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ABSTRACT

We propose the use of entropy, H, as an indicator of the equilibrium state of a seismically active region (seismic system). The relationship between an increase in H and the occurrence of a great earthquake in a study area can be predicted by acknowledging the irreversible transition of a system. From this point of view, the seismic system evolves from an unstable initial state (due to external stresses) to another, where the stresses have dropped after the earthquake occurred. It is an irreversible transition that entails an increase in entropy. Five seismic episodes were analyzed in the south of the Iberian Peninsula, the Alboran Sea (Mediterranean Sea), and the North of Morocco: two of them of moderate-high magnitude (Al Hoceima, 2004 and 2016) and three of them of moderate-low magnitude (Adra, 1993–1994; Moron, 2007; and Torreperogil, 2012–2013). The results are remarkably in line with the theoretical forecasts; in other words: an earthquake, understood as an irreversible transition, must suppose an increase in entropy.

The seismicity of a region contains abundant information that can be used, from different points of view, in an attempt to know when an earthquake is going to occur. The present paper shows how to use entropy to characterize the occurrence of an earthquake. An earthquake, understood as an irreversible transition, must suppose an increase in entropy. Five examples, with earthquakes in the southern of the Iberian Peninsula (Spain), reveal this increase in the entropy. There are two important originalities in this paper: on the one hand, the explained technique is used not only in large-magnitude earthquakes (where the release of energy is evident), but also in moderate and small earthquakes; on the other hand, the method has been successfully applied to two consecutive earthquakes than can be detected by entropic variation.

I. INTRODUCTION

The second law of thermodynamics postulates that only those phenomena for which the entropy of the universe increases are allowed. Thus, in the field of seismology, it is natural to use entropy to ascertain future states that a region of the Earth's crust can access from its current state (Akopian, 2015).

Scientific literature on entropy is huge, and it is frequently used in scientific disciplines such as equilibrium and non-equilibrium thermodynamics or statistical mechanics. However, other fields were entropy is widely used are, for example, cosmology (geometric entropy could help us to understand some black hole features), geosciences (including seismology or climatology), life sciences, chemistry, and even linguistics and social sciences. In fact, entropy has a large number of interpretations, explications, and applications in every of that topics.

Entropy was introduced by Clausius (1865); the new magnitude would tell under what conditions a system could progress from a given initial state to a desired final state spontaneously. It was Boltzmann who introduced entropy into the domain we now call statistical mechanics. After Boltmann's ideas, Gibbs's formulation represents a first generalization of Boltzmann's approach. Von Neumann (1927) defined the density operator to be used on Gibbs formulas. A few years later, Shannon (1948) introduced two main properties in information theory: amount of information and Shannon information entropy. There are two generalized entropies from Shannon's formulation: the Tsallis entropy (Tsallis, 2009) and the Rényi entropy (2010); under some conditions, Renyi and Tsallis entropies are identical to Shannon entropy (Amigó *et al.*, 2018).

From the point of view of thermodynamics, but also statistical mechanics (Vallianatos *et al.*, 2016), variation in entropy has been widely used in seismology as an indicator of the evolution of a system (Rundle *et al.*, 2003; and Sornette and Werner, 2009). Specifically, an attempt has been made to describe in entropic terms the evolution of stresses (and, therefore, of strains) in order to try to predict future seismic activity or, at least, to use parameters (for instance, drop stress, strain changes, or variations of a and b values from Gutenberg–Richter relationship (Gutenberg and Richter, 1944; 1954; 1956) that indicate whether an earthquake will occur. In an important paper, De Santis *et al.* (2019) review and clearly explain the importance of using entropy to understand the seismic phenomenon holistically. Previously, De Santis *et al.* (2011) proposed the use of entropy to characterize the L'Aquila seismic sequence in 2009, understood as an essentially chaotic process (De Santis *et al.*, 2010). Akopian and Kocharian (2014) coined the term "seismic system" (SS) and revealed a growth in entropy during the process of a seismic sequence. We define the seismic system in a similar way to the Akopian and Kocharian (2014) but not only for large earthquakes. These authors define the SS as "an enclosed volume V of lithosphere where strong earthquakes are prepared."

A remarkable application of their hypotheses is revealed in Akopian (2014) with a study of the devastating earthquake in Tohoku (Japan) in 2011, whose magnitude was $M = 9.1$ (Ammon *et al.*, 2011); in that work, the author used entropy to mathematically model the preparatory process that led to the catastrophic earthquake. The same earthquake studied by Sarlis *et al.* (2018) by means of a natural time analysis, which has been found of usefulness to analyze seismicity (Varotsos *et al.*, 2011) and physiological time series (Varotsos *et al.*, 2007; and Sarlis *et al.*, 2018), showed that the entropy of seismicity in natural time under time reversal changed sharply 2 months before the great earthquake. Varotsos *et al.* (2018) by applying also the natural time analysis studied the Tsallis entropy q index (Sotolongo and Posadas, 2004) before the Japanese earthquake, and they found that the index grew before the mega-event. Meanwhile, Lopes and Machado (2016) used entropy to characterize the statistical distribution of earthquakes throughout the world, from 1963 to 2012, and they concluded that entropy H may represent the interrelation between studied data.

Most works that use entropy values to characterize seismicity changes over an area apply their hypotheses to large earthquakes. However, scale invariance of the seismic phenomenon has been revealed in many scientific works (Grinstein, 1991; Lapenna *et al.*, 2000; Kung Lee *et al.*, 2006; Davidsen and Kwiatek, 2013; Li and Xu, 2013; and Mariani *et al.*, 2013). Although the sensitivity of the measuring instrument (in our case, the seismometer network used) will make detection of these changes more or less complicated, those changes must be observed in a seismic system of a few kilometers' length (corresponding to low-magnitude earthquakes) and, of course, in a larger seismic system of several tens of kilometers' length (corresponding to high-magnitude earthquakes).

In this paper, we will show that changes occurring in the entropy value are detectable on small and large scales. We want to emphasize that the proposal of this paper deepens into that of De Santis *et al.* (2011) in two lines: the first, in reference to the spatial and temporal seismic system chosen; the second, with respect to the energy, when considering earthquakes not only of moderate and high magnitude, but also of small magnitude.

The formalism of De Santis *et al.* (2011) was applied to a seismic system that, from a spatial point of view, is a volume of $40 \times 50 \times 20$ km (approximately) of the Abruzzo region in central Italy; it was seismically activated by well-located and large $M_W = 6.1$ earthquake. Moreover, although the fracturing mechanism was not simple (Chiaraluce *et al.*, 2011), from a temporal point of view, the seismicity associated with it was a sequence of 10 000 earthquakes where the cumulative moment release took place in a few days after the mainshock, that is, in a very narrow window of time. In this paper, we will show that this formalism is valid also under other conditions. To do that, several seismic systems will be analyzed in the southern region of the Iberian Peninsula and the Alboran Sea.

