003–2020). A moderate degree of co-movement can be observed at all times. Note that the co-movement is higher during crisis moments or bear markets: 2003 with the end of the dot.com bubble, the end of 2008 and the beginning of 2009 with the financial crisis, mid 2010 with the flash crash, mid 2011 with the Eurozone debt crisis, mid 2015 to early 2016 with the Chinese stock market crash, the end of 2018 with the close to 20% decline on major indexes and mid 2020 with the coronavirus crisis. At some particular days, the co-movement is very close to 1, which is an astonishing value of co-movement. Similar results have been reported by Tseng and Li [53] and Trinidad et al. [54].

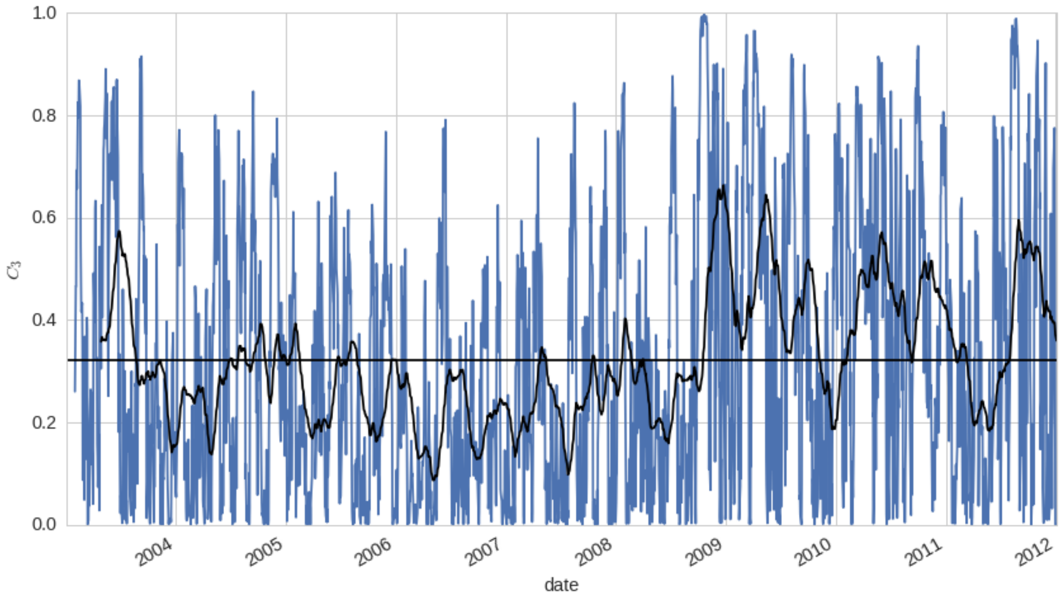


Figure 4. Co-movement ($C_{3t}(20)$) of the whole market from 2003 to 2011. Blue line is the daily co-movement, while black line is a moving average of the blue line with a time window of 60 trading days.

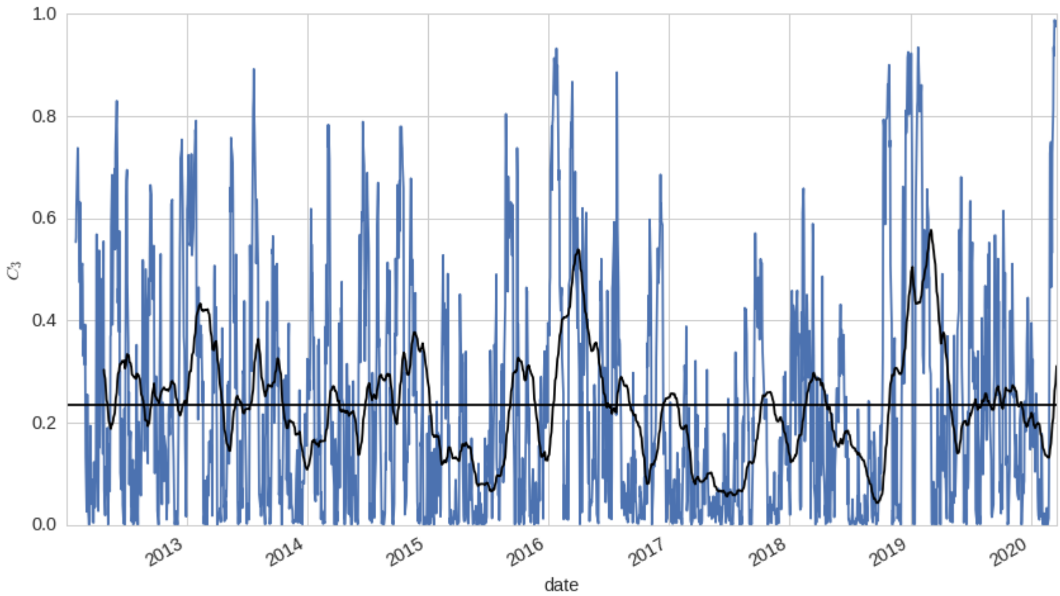


Figure 5. Co-movement ($C_{3t}(20)$) of the whole market from 2012 to 2020. Blue line is the daily co-movement, while black line is a moving average of the blue line with a time window of 60 trading days.

In Figures 6 and 7, we show $C_{3t}(20)$ with the market subtracted, as explained in the previous section. Clearly, no co-movement is detected with this observable. The highest punctual peaks are less than 0.005.

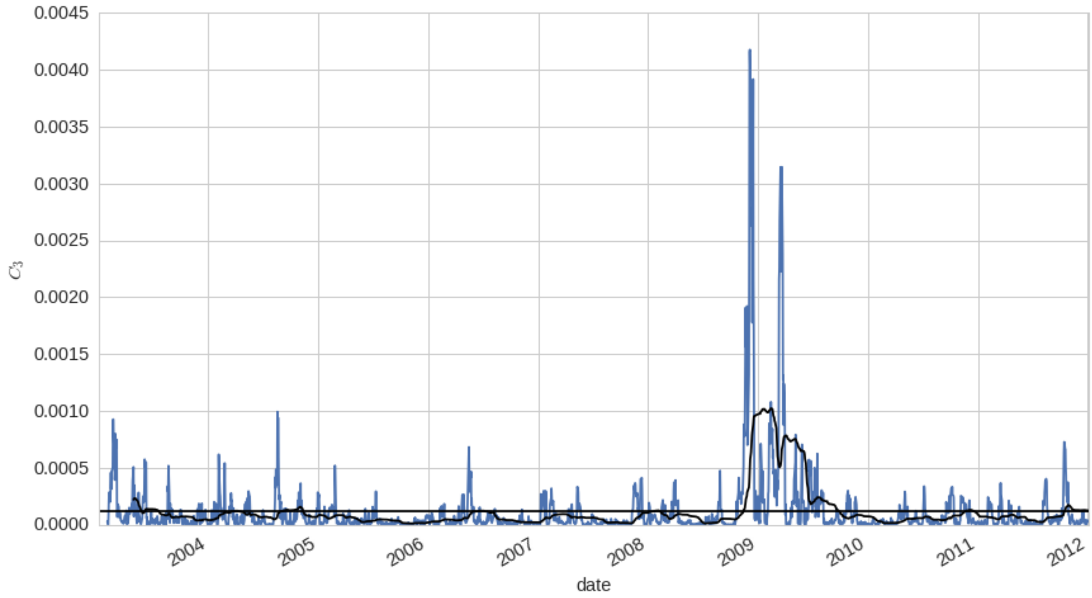


Figure 6. Co-movement ($C_{3t}(20)$) of the whole market from 2003 to 2011 with the market removed. Blue line is the daily co-movement, while black line is a moving average of the blue line with a time window of 60 trading days.

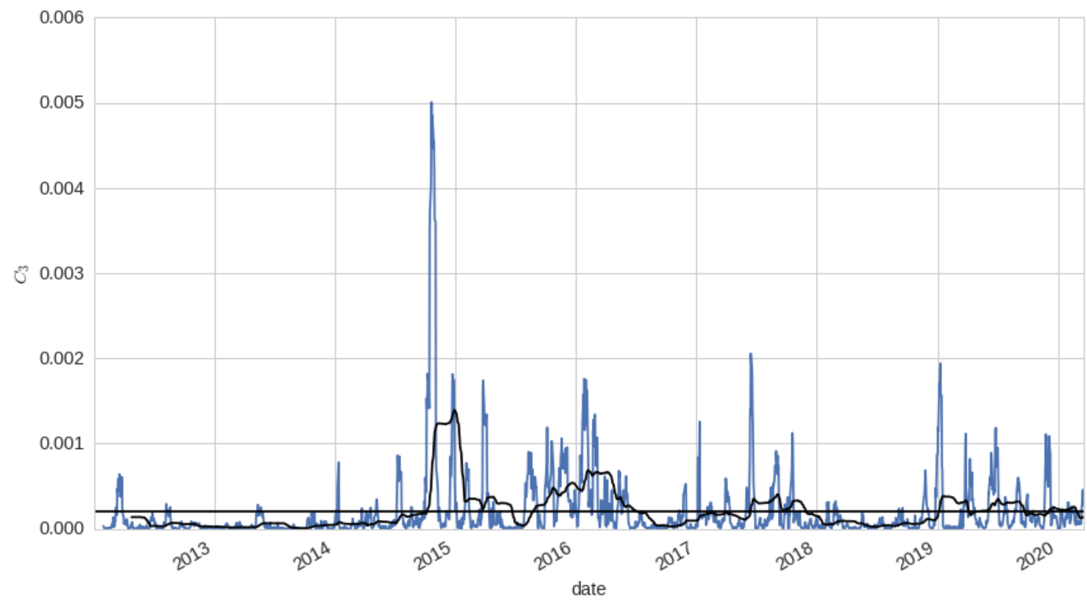


Figure 7. Co-movement ($C_{3t}(20)$) of the whole market from 2012 to 2020 with the market removed. Blue line is the daily co-movement, while black line is a moving average of the blue line with a time window of 60 trading days.

We can conclude that the market completely explains the co-movement among the stocks, which is quite a surprising result, since we have not considered any *beta* relationship.

3.3. Other Ways to Represent the Market

In this section, we have made the calculations taking into account different ways of representing the market, to see in which scenarios the co-movement is more reduced. Therefore, $\delta x_i(t,\tau)$ is calculated as $\delta x_i(t,\tau) = x_i(t+\tau) - x_i(t) - (m(t+\tau) - m(t))$, where x_i is the log price of asset i and $m(t)$ represent the market. The ways of studying the market that we have selected are:

- ew: equal weight. This is the representation described in the previous section, which is, $m(t) = \frac{1}{N(t)} \sum_i x_i(t)$, where $N(t)$ is the number of stocks at time t.
- cap: this representation is calculated as a capitalization-weighted average, which is, $m = \sum_j w_j x_j$ and $w_j = c_j / \sum_k c_k$ with c_j the capitalization of asset j.
- SPY: the SP500 index.
- IWM: the Russell 2000 index.

In the previous section, we studied the co-movement in two periods 2003–2011 and 2012–2020, and found that, if we measure the market with the ew method, the co-movement in these two periods is very close to zero. In Scheme 1, it is shown what happens when we consider other ways to represent the market. It can be seen that the representations of the market that explain the co-movement better than any of the alternatives considered are ew.

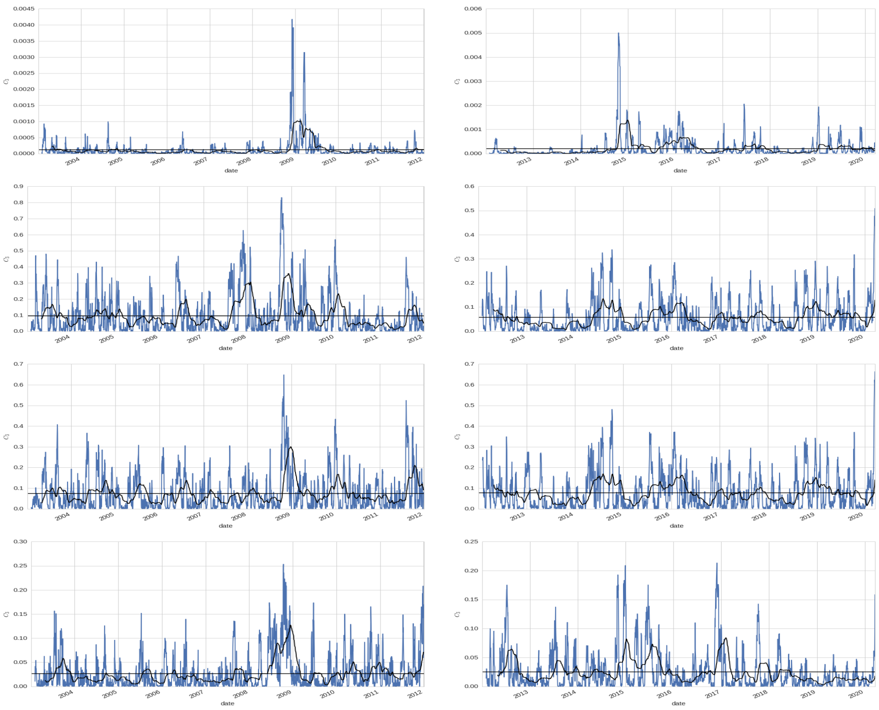
Considering Beta

As we commented in the introduction, the market beta is used in the literature to model the log-return of a stock with respect to the log-return of the market. The simplest model is the Sharpe one, but other models, like the Fama–French factor model, also consider the market beta as a part of the model. Subsequently, the log-return of a stock is modeled as $x_i(t+\tau) - x_i(t) = \alpha_i + \beta_i(m(t+\tau) - m(t)) + \varepsilon_i$, where ε_i follows a fixed stationary distribution with zero mean and is independent of the market. It follows that $\frac{x_i(t+\tau)-x_i(t)}{\beta_i} - (m(t+\tau) - m(t)) = \frac{\alpha_i}{\beta_i} + \frac{\varepsilon_i}{\beta_i}$, and hence it is independent of the market.

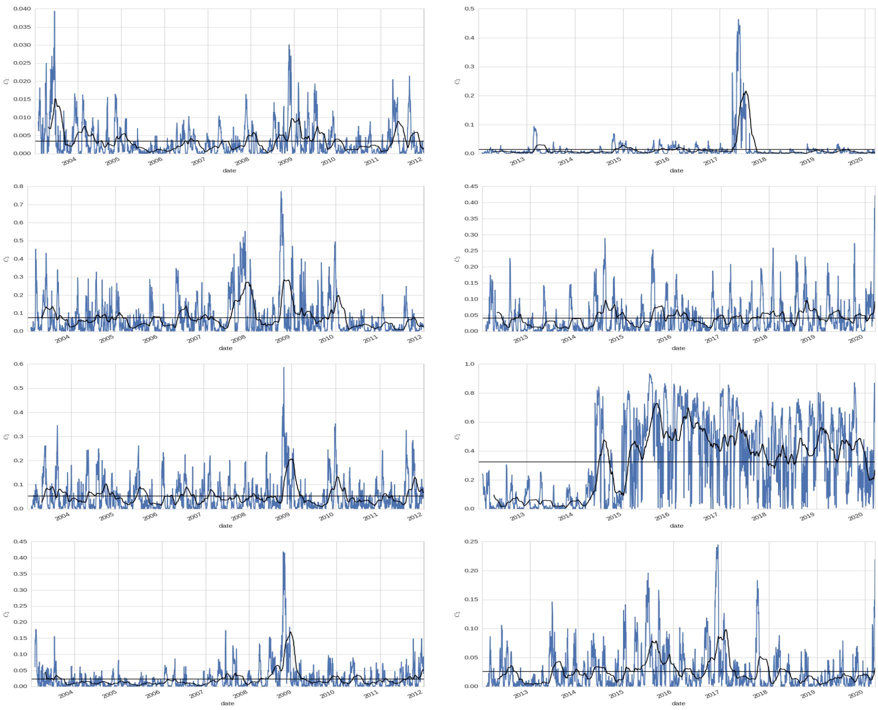
To check this model, in this case, $\delta x_i(t,\tau)$ is calculated as $\delta x_i(t,\tau) = \frac{x_i(t+\tau)-x_i(t)}{\beta_i} - (m(t+\tau) - m(t))$, and m is represented in the different ways described previously (ew, cap, SPY, IWM). Assuming the beta model, the co-movement that is calculated in this way should be lower than the co-movement calculated without considering the beta.

In Scheme 2, we show the co-movement ($C_{3t}(20)$) calculated within the Fama and French model. We can say that this version does not improve the results obtained previously and hence the previous model is preferred, since the results are better and the model is simpler.

Therefore, from the point of view of co-movement, we can conclude that the relationship between the log-returns of a stock and the market is provided by $x_i(t+\tau) - x_i(t) = \alpha_i + (m(t+\tau) - m(t)) + \varepsilon_i$, which implies $\beta = 1$.



Scheme 1. Co-movement ($C_{3t}(20)$) of the whole market during 2003–2011 (left) and 2012–2020 (right) with the market removed. Blue line is the daily co-movement, while black line is a moving average of the blue line with a time window of 60 trading days. The representation of the market is, from top to bottom: ew, cap, SPY, IWM. The horizontal line represent the average co-movement ($C_3(20)$) along the full period.



Scheme 2. Co-movement ($C_{3t}(20)$) of the whole market during 2003–2011 (left) and 2012–2020 (right) with the market removed when considering the beta of each stock. The blue line is the daily co-movement, while black line is a moving average of the blue line with a time window of 60 trading days. The representation of the market is, from top to bottom: ew, cap, SPY, IWM. The horizontal line represent the average co-movement ($C_3(20)$) along the full period.

4. Conclusions

In this paper, a new approach to study co-movement among the stocks of the whole (USA) market is considered, by using some functions that are inspired by cooperative dynamics in physical particle systems. We prove that these functions identify the economic crisis as periods of increased co-movement, which is agreement with the finding of previous research (see, for example, Tseng and Li [53] and Trinidad et al. [54]). From this perspective, the market (represented by an equally weighted portfolio of all the stocks considered) completely explains all of the co-movement among the stocks in such a way that, if we remove the market, the co-movement is almost zero. This fact supposes an important finding considering that one of the most important inconveniences of beta estimations is their instability in time.

The three functions are three different ways to measure the whole co-movement of the market. The fact that the results are similar implies that the measure is quite robust. Therefore, these measures seem to be quite useful, since they provide a way to measure the co-movement among the stocks and we can obtain results that are not obvious. Moreover, we think that these measures open a new door to study the co-movement among a set of stocks for different purposes.

Other ways to represent the market (capitalization weighted, SP500 index and Russell 2000 index) can also explain the co-movement in the market, but to a lesser extent than an equally weighted portfolio. This result is similar to a recent finding of Lopez et al. [20], where the authors show that a market factor represented by an equally weighted portfolio is highly significant in a factor model.

To conclude, the market is also considered taking into account the market beta of the stocks, but again, the results are not as good as the simpler equally weighted portfolio. These results lead us to propose a modification of the CAPM model, removing the beta in the classic model.