On the one hand, we will choose a broader seismic system from both the spatial and also the temporal point of view; in fact, it includes two seismic events of moderate-high magnitude, and we will prove that the entropy increases with the first event, it returns to similar values to those before the earthquake, and it grows again with the second earthquake. We think that there is no objection to applying the proposed technique to a complex system. Actually, we try to show that it is indeed possible to apply it, despite of the complexity of the seismic system. From our point of view, the proposed method is reliable not only for simple sequences but to complex ones. This is the aim of our first application to Al Hoceima series. The first two analyses correspond to seismic systems in which a medium-large earthquake has occurred: the $M = 6.3$ Al Hoceima 2004 earthquake (Stich *et al.*, 2005) and the $M = 6.4$ Al Hoceima 2016 earthquake (Buforn *et al.*, 2017).

On the other hand, we will apply this formalism also to earthquakes of smaller magnitude, where the detection of changes in entropy is not so evident. Hamdache *et al.* (2019) made the largest analysis and up to date of the seismic series in the studied area. They found 23 seismic sequences from 1985 to nowadays; the three seismic series here analyzed are among the 23 explained in that paper. Our choice is based on the qualitative classification from Mogi (1963), who divided seismic sequences in the three well-known types: type I (main shock and aftershocks), type II (foreshocks, main shock, and aftershocks), and type III (swarm). From our point of view, the Adra-Berja (1993–1994) seismic series is type I, the Moron (2007) seismic series is type II, and Torreperogil (2012–2013) is type III. Of course, the limits to include each series in one type or another can be discussed; but, we believe that these three seismic series can be a sample of each type of sequence. The results offer a good correspondence between a sudden change in entropy and the occurrence of a main earthquake in the sequence.

II. SEISMIC SYSTEM AND ENTROPY

Spatial and temporal seismicity patterns should variate as a result of the stress field applied to a volume of the Earth's crust. Modeling of the distribution of earthquakes reflects a physical system characterized by chaotic processes (De Santis *et al.*, 2010), whose level of organization is quantifiable by the physical magnitude of entropy (H) and its evolution, using the second law of thermodynamics. This system, called by some authors a "seismic system" (Akopian and Kocharian, 2014), can be in an equilibrium state or in a non-equilibrium state; nevertheless, reorganization of stresses on that system can lead to an earthquake and a new state. From an entropic point of view, this reorganization is reflected in the values of H.

Entropy, introduced by Clausius almost two centuries ago in the macroscopic context of thermodynamics, was reinterpreted a few years later by Boltzmann in the microscopic field of statistical mechanics. The works from Boltzmann, Gibss, and Szilard in the late 19th century appreciated the connection between entropy and information. However, until works from Shannon (1948) and Shannon and Weaver (1949), there was no clear and explicit discussion of the concept of information and its properties. Recently, Ben-Naim (2017) shows that the Shannon entropy (the author called that concept the Shannon measure of information or SMI) provides a solid and quantitative basis for the interpretation of thermodynamic entropy.

Shannon focused his discussion on the transmission of information from a transmitter to a receiver through a certain channel whose statistical properties are known. Shannon's main idea, in terms of the predictability of a dynamic system, is to assume that the system communicates some information, although not necessarily all information, from its past to its future. Thus, when the objective is to determine the connection between past states of the dynamic system and future states, Fraser and Swinney (1986) suggest that the use of "mutual information" (μ_I) based on Shannon's entropy be used as a measure of the dependence of the current state $S(t)$ on the future state $Q(t) = S(t + \tau)$, where τ is a certain time interval. The mutual information between states, (μ_I), can be deduced in its discrete form using the Kullback–Leibler formalism (Jumarie, 1990),

$$\mu_I = \sum_{i=1}^{n}\sum_{j=1}^{m} P_{SQ}(s_i, q_j) \times \log\left(\frac{P_{SQ}(s_i,\ q_j)}{P_S(s_i) \times P_Q(q_j)}\right), \tag{1}$$

where $P_S(s_i)$ is the probability of the initial or past states, $P_Q(q_j)$ is the probability of the final or future states, and $P_{SQ}(s_i\ ,\ q_j)$ is the joint probability of the S and Q states. Some authors (Posadas *et al.* 2002; and Machado and Lopes, 2013) have used Eq. (1), and the principle of maximum entropy to find the population of events in future states from current states in different regions of the world.

In the present work, the initial and final states are represented by a distribution of earthquakes with magnitudes associated with time t; Aki (1965) and Utsu (1965) proved that, under certain conditions (e.g., the maximum magnitude is much greater than the minimum magnitude allowed), the probability density function for an earthquake distribution with magnitudes M is given by

$$P(M) = b \times \ln(10) \times 10^{-b\times(M-M_0)}, \tag{2}$$

where M_0 is the minimum magnitude of the dataset and b is the known slope of the Gutenberg–Richter relationship,

$$\log\ n(M >) = a - b \times M, \tag{3}$$

where $n(M >)$ denotes the number of earthquakes having a magnitude $\geq M$. Marzocchi and Sandri (2003) prove that only if the difference between the maximum and minimum values of the magnitude is greater than or equal to 3, it is possible to obtain Eq. (2). Page (1968) and Bender (1983) proved that using a simple linear least-squares fit biased values for a and b parameters are obtained; for instance, Bender (1983) carries out an in-depth study of obtaining parameter b from two maximum probability fits, two least-squares fits, and one minimum fit of χ^2, and he warns about the right choice of the fit method. Recently, Kijko and Smit (2012) offered a new and simpler way of estimating b. De Santis *et al.* (2011) carry out a very elegant mathematical proof considering the mean value of all possible magnitudes, $\bar{M}$, in a given time interval T, over which M is defined, using Eq. (2) and, finally, solving by integration per parts; they conclude that the b value is the one deduced by Aki (1965),

$$b = \frac{1}{\ln(10) \times (\bar{M} - M_0)} = \frac{\log(e)}{\bar{M} - M_0}, \tag{4}$$

subsequently, slightly improved by Utsu (1965),

$$b^U = \frac{\log(e)}{\left[\bar{M} - \left(M_0 - \frac{\Delta M}{2}\right)\right]}, \tag{5}$$

where $\bar{M}$ is the average value of the magnitude in the considered time interval and ΔM is the resolution of the magnitude (usually $\Delta M = 0.1$). The uncertainty associated with each value, interpreted as the error in the value's determination, is given by

$$\sigma_b = \frac{b}{\sqrt{N}}, \tag{6}$$

where N is the number of earthquakes occurring in the same time interval. Other estimations of b uncertainty are on Shi and Bolt (1982) or Amorèse *et al.* (2010).