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Volatility co-movement in stock markets

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Volatility Co-Movement in Stock Markets

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Abstract: The volatility and log-price collective movements among stocks of a given market are studied in this work using co-movement functions inspired by similar functions in the physics of many-body systems, where the collective motions are a signal of structural rearrangement. This methodology is aimed to identify the cause of coherent changes in volatility or price. The function is calculated using the product of the variations in volatility (or price) of a pair of stocks, averaged over all pair particles. In addition to the global volatility co-movement, its distribution according to the volatility of the stocks is also studied. We find that stocks with similar volatility tend to have a greater co-movement than stocks with dissimilar volatility, with a general decrease in co-movement with increasing volatility. On the other hand, when the average volatility (or log-price) is subtracted from the stock volatility (or log-price), the co-movement decreases notably and becomes almost zero. This result, interpreted within the background of many body physics, allows us to identify the index motion as the main source for the co-movement. Finally, we confirm that during crisis periods, the volatility and log-price co-movement are much higher than in calmer periods.

Keywords: co-movement; volatility; econophysics; stock market

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1. Introduction

Since the 1980s, asset co-movement has attracted the attention of financial researchers (see the pioneer paper of Meese and Rogoff [1]). Co-movement plays a critical role for asset allocation, portfolio diversification or risk management, and its causes have been studied from many points of view. Some authors found that it depends on market information capitalized in asset prices (Roll [2]; Katsiampa [3]). Others attributed the co-movement to market order flows and order type (Domowitz et al. [4]). Byrne et al. [5] concluded that global inflation explains most of the global yield co-movement. Some researchers also considered that co-movement is affected by variables that reflect different institutional aspects, such as international macroeconomic policy (Parsley and Popper [6]). Other groups attributed the co-movement to market proximity (Edwards and Susmel [7] or Lee [8]).

In this paper, we focus on the growing interest in the study of the co-movement of volatility, as its movement affects financial assets in various ways, exerting a great influence on risk management, portfolio selection, pricing of derivatives and for setting regulatory policy.

The pioneer paper of Hamao et al. [9] documented the existence of price volatility effects across Japanese, London and New York financial markets. The authors found that the spillover effects are only significant in the case of the Japanese market. Recently, Susmel and Engle [10] examined the timing of mean and volatility spillovers between New York and London equity markets. The authors reported no evidence of significant volatility spillovers.

Fleming et al. [11] used a simple model of speculative trading to study the role of information in creating volatility linkages between markets. In the same line, Dávila and Parlatore [12] found that when prices are uninformative, there is a positive (negative) co-movement between price informativeness and price volatility, and they concluded that stocks with more volatility prices are likely to be less informative, and vice versa.

Edwards and Susmel [7] provided evidence of significant volatility co-movement across financial markets in the emerging nations. The authors found evidence that these connections go beyond geographical proximity.

Jondeau and Rockinger [13] reported evidence of covariability of volatility between five stock-index returns and six foreign-exchange returns sampled at a daily frequency. The authors concluded that extreme realizations tend to occur simultaneously on different markets.

Gabudean [14] examined the difference in volatility behavior between a stock that is part of an index and one that is not. The author found that volatilities co-move more after a stock becomes part of the index, mainly at hourly frequency and less at a daily one. Calvet et al. [15] found strong patterns in volatility co-movement between currencies. Volatility components tend to have high correlation when their durations are similar, and low correlations otherwise.

Lee [8] studied the volatility spillover effect within six Asian countries for the period from 1985 to 2004, finding that there are significant volatility spillover effects between them. Modi et al. [16] used various alternative techniques for recognizing co-movement resulting among the selected developed stock markets and the emerging stock markets of the world. These authors found that all the markets showed positive average daily returns and that there was considerable volatility in the correlations between the eight stock markets over time. Regarding the co-movement between markets, it was concluded that the eight stock markets are fragmented into two major components.

Chen et al. [17] reported evidence of a common time varying volatility factor in the United Kingdom, Singapore and Australia. In the same line Zhang and Ding [18] analyzed the volatility co-movements in different commodity futures markets, finding that various volatility measures of commodity return share a common trend, which can be interpreted as a common market volatility factor. Apparently, liquidity is an important transmission channel for volatility shocks.

Bašta and Molnár [19] showed that the implied volatility of VIX index and the implied volatility of the oil market are highly correlated. The authors also found that the correlation between the stock and oil market volatility is time-varying and depends on the time scale. Using vector autoregressive models–multivariate generalized autoregressive conditional heteroscedastic models (VAR-MGARCH), Huang and Wang [20] investigated the systemic importance of the volatility spillover.

Katsiampa [3] investigated the volatility dynamics of Bitcoin and Ether, finding that the volatility of the two cryptocurrencies is responsive to major news. Fernandez-Aviles et al. [21] did not find clear co-movement patterns in volatility of commodity markets in extreme financial episodes worldwide.

In a different perspective, Zheng et al. [22] proposed a dynamic conditional correlation-mixed data sample (DCCMIDAS) model to analyze the contagion between the business cycle and financial volatility. Wang and Guo [23] used a DCC-MGARCH model to study the stock market volatility co-movement of China and other G20 members, finding that the performance and influencing factors of co- movement are time varying.

Recently, Liu and Jiang [24] have introduced a propagation dynamics model, called WSI mode, based on the classical discrete virus propagation mechanism, to study the volatility co-movement through different market indexes. Qiaoa et al. [25] used wavelet coherence and the correlation network to examine the co-movement relationship among representative cryptocurrencies from the perspectives of returns and volatility. These authors reported evidence of co-movement and hedging effects.

To document co-movement, financial literature has proposed different variations of ARCH [10,17] and GARCH models [13]. Other approaches have been the GARCH-M model [9], the GMM model [11], the SWARCH model [7], the Multiplicative Error models (MEM) [14], the Markov-Switching Multifractal (MSM) [15] , the bivariate Diagonal BEKK model [3], the DCC-MGARCH model [23], the DCCMIDAS model [22] or the VAR-MGARCH model. More complex methods were introduced by Baštaa and Molnár [19] where the authors used a Continuous Wavelet Transform, Lee [8] where Bivariate Vector Autoregression-Generalized Autoregressive Conditional Heteroskedasticity Model is used, Fernandez-Aviles et al. [21] where the authors proposed a combined ES-MDS procedure, the wavelet coherence analysis of Qiaoa et al. [25] and the WSI model introduced by Liu and Jiang [24].

In this paper, we propose to look at the co-movement in volatility from a different approach based on a function of physical particle systems and the previous works of Clara Rahola et al. [26], Sánchez Granero et al. [27], Puertas et al. [28] and López García et al. [29]. Unlike previous literature, this paper looks to several aspects of the volatility co-movement among the stocks of the same market and region. First, we study the volatility co-movement dynamically over time. This allows us to detect those periods of time when the volatility co-movement is higher or lower, and we can try to identify the source of the co-movement. Then, we focus on the volatility and log-price co-movement of stocks with similar volatility. We find that this co-movement is higher than the full market co-movement. In addition, an inverse relationship between co-movement and volatility is found. This is a pattern that repeats most of the years. Moreover, by repeating the calculation subtracting the market, we find that almost all the volatility and log-price co-movement is explained by the market. Finally, we study the co-movement between stocks of different volatilities. The analogy with physical systems allows us to conclude that most of the co-movement is originated by herding, with uncorrelated motion around the market. In crisis periods, however, a higher degree of co-movement beyond the market can be identified.

2. Methodology

In order to study co-movement, we borrow the analysis of physical many-body systems and adapt it to the financial markets. It is well known that the movement of assets can be described, as a first approach, with Brownian motion, initially developed for suspended particles or macromolecules in a solvent [30]. The most simple system showing Brownian motion is hard particles, without any internal degrees of freedom, realized experimentally as sub-micrometer particles suspended in water or other solvents, generally known as colloidal systems [31]. These can be directly observed through the microscope allowing direct access to their time-resolved positions. Our approach from physics considers a portfolio as if it were a system of many bodies [28], using the index to represent a characteristic point that characterizes the whole system, such as the center of mass. Following the classical mechanics of multi-body systems, internal forces only affect the relative motion of a body with respect to the center of mass, so a proper estimate of co-motion due to interactions, within a physical approach, can only be made by subtracting the center of mass (or index). However, we are also interested here in the co-movement of the full market, without subtracting the center of mass.

Cooperative motion in colloids has been studied in connection to vitrification. Upon cooling down the system, its dynamic slows down significantly until the structural relaxation time scale becomes larger than the observational time scale, and the system becomes effectively a solid; it is said to have crossed the glass transition, or vitrified. Cooperative motions appear then as a route for relaxing the density or temperature fluctuations, when single particle dynamics is hindered [32]. In fluids, on the other hand, collective motions are negligible as single particle diffusion is enough to relax these functions. Several parameters have been devised for accessing these collective motions [33,34], which have been used to study the co-movement in stocks [28].

Based on this analysis, three functions were proposed in [29] to monitor cooperative dynamics in financial systems using the log-price. As the results applying the different functions were very similar, it has been decided to use the C_3 function in this work, as it was the one that provided more information in the research of co-movement in [29]. This function is defined as follows:

$$C_{3t}(\tau) = \frac{\sum_{i,j} \delta y_i(t,\tau)\delta y_j(t,\tau)}{\sum_{i,j} |\delta y_i(t,\tau)\delta y_j(t,\tau)|}$$

$$C_3(\tau) = < C_{3t}(\tau) >$$

where $y_i(t)$ is the variable under study (log-price, $x_i(t)$, or volatility, $v_i(t)$, in this work) of asset i at time t, $\delta y_i(t,\tau) = y_i(t+\tau) - y_i(t)$. The summation runs over all pairs of assets i, j, excluding $i = j$. $C_3(\tau)$ is the average of $C_{3t}(\tau)$ over time origins t.

$C_{3t}(\tau)$ compares the change of variable y for stocks i and j in the time interval τ, averaging the product of changes for all pairs of particles. The function thus reaches values from -1 to 1. When the values are close to 0 it means that there is no co-movement; if they are greater than 0 it means that the assets are moving in the same direction; if, on the contrary, the results are less than 0, it means that the assets are moving in the opposite direction. Furthermore, following the analogy with physical systems, we consider the center of mass of the system as the average position, $m(t) = \langle y_i(t) \rangle$. This index $m(t)$ is affected only by external forces and represents a particular point in the system.

In this work, a set of 3577 American shares for the period from 2008 to 2020, sampled daily, has been used. The volatility of stock i at time t, $v_i(t)$, is calculated as the standard deviation of the log-returns of one year (250 trading days) ending at t. Other values of the time interval provide qualitatively similar results.

The calculations of $C_{3t}(\tau)$ and $C_3(\tau)$ are performed with the bare data $y_i(t)$ and with $y_i(t) - m(t)$. This can serve then to identify the origin of co-movement: if it disappears when the motion of the center of mass is subtracted, there are no important dynamic correlations between individual stocks; on the other hand, if it remains after the center of mass is subtracted, it indicates the existence of dynamic correlations among stocks or groups of stocks.

It must be noted that, different from other techniques in the literature, the present method allows us to analyze the whole set of stocks in a given market, or a particular subset.

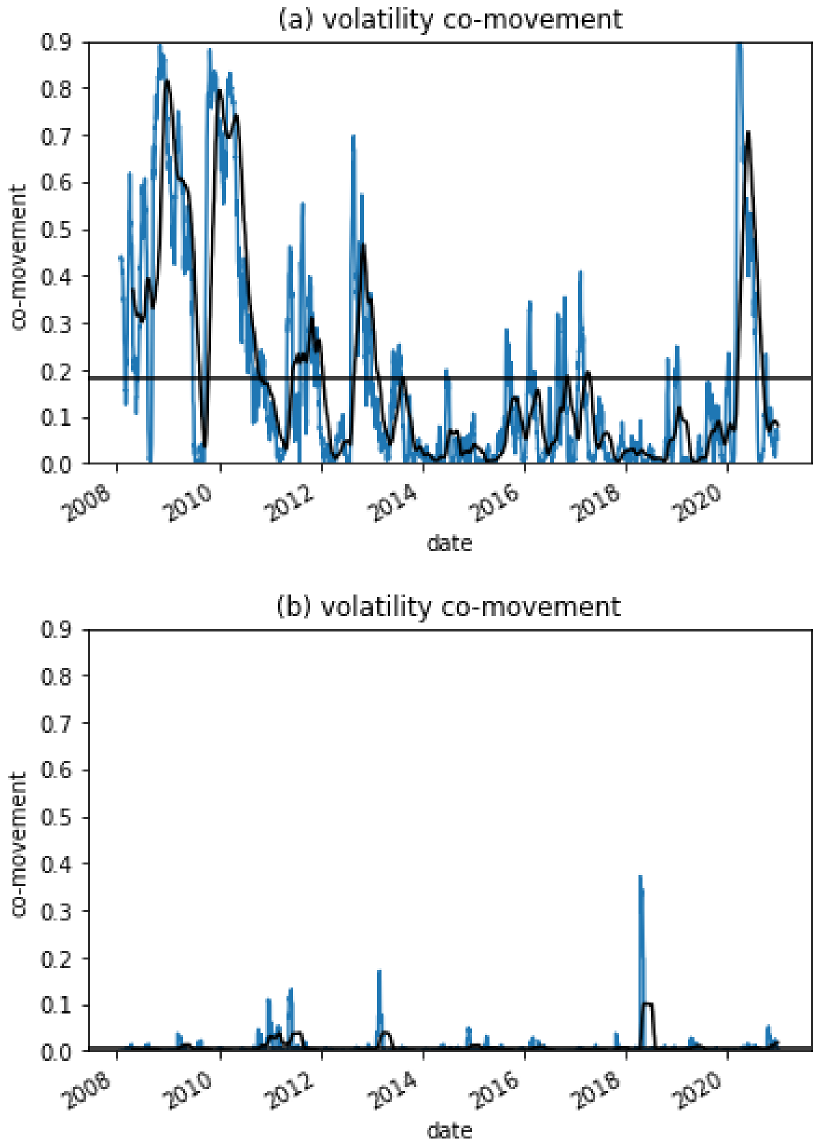
3. Results

Previous results have shown that $C_3(\tau)$ indeed captures the co-movement in the stocks log-price, but also that the overall co-movement can be corrected almost completely subtracting the index [29], defined as the average log-price. Here, we focus on the co-movement in volatility of the sample of stocks using the function C_3 defined above. In this case, y_i stands for the volatility. Bare volatility or the deviation from the mean (termed market) is also considered to identify the source of co-movement. Furthermore, in addition to averaging $C_{3t}(\tau)$ over all pairs of particles, partial averaging over stocks i of similar volatility are also performed. This allows us to conclude correlations, or absence of correlations, between stocks with different volatility.

3.1. Volatility Co-Movement Along Time

For the analysis of volatility co-movement over time, we have studied the value $C_{3t}(20)$ over time t, which is the degree of co-movement after 20 trading days (around one month). Scheme 1 shows the function $C_{3t}(20)$ vs. t in the period 2008 to 2020 using the bare volatility (upper panel), or subtracting the mean (lower panel). Scheme 1a shows that the volatility co-movement varies strongly over time, with the highest values occurring in periods where some type of crisis has occurred. In the 2008–2009 period, the volatility co-movement reached values above 0.8, which corresponds to the last financial crisis experienced. In mid-2010 and 2011, values close to 0.8 were also observed, corresponding to the flash crash and the Eurozone debt crisis. In 2012–2013, values between 0.6 and 0.8 were also quite high, corresponding to a year of high volatility and recurrent uncertainty due to the new US fiscal policies. Finally, in 2020, values of the volatility of 0.8 were again reached due to the recent crisis from COVID 19. The results obtained are in line with the work of Tseng and Li [35], Trinidad et al. [36] and López García et al. [29], where the co-movement was analyzed with respect to price.

In Scheme 1b, the co-movement is calculated by subtracting the market from the volatility, i.e., using $y_i(t) = v_i(t) - m(t)$ instead of $v_i(t)$, where the market (equivalent to the center of mass in physical systems) is calculated with an equal weight, $m(t) =< v_i(t) >_i$. In this case, the co-movement is very close to 0, indicating that there is hardly any co-movement in volatility in the sample; thus, we can conclude that the market explains almost completely the volatility co-movement of the stocks. This finding follows a similar result with the log-price co-movement found in [29], though the effect is a bit stronger in log-price co-movement than in volatility co-movement.



Scheme 1. Volatility monthly co-movement ($C_{3t}(20)$) of the whole market from 2008 to 2020. Blue line is the daily co-movement, while the black line is a moving average of the blue line with a time window of 60 trading days; (**a**) the market is not subtracted; (**b**) the market is subtracted.

3.2. Co-Movement as a Function of Stock Volatility

Figure 1 shows the histogram of the volatility of all stocks for the period 2008–2020. It is noticed that most stocks have volatilities between 0 and 1, but the distribution extends to values above 4. Given this broad range of volatilities, one may ask if the co-movement shows some dependence on the stocks, classified according to their volatility. Even more, since the volatility changes notably over time, this dependence may have changed in some years, especially in crisis periods. In this section, thus, we study the volatility co-movement for our entire sample of American stocks, classifying the stocks according to their volatility, and we repeat the analysis carrying out the study on an annual basis.

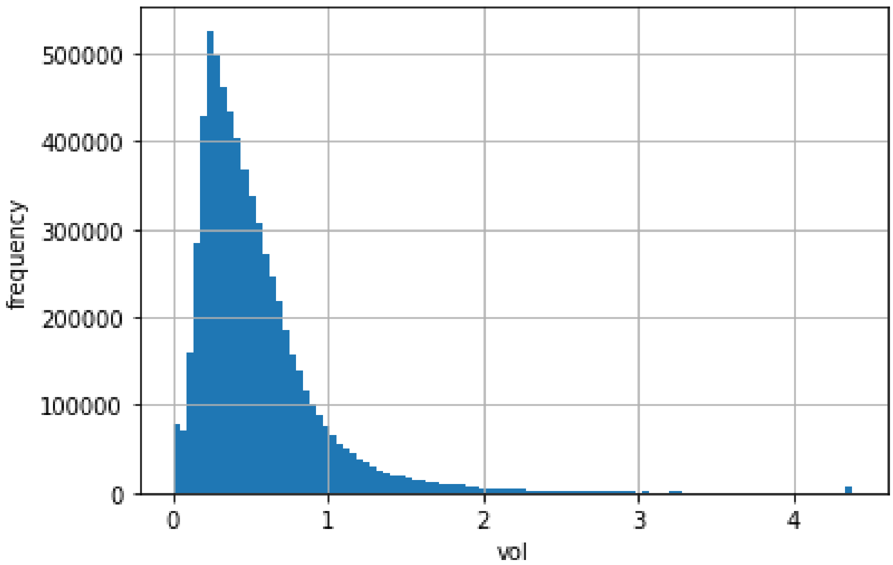


Figure 1. Histogram of the volatility of all stocks in the period 2008–2020.