Finally, if the states of the seismic system at time t are given by a probability distribution $P(t)$, the entropy postulated by Shannon associated with the information is given by (Fraser and Swinney, 1986)

$$H\ (t) = -\int P(t) \times \log(P(t))dt. \tag{7}$$

Following Telesca *et al.* (2004), in the extreme case, when $P(t)$ is equiprobable for all t, there is no variable for which we have more knowledge than another; in this case, $H = 1$ (if H is normalized to 1) (Fig. 1). Shannon's entropy or information entropy has been widely used in the study of seismicity by different authors (Bressan *et al.*, 2017). Combining it with the natural time analysis, an explanation of the b-value from first principles was obtained without using any adjustable parameter (Varotsos *et al.*, 2004; 2006); in particular, it was found that the almost constant value $b \approx 1$ is just a consequence of the physical expectation that the information entropy associated

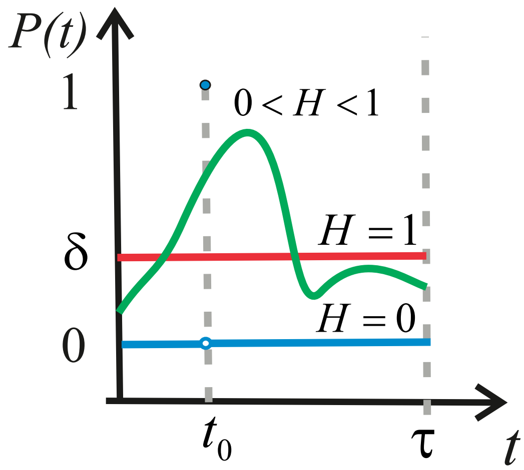

FIG. 1. The probability distribution $P(t)$ of the states of the seismic system as a function of time t determines the value of entropy $H(t)$. In extreme cases, when $P(t) = \delta$ for all t, all states are equiprobable (red line) and H reaches its maximum value (if H is normalized to 1, then $H = 1$); H expresses our ignorance about the system's state. On the other hand, if $P(t = t_0) = 1$ and $P(t \neq t_0) = 0$, we have $H = 0$ (blue line); in that case, we have complete information about what state the system is in. Other possibilities (green line) are represented.

with the probability distribution of the order parameter of seismicity (emerged from natural time analysis) should become maximum. De Santis *et al.* (2011) particularized Eq. (7) for a distribution of earthquakes with magnitudes M in a time interval t as

$$H\ (t) = -\int_{M_0}^{\infty} P(M,t) \times log(P(M,t))dM \tag{8}$$

from which, together with Eqs. (2) and (4), it is concluded that

$$H(t) = \log(e\ \times \log\ e) - \log\ b = \log e + \log\left(\bar{M} - \left(M_0 - \frac{\Delta M}{2}\right)\right). \tag{9}$$

Obviously, last equation only can be used if $b \leq \log(e \times \log\ e) \approx 1.18$ in order to have $H \geq 0$ (De Santis *et al.*, 2011).

To sum up, the steps of the analysis are as follows:

1. The magnitude of earthquakes must verify that
$$M_{max} - M_0\ \geq 3.0. \tag{10}$$
2. The value of M_0 is estimated using the Gutenberg–Richter law and, if necessary, that law is used without accumulating magnitudes (sometimes this method is clearer than the accumulative method).
3. The time interval W (the minimum number of earthquakes which is used to calculate H) is determined for the calculation of entropy; this can be done with a cumulative, moving, or overlapping earthquake window. In this paper, the results will be presented with a cumulative window, which presents greater stability; nevertheless, as De Santis *et al.* (2011) pointed out, the results are substantially the same in all cases; in fact, the results that we got are almost similar with different types of windows. The width of the window can be chosen following the criteria of these same authors based on meaningful values of b.
4. Finally, the entropy function is obtained for each time t following Eq. (9). By convention, the time attributed to each point of the analyses is the time of the last seismic event considered in each window. The occurrence of a large earthquake (or the accumulation of several important ones) is expected to lead the seismic system to a state of greater disorder; that is, the earthquake is an irreversible transition to a new state, which means an increase in entropy. Once the mainshock is over, entropy returns to stable values.

III. SEISMOLOGICAL SETTING AND DATASET

The study region is defined by the South of the Iberian Peninsula, the Alboran Sea, the North of the African margin (Morocco, Algeria, and Tunisia), and the Atlantic side of the Gulf of Cádiz. Geographically, it is located between 34° and 39° North latitude and between 2° East longitude and −12° East longitude. The area is located on the boundary of the large Africa (Nubia) and Eurasian tectonic plates, where the seismicity is dominated mainly by the release of small and moderate earthquakes ($M < 6.0$). However, earthquakes of magnitude greater than 6 in the north of Algeria (e.g., El Asnam, 1980, with $M = 7.0$ or Bourmedes-Zemmouri, 2003, with $M = 6.9$), in the Atlantic (Gulf of Cádiz, 1964, with $M = 6.6$, Cabo San Vicente, 1969, with $M = 7.8$, and also Cabo San Vicente in 2007 with $M = 6.0$) are present too, whereas in the Alboran basin and the Betic–Rif Cordilleras sectors the elastic deformation is released mainly by $M < 5.5$ earthquakes, with higher magnitude earthquakes more separated in time (Stich *et al.*, 2019). Seismicity in the region is mainly due to the slow (≈5 mm/year) oblique convergence between the above-mentioned (Nubia and Euriasia) tectonic plates (Serpelloni *et al.*, 2007; and Nocquet, 2012). The seismicity driven by this geodynamic context draws a broad (not diffuse) plate tectonic boundary. The increase in seismic instrumentation deployed in the region in the last two decades and the higher quality of the resulting data allow to better understand that the seismic sequences (main event with precursor and aftershocks; main earthquake with aftershocks; or seismic swarm releasing large number of microearthquakes) in Southern Spain not fully follow the Poisson law of independent events, since these are grouped into different types of seismic sequences and must be considered as interdependent phenomena (Stich *et al.*, 2019). Most of the seismic activities in southern Spain are characterized by earthquakes of moderate magnitude ($M < 5.5$) as the $M = 5.2$, 2011 Lorca earthquake (Morales *et al.*, 2014) and also with intense (in time) microseismic ($M < 3$) activity clustered in series or swarms (Morales *et al.*, 2015; and Hamdache *et al.*, 2019). The area under study has been deeply studied from several points of view. Thus, from the seismic source and seismotectonic framework (Stich *et al.*, 2006; 2010; and Martín *et al.*, 2015), the fine source area structure through relocation techniques (Stich *et al.*, 2001; Ocaña, 2009; Ocaña *et al.*, 2008; and Morales *et al.*, 2014; 2015), applying principal component and three-point methods (Posadas *et al.*, 1993a; 1994b) or carried out an analysis on the determination of the next most probable earthquake in a seismic series (Torcal *et al.*, 1999a; 1999b). On the other hand, some authors

(Sotolongo *et al.*, 2000) analyzed the distribution of earthquakes in southern Spain by using Lévy functions, and they concluded that it is possible to model the occurrence of earthquakes over time as a Lévy flight. Most seismic series and swarms (up to 23 of them) have been studied using a fractal approach by Hamdache *et al.* (2019).