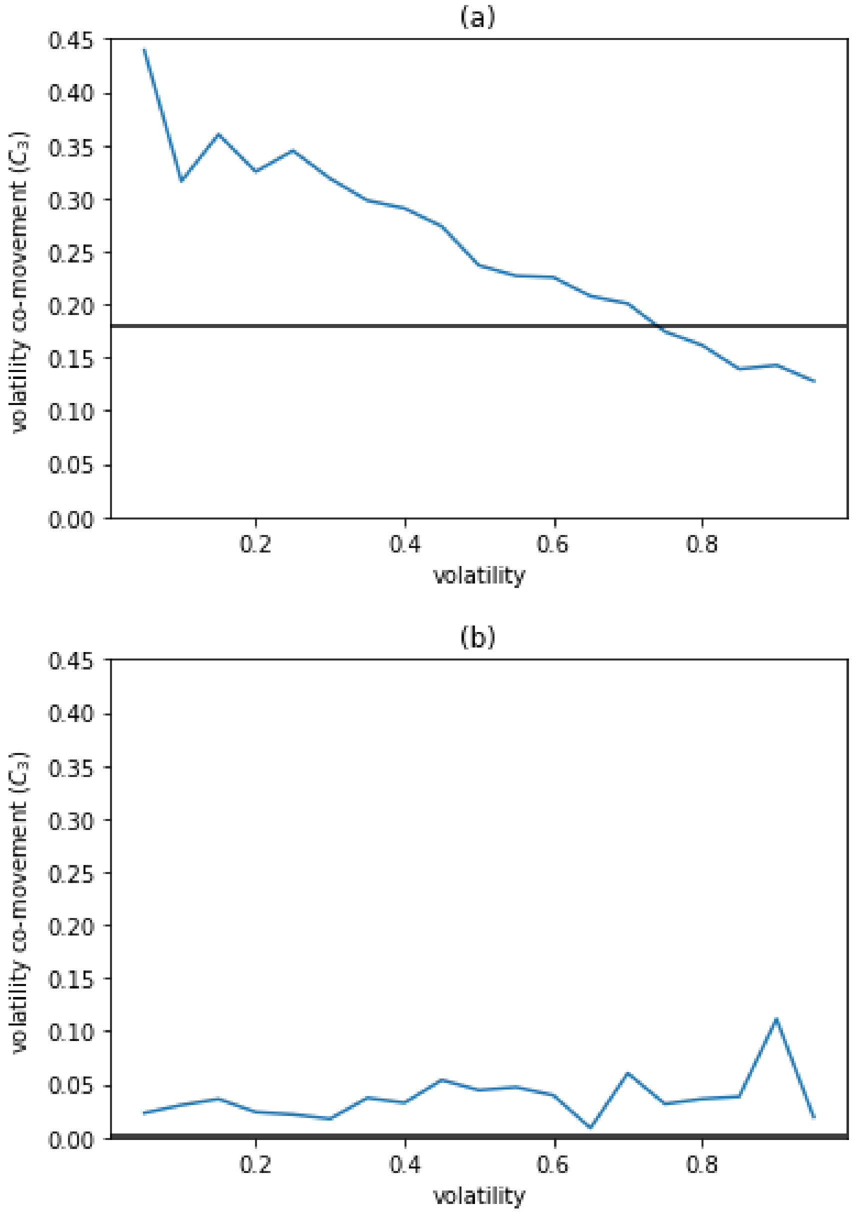
Scheme 2 shows the volatility co-movement for stocks of similar volatility averaged over the whole period without subtracting, and subtracting, the market (upper panel and lower panel, respectively). For this purpose, stocks are classified according to the volatility in bins of width 0.10, and the summation in the calculation in $C_{3t}(20)$ is restricted to stocks within the same bin.

In Scheme 2a, the volatility co-movement decreases with the volatility, that is, the stocks with smaller volatility reflect a greater co-movement, and as the value in volatility increases the co-movement is lower. In particular, the co-movement of stocks with low volatility is significantly higher than the average value (considering all stocks and the whole period), marked by the horizontal black line at 0.18. This graph shows that volatility is indeed a relevant factor to consider when studying volatility co-movement.

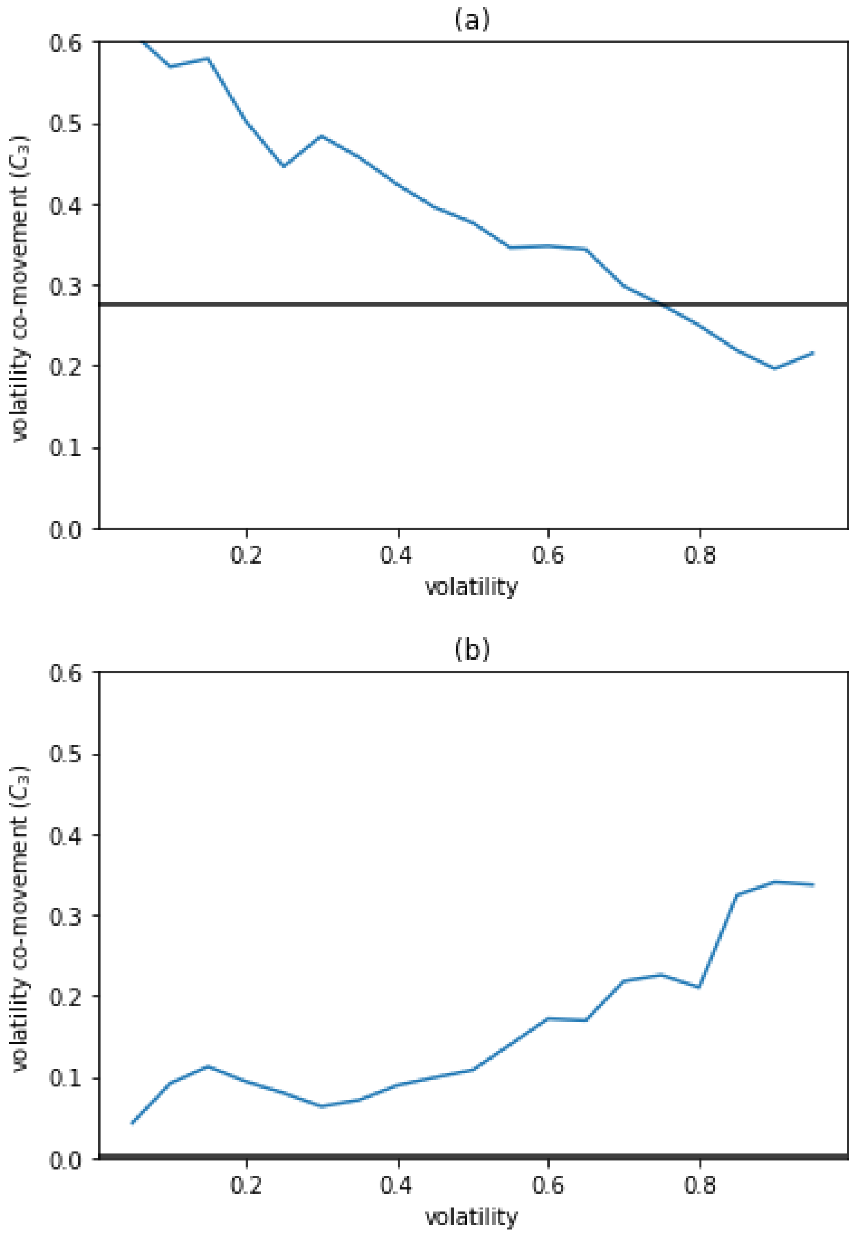
In Scheme 2b, the average volatility is subtracted for the calculation of the co-movement. The results are very close to zero, and this is reflected in its average (black horizontal line). Nevertheless, for stocks with similar volatility the co-movement is above the average, but still close to zero. Therefore, it can be concluded that the market drives the volatility co-movement of the whole market almost completely, replicating the results of the co-movement in price [29].

The robustness of these results is tested in Scheme 3, where the same analysis is repeated for a much longer time interval for the co-movement; in this case 250 trading days (around one year) are used, in contrast to the 20 days used in Scheme 2. When the market is not subtracted (Scheme 3a), the pattern is similar, or even stronger, to that obtained for 20 trading days (see Scheme 2a). On the other hand, when the market is subtracted, volatility co-movement in 20 trading days (Scheme 2b) is a bit noisier than in the 250 trading days case (Scheme 3b). In the latter, the volatility co-movement cannot be fully explained by the market for stocks with similar high volatility.

We focus now on changes of the distribution of the co-movement as a function of the volatility in time. For this purpose, $C_3(\tau)$ is calculated as the average of $C_{3t}(\tau)$ in natural years, and we present the distributions in Scheme 4 for all years, together with the average within this year for all stock pairs (horizontal black lines). As discussed above, periods of crisis can be identified by the high value of the average co-movement (in 2008–2010 it reached 0.4, and in 2020 it was above 0.3), whereas stable years yield a smaller value, close to zero. Even more, the distribution of co-movement also changes notably from crisis to stable periods, being generally higher and flatter in the former, and lower and decreasing in the latter.

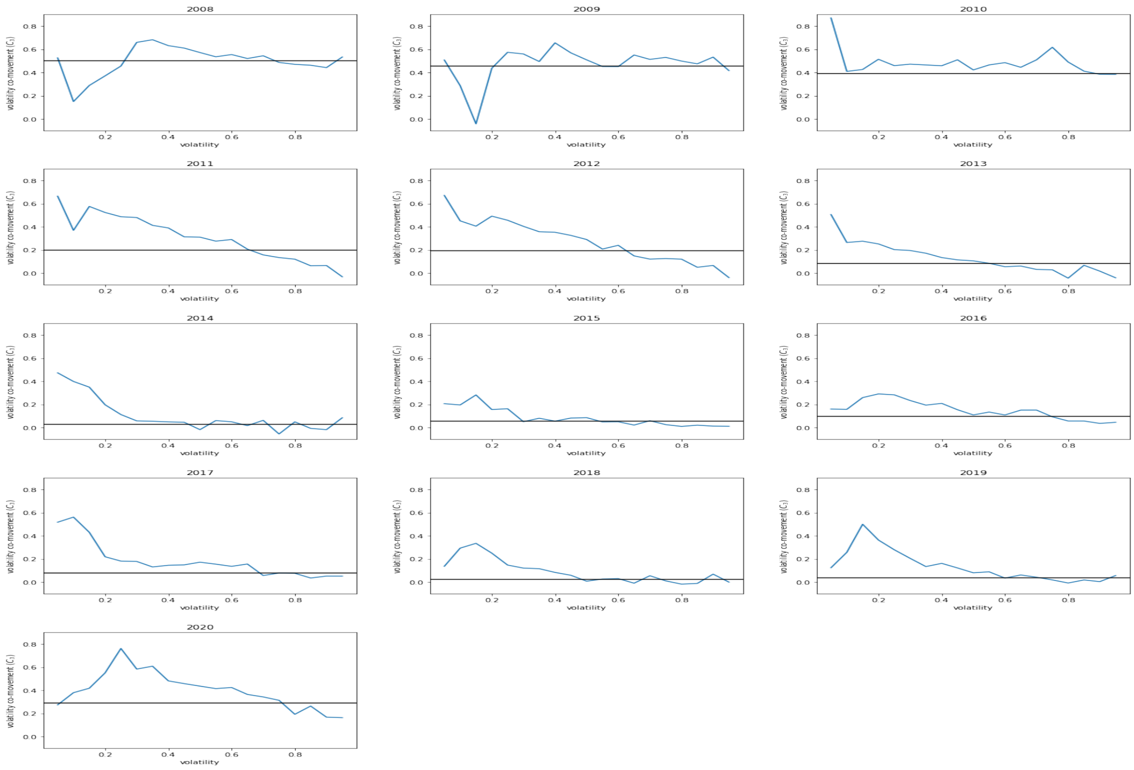


Scheme 2. Volatility monthly co-movement ($C_3(20)$) for stocks of similar volatility for the period 2008–2020; (**a**) the market is not subtracted; (**b**) the market is subtracted.



Scheme 3. Volatility monthly co-movement ($C_3(250)$) for stocks of similar volatility for the period 2008–2020; (**a**) the market is not subtracted; (**b**) the market is subtracted.

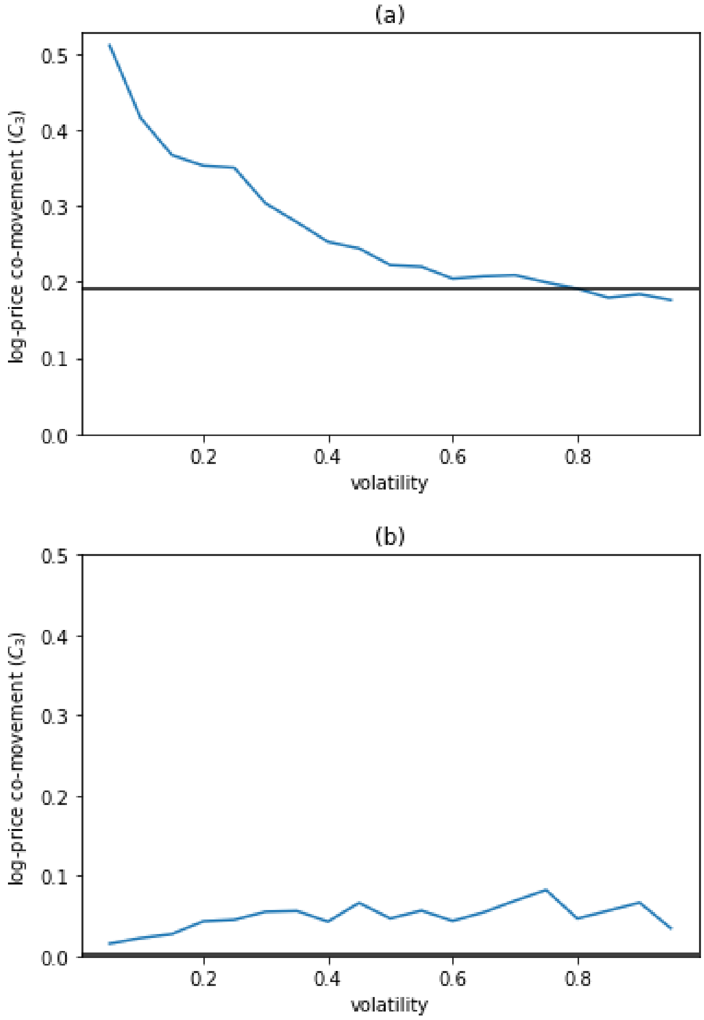
In Scheme 5, the volatility co-movement is calculated subtracting the market. As in Scheme 2b, the average of the volatility co-movement remains very close to zero, regardless of the year, even in 2008, 2009, 2010 and 2020. Accordingly, the distribution of co-movement stays close to zero for all volatilities and years. This result confirms that the co-movement among stocks of similar volatility can be interpreted as uncorrelated fluctuations around the mean.



Scheme 4. Volatility monthly co-movement for each year in the period 2008–2020.

Scheme 5. Volatility monthly co-movement (subtracting the market) for each year in the period 2008–2020.

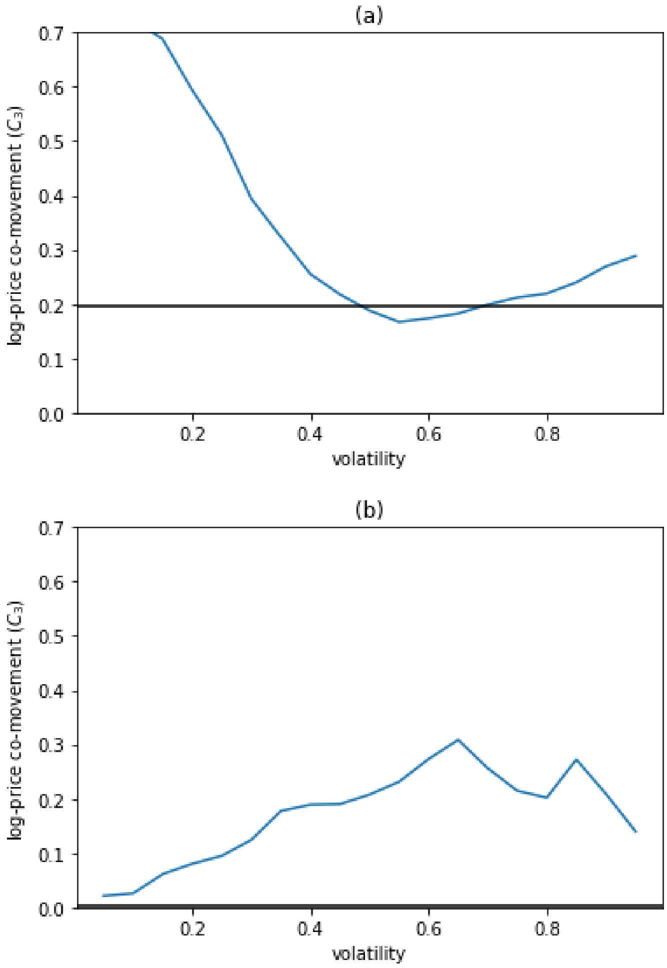
To get further insight into the cooperative dynamics of stocks, and how this changes between crisis and stable periods, we study the co-movement in log-price as a function of stock volatility for the same period. This is performed with the function $C_{3t}(\tau)$ calculated using the log-price as $y_i(\tau)$, and restricting the summation to stocks with the volatility within the same bin. The results are shown in Scheme 6: in the upper panel the bare co-movement in the log-price is presented, whereas the lower panel presents the results for the log-price corrected with the mean. A rather high degree of price co-movement in stocks with low volatilities is noticed in Scheme 6a, which decreases as the volatility increases. Again, the co-movement is practically zero when the market is subtracted (Scheme 6b).



Scheme 6. Log-price monthly co-movement ($C_3(20)$) as a function of volatility during the period 2008–2020; (**a**) the market is not subtracted; (**b**) the market is subtracted.

It is interesting to note the similarities between this Scheme and Scheme 2, despite the co-movement of different quantities being studied. Even more, the year-by-year study (not shown), that is, the equivalent to Schemes 4 and 5, with log-price co-movement instead of volatility co-movement is very similar to Schemes 4 and 5. In addition, if the log-price co-movement in 250 trading days is considered, see Scheme 7, the pattern is again similar to the case of volatility co-movement (Scheme 3) and also similar to log-price co-movement in 20 days (Scheme 6), especially in the case when the market is not subtracted. Note also the very high degree of log-price co-movement for stocks with similar low volatility.

These results, concerning the stability of the distributions, similarity of the results of the price or volatility co-movement, as well as the identification of crisis and stable periods, indicates that the volatility is a key factor for assessing and classifying the dynamics of stocks, and in particular the collective motions. Additionally, the average volatility (or market) drives most of the co-movement in the market.

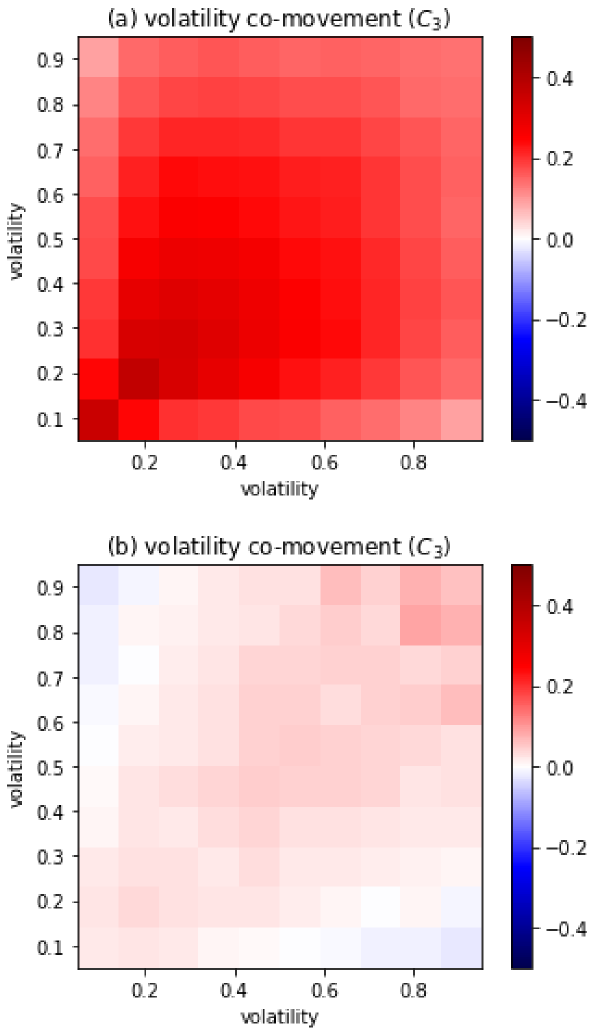


Scheme 7. Log-price yearly co-movement ($C_3(250)$) as a function of volatility during the period 2008–2020; (**a**) the market is not subtracted; (**b**) the market is subtracted.