In this paper, two seismic sequences associated with a seismic episode of considerable size are used on the northern margin of the African continent (Al Hoceima, 2004, with $M = 6.2$ and Al Hoceima, 2016, with $M = 6.3$), as well as three moderately sized series or swarms in the south of the Iberian Peninsula (the Adra series from 1993–1994 with $M = 5.1$, the Moron series from 2007 with $M = 4.6$, and the Torreperogil series from 2012–2013 with $M = 3.8$); data from the National Seismic Network, provided by the National Geographic Institute (www.ign.es), are used in the Al Hoceima earthquakes, while data from the Andalusian Seismic Network, belonging to the Andalusian Institute of Geophysics and Seismic Disaster Prevention (https://iagpds.ugr.es/), are used for Adra, Moron, and Torreperogil seismic series or swarms.

IV. VARIATION OF ENTROPY IN THE SEISMIC SEQUENCES OF AL HOCEIMA 2004 AND 2016

The Al Hoceima region has suffered three earthquakes of relevant magnitude in the last 25 years. Kariche *et al.* (2018) studied the three earthquakes that occurred in 1994 ($M = 5.7$), 2004 ($M = 6.2$), and 2016 ($M = 6.3$). Due to the wide range of dates, a homogeneous magnitude catalog is not available for the three seismic sequences after the major events mentioned. In the case of the NEIC catalog (National Earthquake Information Center, U.S. Geological Survey), of course, there is homogeneity, but there are only 500 earthquakes with magnitudes greater than 2.5–3.0. Although this amount of data is enough for our analysis, larger catalogs will provide the best results. Therefore, it is necessary to retrieve data from local seismic networks, such as the National Geographic Institute (IGN, Spain) or the Andalusian Seismic Network (RSA, Spain); in the first case, the earthquake catalog is homogeneous for the 2004 and 2016 sequences (magnitudes greater than 1.0) but not for the 1994 series (threshold magnitudes above 2.5), while, in the second case, there is homogeneity for the earthquakes of 1994 and 2004 (magnitudes greater than 2.0) but not for that of 2016, where the sensitivity already reaches magnitude 1.0. Due to the temporal proximity and size of catalogs, the results of the most current earthquake sequences (2004 and 2016) are presented here using the IGN database.

The Al Hoceima seismic sequences of 2004 and 2016 are examples of a moderate-large earthquake in the area. Both events have been studied from a tectonic, seismic, and engineering point of view [for the 2004 earthquake, e.g., please refer to Stich *et al.* (2005), and for the 2016 earthquake, e.g., please refer to Buforn *et al.* (2017)]. The epicentral region is located between 34.5° and 36° North latitude and 3.5° and 4.5° West longitude (Fig. 2).

The first earthquake took place on February 24, 2004, reaching a magnitude of $M = 6.2$; it is located at 35.156° North latitude and 3.984° West longitude, about 10 km from the city of Al Hoceima. The main event was followed by an extensive sequence of aftershocks, although little previous activity was seen. For the analysis that has been carried out here, and in order to have previous data with which to compare the entropy before and after the event, a catalog with more than 2300 earthquakes occurring between 1996 and 2010 was used. This sequence is referred to as "AlHoceima01."

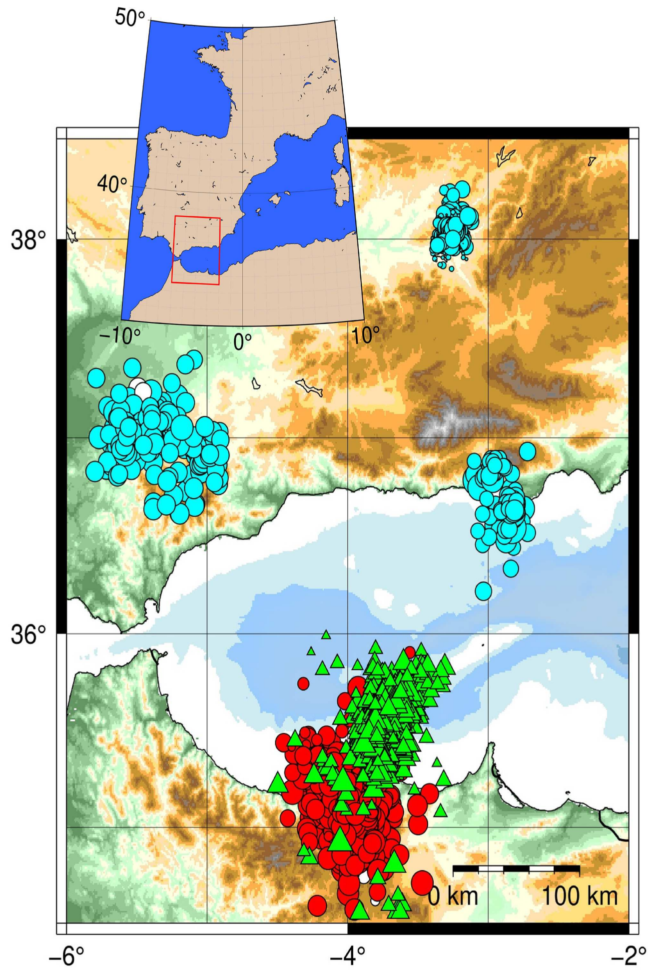

FIG. 2. The epicentral area studied in this paper corresponds to the African and Eurasian plates border in the south of the Iberian Peninsula, the Alboran Sea (Mediterranean Sea), and the north of Morocco. The five seismic sequences studied are shown: in red circles and green triangles, the Al Hoceima earthquakes of 2004 and 2016, in cyan, from left to right, the sequences of Morón (2007), Torreperogil (2012), and Adra (1993–1994). Earthquakes plotted correspond to the whole catalog.

The second earthquake occurred on January 25, 2016, in the same area, although this time its epicenter (35.600° North latitude and 3.806° West longitude) was located in the sea, about 30 km from the African coast. The magnitude was $M = 6.3$ and a catalog with just over 3700 events between 2010 and 2019 was used. This sequence is referred to as "AlHoceima02."