Co-Movement Maps

To take a step further in the study of co-movement in volatility, maps have been created to show the volatility co-movement among stocks of different volatilities. As discussed previously, stocks are classified according to the volatility in bins of width 0.1, and the summation over i and j in the definition of $C_{3t}(\tau)$ is restricted to stocks belonging to the first and second bins, respectively. In these maps the intensity of co-movement is graduated with colors ranging from intense red (high co-movement in the same direction) to intense blue (high co-movement in the opposite direction), leaving white as the color that expresses the non-existence of co-movement.

The first maps are shown in Scheme 8, where we study the co-movement in volatility using the two calculation methods (subtracting and without subtracting the market, for the upper and lower panel, respectively), for the period from 2008 to 2020. Scheme 8a shows in all its boxes the red color, indicating a direct co-movement in volatility between stocks with different degrees of volatility. Stronger co-movement is observed in the diagonal, showing that stocks of similar volatility change more cooperatively than stocks of different volatility, as anticipated in Scheme 2a. Accordingly, the lowest degree of co-movement is observed between stocks of large and small volatility, and vice versa.

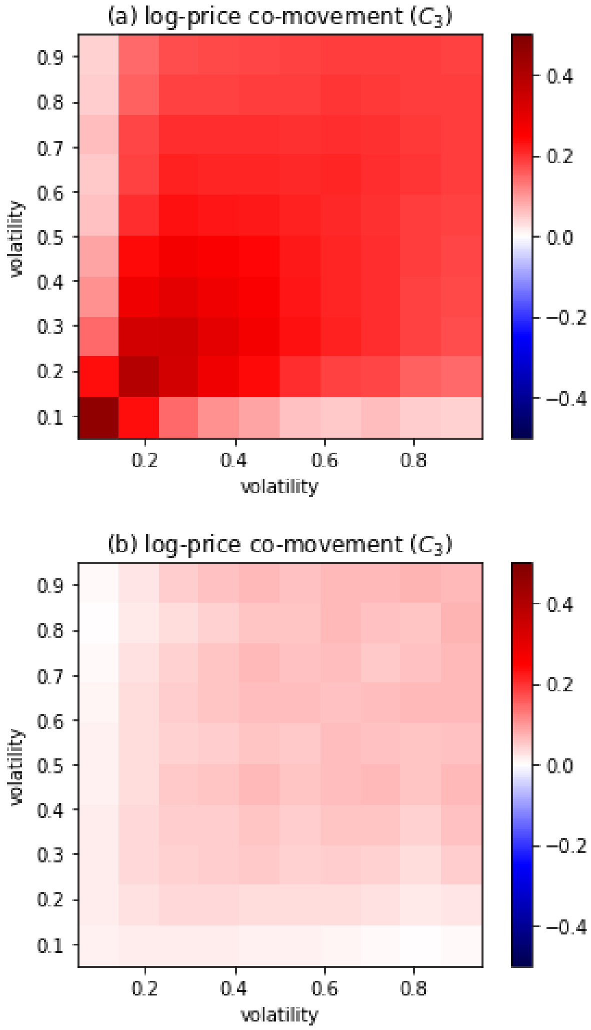


Scheme 8. Volatility monthly co-movement among stocks of different (and similar) volatilities. (**a**) Without subtracting the market; (**b**) subtracting the market.

On the other hand, Scheme 8b is almost white, so it hardly shows the existence of co-movement in volatility, which supports the results obtained in Scheme 2b when we subtract the market, now for the full sample of stocks. This result confirms that most of the cooperative motion is in fact driven by the market, calculated as the simple average over all stocks, but not all of it. In fact, an increase in the co-movement can be noticed around the diagonal for large values of the volatility.

In Scheme 9, we again perform the study of log-price co-movement as a function of volatility to check whether the same patterns as in the previous section apply. The results again reflect very similar results as in the volatility co-movement, showing in Scheme 9a a predominance of red, indicating a direct log-price co-movement among stocks with different degrees of volatility, with the maximum co-movement among stocks of low volatility. On the other hand, in Scheme 9b, the colors are very soft and almost white, reflecting the scarce existence of log-price co-movement once the market is subtracted. Compared to the volatility co-movement, however, these distributions are flatter, but the lowest values are still obtained for stocks with very different volatility.

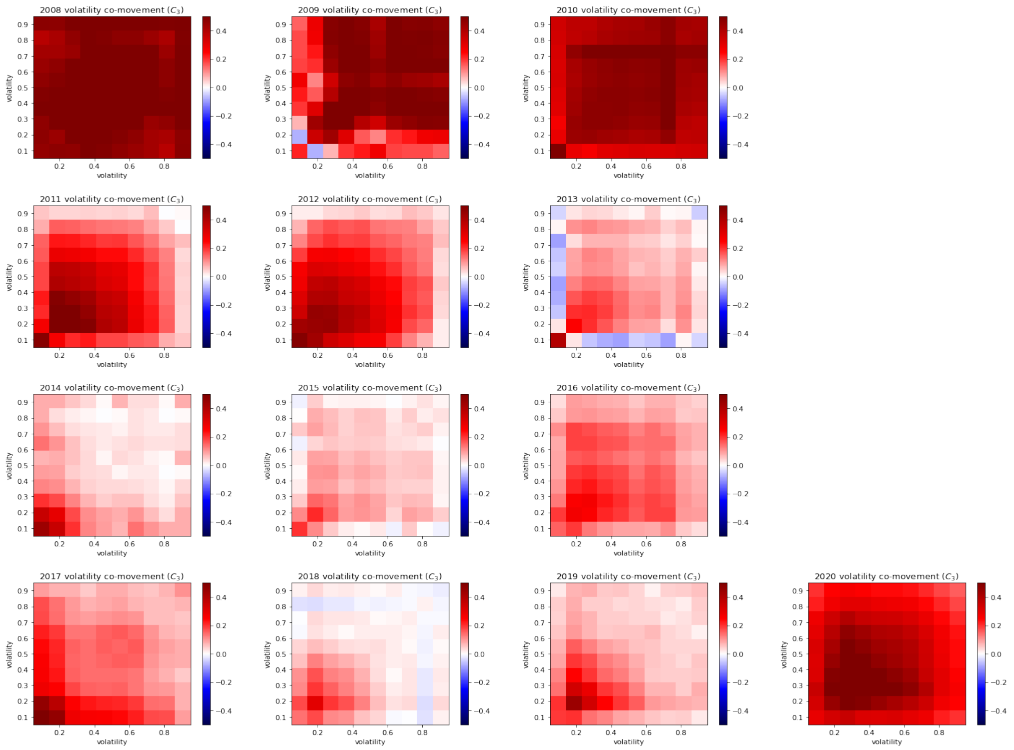
In Scheme 10, we present the volatility co-movement maps year by year without subtracting the market. It is clearly noted that the years more affected by crises, which presented a higher average of co-movement in Scheme 4, are the years with the most intense reds; these are 2008, 2009, 2010 and 2020. In most cases, the maximum values of $C_3(20)$ are obtained among stocks of low and similar volatility, whereas the minima are usually found in stocks of large volatility. Some peculiar cases are 2013, where the co-movement between stocks of very low volatility and any other stock is negative or very small. On the other hand, 2014 and 2017 show an increased co-movement in this case.



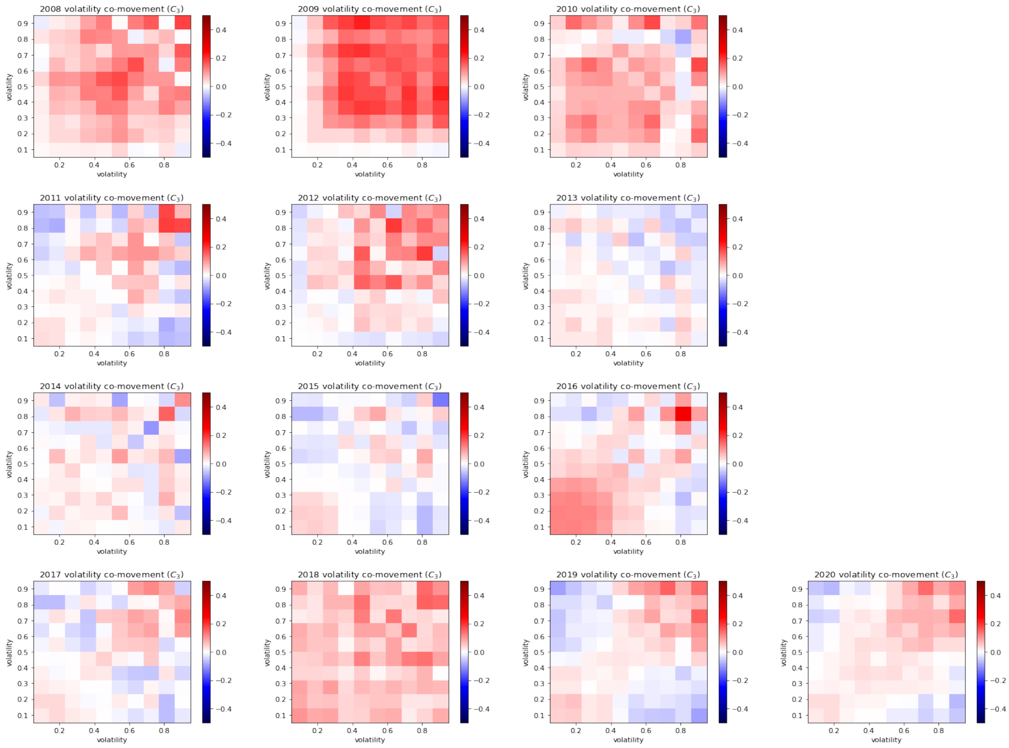
Scheme 9. Log-price monthly co-movement among stocks of different (and similar) volatilities. (**a**) Without subtracting the market; (**b**) subtracting the market.

In Scheme 11, the volatility co-movement subtracting the market is studied, which again is close to zero (with values between -0.2 and 0.2). In this occasion, blue colors appear in some locations, particularly when stocks of small and large volatility are correlated, indicating an inverse co-movement between the stocks. Even so, crises years are not fully corrected with the market, and the maximum values of $C_3(20)$ are observed for these years. Additionally, within a year, the largest co-movement is observed for stocks of similar but large volatility.

Comparing Schemes 10 and 11, co-movement in volatility appears as a powerful and robust tool to identify crisis periods, either with the market accounted for or without it. However, differences are noticed in particular years, where the shape of the distribution changes notably. For instance, year 2013 showed a peculiar behavior using the bare volatility, whereas in Scheme 11 this is fully corrected, and all co-movement has disappeared, indicating that most stocks follow the average with no other correlated motion. More interestingly, year 2020 can be identified as a crisis period, but when subtracting the market, it shows the typical behavior of stable years (such as 2019 or 2017). This points to a different dynamic, and origin, in the crisis in 2020 from the crisis in 2008–2010.



Scheme 10. Volatility monthly co-movement among stocks of different (and similar) volatilities for each year from 2008 to 2020 (without subtracting the market).



Scheme 11. Volatility monthly co-movement among stocks of different (and similar) volatilities for each year from 2008 to 2020 (subtracting the market).

4. Conclusions

In this paper we use a function borrowed from the analysis of cooperative dynamics in physical systems of colloidal particles to study the co-movement in the volatility of stocks within a single market. A total of 3577 American shares, sampled daily from 2008 to 2020, are analyzed. The period includes the financial crisis in 2008–2010, and COVID19 crisis in 2020. We focus on the co-movement in two main factors: volatility and market. The contribution to the total co-movement from different stocks is classified according to the stock volatility. The methodology allows us to identify the origin of the co-movement and conclude if it is caused by an external agent or correlation among stocks.

Globally, stocks with low volatility have a greater volatility and log-price co-movement than the general market co-movement. In addition, co-movement between stocks of similar volatility is typically larger than co-movement between stocks of different volatility. Negative co-movement is found, in some years, between stocks of very different volatility.

Interpreting this analysis within the background of many-body physics, the market index is defined as the average over all stocks, similar to the center of mass. We study, thus, the volatility (and log-price) co-movement after subtracting the market. The results for the co-movement are very close to zero in both cases, indicating that the market explains the general co-movement to a great extent. These results are similar to those reported by Lopez-Garcia et al. [29] for the log-price co-movement. Our results also show that the market factor is highly significant, as Lopez-Garcia et al. [37] suggested, for asset pricing and portfolio risk and indicates that dynamic correlations between stocks are rare, but relevant only in selected periods, mainly the crisis.

On the other hand, it is observed that both volatility and log-price co-movement of the general market are much higher during crisis periods. Similar results are shown by Trinidad et al. [36] or Tseng et al. [35]. In these periods, stocks with low volatility have a great co-movement, not only with other stocks with low volatility (that they have in general), but also with stocks with greater volatilities, resulting in different distributions of co-movement in crisis and stable periods. Studying the volatility distribution of the co-movement thus serves to identify and characterize crisis periods as well as provide further insight into the dynamics of stock markets.

Finally, the proposed method to study co-movement should be seen as a method to study the co-movement of the full market (or a subgroup of stocks) and can be used to study the influence of a factor or variable (in this paper we have used the volatility) in the log-price or volatility co-movement.

Our findings could also be helpful to approach other problems, such as portfolio diversification. We showed that stocks with low volatilities usually have a high degree of co-movement (both in log-price and volatility); thus, some questions arise with respect to the portfolio: Is a portfolio with stocks of low volatility riskier since they will have a high degree of co-movement? Is it better to construct a portfolio with stocks of different volatilities, as suggested by the fact that they will have a lower degree of co-movement? What about the risk of a portfolio of stocks with high volatility, which tend to have a lower degree of co-movement? These questions represent future promising research lines.

Another implication of our findings is that during crisis periods, the co-movement of the general market increases significantly, and the co-movement between stocks with different volatilities is very high, in comparison with the co-movement between stocks with different volatilities in other periods. Hence, a portfolio that can be considered well diversified in non-crisis periods will probably be much riskier during a crisis, since the diversification fails to work during crisis periods.

To conclude, another future research line is to study with the proposed method the co-movement of other asset pricing factors to see if stocks with a similar factor will have higher or lower co-movement than the general market.

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Chapter 3

Conclusions

Since the 1990s, the application of statistical mechanics models in the field of financial markets has aroused great interest among the community of physicists, mathematicians and economists. The thesis that has been developed continues in this line of work and is a continuation of the work previously developed by my directors and some of my colleagues. Two clear lines of research that have been developed in three published works, and one that is in development, are presented. All of them show a common link, the APT.

In the first work, the starting point is one of the algorithms developed by Sánchez et al. [59], Known as the Fractal Dimension (FD) algorithm, it is used to calculate the Hurst exponent, proposing its incorporation to the Fama and French factor model ([24], [27]). This is done with a double objective: on the one hand, to see if it is an explanatory factor for the performance of stock market securities and, on the other hand, to see if it can be considered a substitute factor for the well-known momentum [14]. The results showed that the H factor is one of the most significant factors in the study model, obtaining appearance values quite similar to the classic factors of size (SMB) and value (HML).

Once compared the results of the H factor with those obtained by the momentum factor, and although at first it was supposed that they were factors that could be related, since impulse and memory may seem to be related phenomena, it was shown that they are not substitute factors and that each contributes different information to the study of market performance.

Finally, the change in the results depending on the way of calculating the market factor was studied, and we concluded that, depending on the methodology used, the results can be totally opposite. Thus, it was found that when the market factor is calculated with an equally-weighted portfolio (EW), the market factor by itself is capable of explaining most of the market returns. On the other hand, when this factor is calculated with a portfolio weighted by capitalization, in this case, according to the criteria of the S&P 500 index, the factor becomes irrelevant within the model and it can be removed.

The second line of research tries to incorporate a new methodology to study the market comovement from some functions that are inspired by the cooperative dynamics of physical particle systems. Initially, three possible functions were created to measure market comovement. When they were put to the test, quite similar results were obtained, which confirmed that the proposed model for measuring comovement is quite robust.

In the first article, all the research was carried out comparing the results of the three proposed functions, while in the second one it was decided to select only the third way of calculating comovement (C_3) due to its simplicity.

The study of the market comovement was started using the logarithm of the price as the basis for the calculation of the comovement and its behavior over time was studied. The results obtained show that comovement is capable of identifying economic crises as periods of greater comovement.

To test the effect of the market within the comovement, the market was calculated in various ways (EW, CAP, SPX, IWM) and subtracted from the comovement functions. The obtained results show that the market is able by itself to explain almost all the comovement, yielding the best results when the market is calculated using an equally weighted portfolio (EW) for all securities. Thus, results that support those obtained in the study of market memory were found.

As a result of these findings, and based on the results obtained in the first line of work, a new way of calculating the market was proposed using the market beta, a factor widely used in classical models. The obtained results were very similar to those obtained with the EW method. The great representativeness of the market in the explanation of comovement leads us to propose a modification of the CAPM model, eliminating the beta in the classic model.

Research was continued using volatility to calculate comovement. The study was started confirming that with this new form of calculation continues good results continue to be yielded in the study of comovement over time, and that by subtracting the market (measured with the EW representation) the comovement is again almost zero, giving it the same weight of significance to the market.

Subsequently, an analysis was carried out on the behavior of the comovement according to the volatility values and the results were compared with the comovement according to the logarithmic price. The results show that securities with low volatility have a greater logarithmic volatility and price comovement than the overall market comovement. In addition, the comovement among securities of similar volatility is usually greater than the comovement between securities of different volatility.

The results of how the comovement behaves could be useful to address various problems, such as portfolio diversification. It has been proved that low volatility securities usually have a high degree of comovement, so based on this result various questions can be raised: Is a portfolio with low volatility securities riskier, since they will have a high degree of comovement? Is it better to build a portfolio with securities of different volatilities, as suggested by the fact that they will have a lower degree of comovement? What happens to risk in a portfolio of securities with high volatility, which tend to have a lower degree of comovement? These questions represent promising future lines of research.

Future lines of research

After reviewing the research carried out to date, the aim is to continue the study using the new methodology created to measure market comovement, since, as mentioned in the previous section, we believe that this methodology can help in the formation of portfolios by helping in their diversification from a different perspective.

The study of the comovement from different factors is very interesting to be able to study the composition of a portfolio. For this reason, the study of cluster formation which emulates the formation of investment portfolios from classifications made according to the comovement studied from different factors is proposed for future lines of research.

As a starting point, the volatility factor will be used since it yielded very good results in the last study and, as seen, raises several questions about the formation of investment portfolios, although starting with the volatility factor does not close the way to carry out studies on other factors. It would be very interesting to see the evolution in the composition of an investment portfolio according to various economic factors, which could be the sector, capitalization, book-to- market ratio, etc.

The verification of the importance of the market will continue to be carried out, since it is very interesting to see if, even if the object of study is changed, the representation of the market is still important in the explanation of the proposed model.