The Gutenberg–Richter law for Alhoceima01 was calculated, and the result is presented in Fig. 3; M_0 has an approximate value of $M_0 = 1.5$, and it is estimated that the value of b is significant with

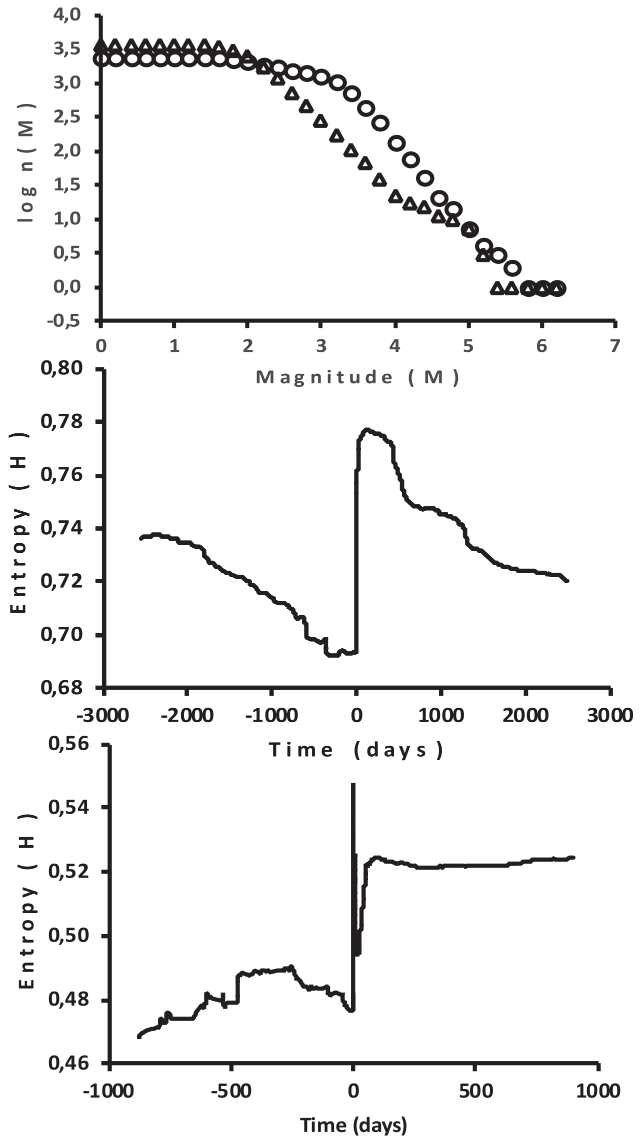

FIG. 3. (Top) The Gutenberg–Richter relationship for the two sequences of Al Hoceima 2004 and 2016 (circles and triangles, respectively) allows us to estimate the values of M_0 as 1.5 and 1.25, respectively. (Center) Variation in entropy, *H*, for Al Hoceima (2004); an abrupt increase in *H* occurs before the day of the main event and a decrease occurs after it (time, in days with respect to the date of the main event). (Bottom) Variation in entropy *H* before and after the 2016 event.

$W = 300$ events. The results of entropy vs time ($t = 0$ when the main event occur, i.e., in days with respect to the main event) are shown in Fig. 3. The abrupt variation in the entropy values clearly fits with the occurrence of the 2004 earthquake. A similar analysis was carried out with the sequence AlHoceima02. In this case, the values of M_0 and W were, respectively, 1.25 and 300. The variation in entropy is also clear from Fig. 3.

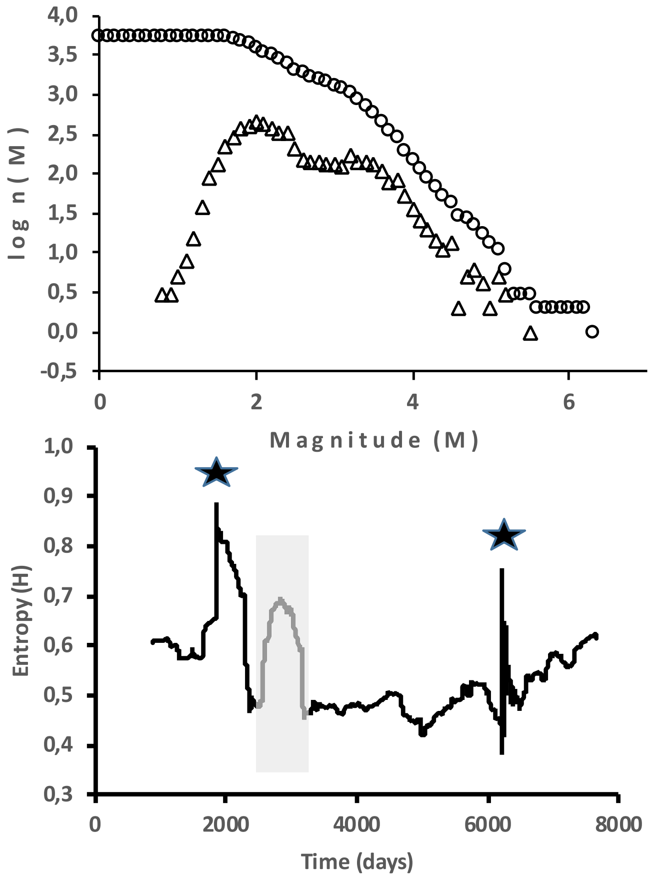

FIG. 4. (Top) The Gutenberg–Richter relationship (cumulative in circles, non-cumulative in triangles) for the whole set of Al Hoceima earthquakes (2004 and 2016) us to estimate $M_0 = 1.5$. (Bottom) Variation in entropy for the whole set of Al Hoceima earthquakes. The days of occurrence of the two main earthquakes are marked with a star, showing the correlation with the great change in *H*. The shaded area corresponds to a period of great seismic activity (including up to 15 events with magnitudes greater than 4.0). Time, in days, with respect to the date of the main event.

The methodology presented here is capable of detecting, from the abrupt change in the values of *H*, an irreversible transition represented by an earthquake; however, the clear visualization of these results is achieved with different M_0 (and equal *W*) values in each case. The next step would be to use the Al Hoceima seismic system as a whole set (for the available data) and to try to identify both events at once. An estimation of M_0 is made from the Gutenberg–Richter law (Fig. 4), which has now been carried out cumulatively and non-cumulatively for a better estimation of the threshold magnitude. This time, a value of $M_0 = 1.5$ is taken. With respect to *W*, the accumulated display of *H*, as we have previously stated, is always more stable than other types of windows (moving or totally or partially overlapped); however, in the case of accumulative window, there is a memory maintenance effect of the entire series that now is undesirable because a major new earthquake is expected after the first.

Although the results are very similar with all types of windows, the sharpest display is now obtained with a moving window. The results of this test can be seen in Fig. 4; the value of H increases considerably in the days before both earthquakes and then decreases in the following days (new stable state). However, the growth in entropy between days 2300 and 3200 (from the beginning of the catalog) is striking. The first earthquake corresponds to day 1867 (higher peak of entropy in Fig. 4), and the relaxing process extends for 1 year; in fact, day 2363 (1 year after the 2004 earthquake) corresponds to a lower value of entropy. However, entropy begins to increase again until day 2873 and then decreases until day 3200. This unexpected growth can be explained by the great concentration of earthquakes with magnitudes between 4.0 and 5.0 in those days.

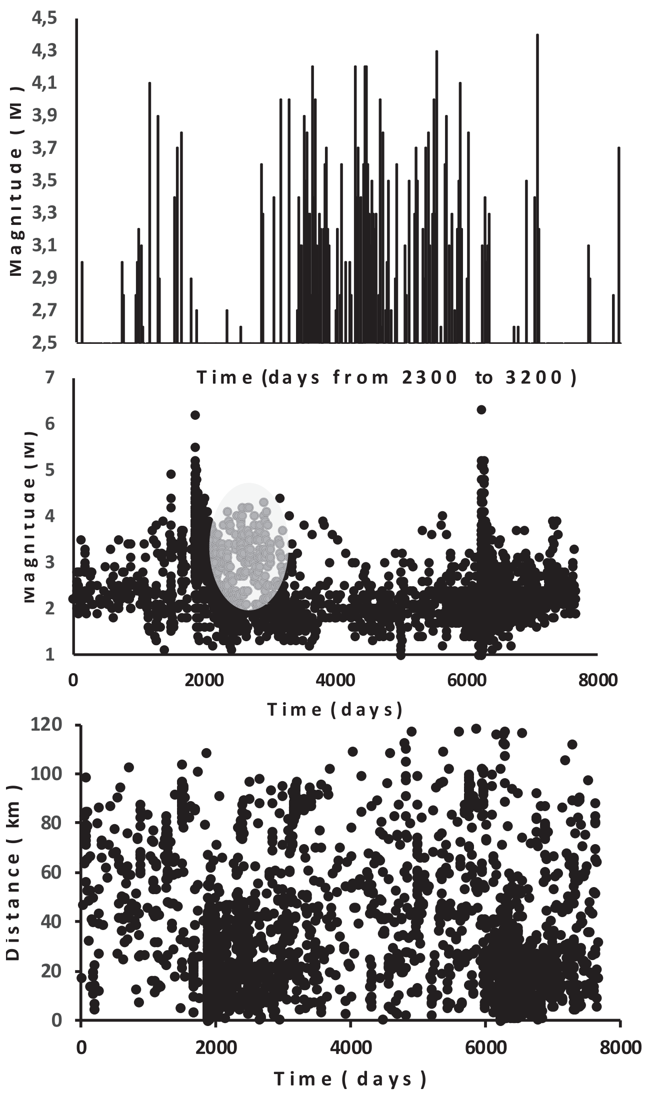

FIG. 5. (Top) Frequency of earthquakes in the shaded area of Fig. 3 (approximately from day 2300 to day 3200). (Center) Evolution of magnitude in the whole sequence of Al Hoceima; the shaded area corresponds to the high activity recorded between 2300 and 3200 days after the first major earthquake (not immediately afterward—see the main text for an explanation). (Bottom) Distances from earthquakes to the epicentral location of the main earthquakes, showing how these distances converge to values lower than 20 km.

Figure 5 shows the frequency of these earthquakes, and we observe the occurrence of up to 15 events with magnitudes greater than 4.0 (in addition to relevant seismic activity in lower magnitude ranges, fundamentally with $M > 3.0$). This phenomenon is also verified if the temporal evolution of magnitudes is analyzed (Fig. 5). Clearly, between approximately 2500 and 3500 days after the first earthquake, a large concentration of events with magnitudes between 3.0 and 4.5 took place, which is reflected in the sudden rise in the values of H. Finally, the distances from all earthquakes to the epicentral location of the main earthquakes before and after the main events were calculated, and the results are shown in Fig. 5. Notably, the distances to both main events converge to values of less than 20 km after the occurrence of the main earthquake, clearly distinguishing the two clusters of seismic activity.

V. SEISMIC SEQUENCES OF ADRA (1993–1994), MORON (2007), AND TORREPEROGIL (2012–2013)

As previously outlined, the south of the Iberian Peninsula is characterized by a moderate or low seismicity with some frequent seismic crises grouped into more or less numerous clusters. Up to 23 of these swarms or series have been analyzed, among others, by Ocaña (2009) and Hamdache *et al.* (2019).

The Adra seismic sequence (1993–1994) began with the December 23, 1993 earthquake of magnitude $M = 5.0$, located at 36.785° North latitude and 3.080° West longitude, close to the town of Adra. More than 500 earthquakes occurred following the first one. A notable earthquake was that of January 4, 1994 with $M = 5.1$ at 36.543° North latitude and 2.833° West longitude, 34 km from the December 23 event, this time close to the town of Berja. The epicentral zone is presented in Fig. 2, and the evolution of the magnitude is shown in Fig. 6. To estimate M_0, the Gutenberg–Richter law is used, showing that $M_0 = 1.25$, while $W = 40$. The calculation of the entropy leads to the graph shown in Fig. 6, where the time in days is expressed with respect to the occurrence of the main event.

The Moron sequence (2007) began with a magnitude 4.6 earthquake on June 30, 2007 at 37.071° North latitude and 5.445° West longitude, a place close to the town of Moron; 9 h later, a new magnitude 4.0 earthquake occurred just 2 km from the first. A year later, on October 2, 2008, two successive earthquakes occurred within 3 min of each other, both with a magnitude of 4.1, occurring practically in the same place as the first two. The same fault system generates some sporadic current earthquakes of some relevance (for example, on October 5, 2017 with $M = 4.0$). To our knowledge, the different series that have occurred in the Morón region are dominated mainly by thrust or reverse faulting either the same fault or parallels) for the events of greater magnitude, although other fault regimes (strike-slip faulting) also released microseismicity in the series. It should be noted that, in the same region and 5 years earlier, there was a small outbreak of seismic activity with just 77 earthquakes, the largest of which occurred on September 15, 2002 with a magnitude of 4.1.

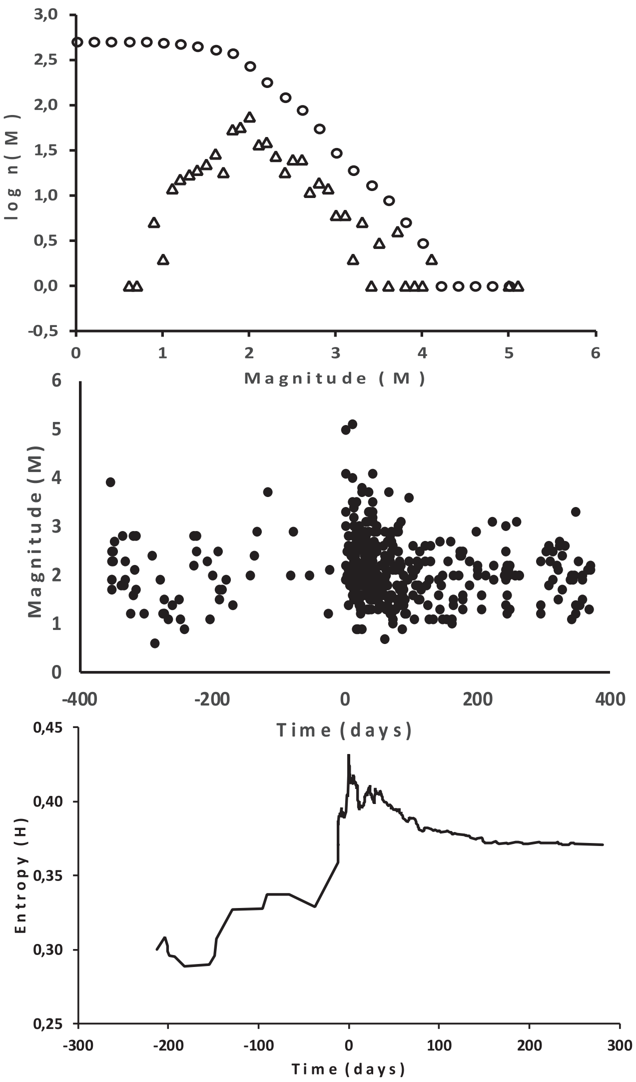

FIG. 6. Gutenberg–Richter relationship for the Adra seismic sequence (1993–1994), the evolution of magnitude, and the result of the estimation of entropy (top, center, and bottom, respectively). A correlation between increasing H and the day of the main earthquake on January 4 is clearly evident. Time, in days, with respect to the date of the main event.

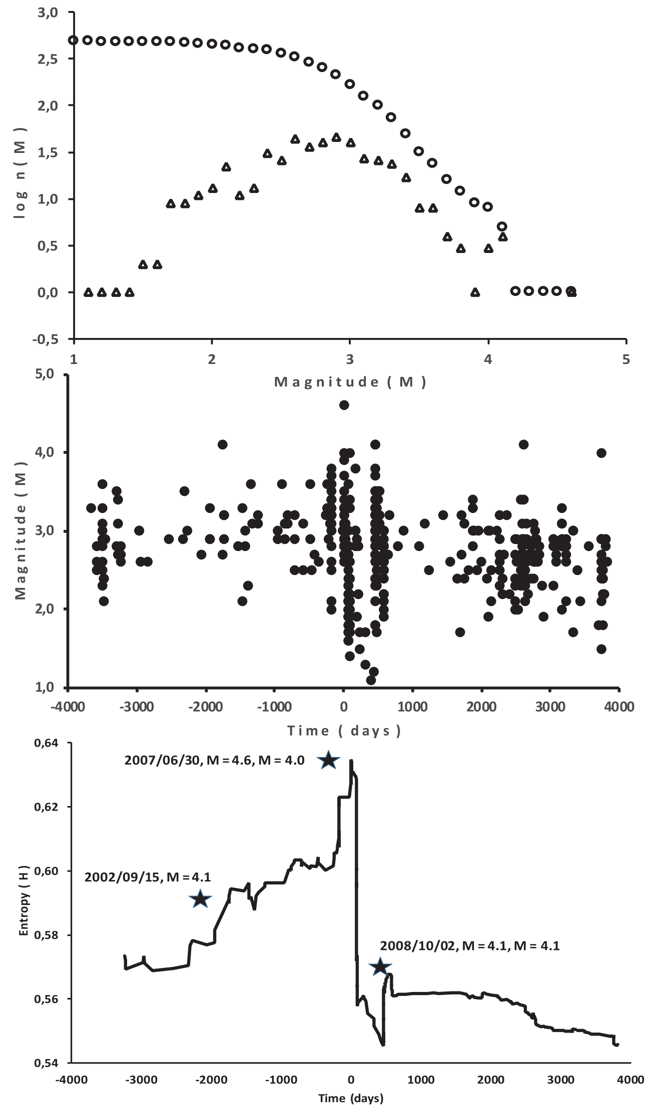

FIG. 7. Gutenberg–Richter relationship for Moron seismic sequence (2007), the evolution of magnitude, and the result of the estimation of entropy (top, center, and bottom, respectively). The lower graph shows the occurrence of the earthquake on September 15, 2002 ($M = 4.1$), the two consecutive earthquakes with magnitudes 4.6 and 4.0 on June 30, 2007 (main episode), and the two successive earthquakes on August 2, 2008 both with magnitude 4.1 (marked with stars).

Figure 2 shows the epicentral region, and Fig. 7 shows the results obtained for entropy. The variation in H is clear on the day of the main event and the days after, but significant changes in entropy can also be detected for the two consecutive earthquakes of magnitude 4.1 in 2008, as well as the earliest of 2002, also with 4.1 magnitude.

The epicentral area (Fig. 2) of the Torreperogil sequence (2012) is centered at 38.090° North and 3.280° West, and it extends over an area of about 300 km^2 (Morales *et al.*, 2015; and Hamdache *et al.*, 2019). From 1980 to October 2012, this area had not registered more than 24 earthquakes, all of them moderate, and only one occurred in August 2005 with a magnitude of $M = 3.4$; but, suddenly, it is from October 2012 when 130 microearthquakes were recorded, with an average magnitude of 2.0, which ended on December 15 with an earthquake of magnitude $M = 3.5$. After this date, more than 5000 events took place in the area until February 5, 2013, when the largest earthquake occurred ($M = 3.7$). Subsequently, an additional 2000 microearthquakes occurred until the end of 2013. On the whole, the seismic sequence includes just over 7000 events with magnitudes between −1.5 and 3.7.

The Torreperogil seismic sequence can be classified to a first approximation as a seismic series or swarm, where in the absence of external perturbation (static or dynamic) that trigger it, the microearthquake rate activity increases very fast (in a small Earth crust volume) related to the background seismicity level. It can generate thousands of microearthquakes in weeks or months without being able to clearly distinguish a main earthquake. At present, how they start and dissipate remain unclear. The sequence shows a hundred precursors prior to the December earthquake ($M = 3.5$), and from here, an explosive increase in activity (5000 events) until

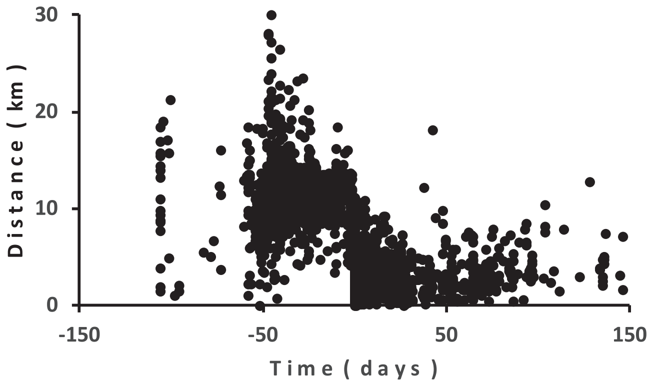

FIG. 9. Gutenberg–Richter relationship for Torreperogil seismic sequence (2012), the evolution of magnitude, and the result of estimation of entropy (top, center, and bottom, respectively). Again, the entropy increases before the day of the main event ($M = 3.7$), recovering its original value days after the earthquake.

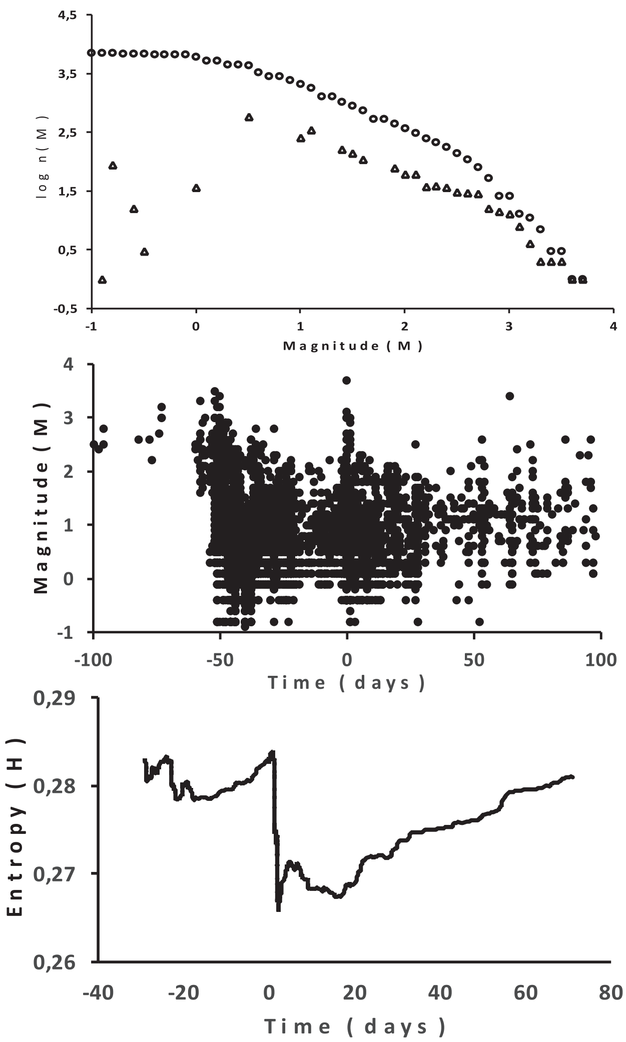

FIG. 8. Distances from all earthquakes to the earthquake of December 2012 ($M = 3.4$) and to that of February 2013 ($M = 3.7$). The graph reveals that earthquakes prior to the main event are between 5 and 15 km near the first earthquake, while those after the main event are less than 5 km from its epicenter.

reaching the 3.7 magnitude earthquake in February. The daily seismic activity was remarkable, with 798 earthquakes taking place on the same day, February 5. The absence of a larger earthquake in the sequence indicates that maybe the series is a swarm rather than a seismic series.

The Torreperogil earthquakes seem to tend to cluster around the main event, as shown in Fig. 8. The distances from each event to the first earthquake and to the main earthquake were calculated. In the first earthquake, the distance ranges from 5 to 15 km, and in the second case, the events are clustered at distances of less than 5 km. A detailed spatial analysis using the double-difference method to relocate 500 events was used by Morales *et al.* (2015), revealing a fine structure of up to three clusters in the epicentral zone.

Despite the low magnitude of the seismic sequence (the largest event only reaches 3.7), due to the large number of events that occur, seismic relaxation in the region is evidenced in variation in H. With values of $M_0 = 0.2$ and $W = 1000$, the entropic analysis shown in Fig. 9 is carried out, revealing an increase in entropy before the main event.

VI. CONCLUSIONS

In this work, we use a methodology proposed by De Santis *et al.* (2011) through which it is possible to associate changes in entropy H with the occurrence of a significant event in the analyzed area. A relationship between H and the value of the parameter b in the well-known Gutenberg–Richter law was introduced by De Santis *et al.* (2011) at a theoretical level, providing two tests on Italian seismic sequences with large earthquakes, such as that of L'Aquila 2009, with magnitude 6.3, and Colfiorito 1997, with magnitude 6.0. In the present paper, the robustness of this technique has been confirmed, allowing it to be extended to (a) various events in the same area and (b) seismic sequences of lesser magnitude. The method has been applied to the contact area between the African and Eurasian plates in the south of the Iberian Peninsula and north of Morocco where two earthquakes of moderate-high magnitude (Alhoceima 2004 and 2016) and three seismic sequences of moderate-low magnitude (Adra 1993–1994, Moron 2007, and Torreperogil 2012) were analyzed. The results confirm a strong correlation between the value of entropy H and the occurrence of an earthquake of relative relevance in the studied area. An increase in entropy H, from a thermodynamic point of view, is associated with an irreversible transition from one state to another, on a small scale [Scholz's (1968) pioneering work confirmed this] but also on a large scale [e.g., recently the work of Parsons *et al.* (2008)]. In the case of quiescent seismic sequences, that is, a relative decrease in the number of earthquakes or energy within a certain time interval in comparison with long-term observations in the same region, entropy could decrease and it could be used as a precursor parameter (Hainzl *et al.*, 2000; and Rudolf-Navarro *et al.*, 2010).

Although the methodology used here may seem useful in the field of seismic prediction, two important considerations must be taken into account. The first is that changes in H are detected with the use of a catalog of earthquakes before the large one but, also, after the main earthquake. In this sense, further studies are necessary to determine, without prior knowledge of how a series will continue, with which H values the occurrence of a significant event should be expected in the immediate future. In other words, an absolute scale of entropy is necessary. An answer to this question has been recently achieved (Varotsos *et al.*, 2020) by using the fluctuations of the entropy change under time reversal of the entropy in natural time (Varotsos *et al.*, 2004) which considers from its definition the sequential order of the events, thus being a dynamic entropy capturing the characteristics of the dynamics of the system and hence differing essentially from a statistical entropy (Shannon entropy). The second consideration falls within the spatial domain. The description presented in this work, and in other work, always takes into consideration a spatially limited seismic system. The expansion of the technique over a large area (e.g., the entire Mediterranean arc) still presents difficulties, since it is necessary to connect the variations in H to a point, that is, it is not enough merely to detect an increase in H; one must also determine *where* the increase is located. The authors of this work are currently working on a "microzone" of regions within the south of the Iberian Peninsula and the Alboran Sea that could allow spatial, and not just temporal, monitoring of H.

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DATA AVAILABILITY

The data that support the findings of this study from IGN are openly available in https://www.ign.es/web/ign/portal/sis-catalogo-terremotos. The data that support the findings of this study from IAGPDS are available from the corresponding author upon reasonable request.

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