Article

New Approximate Analytical Solution of the Diode-Resistance Equation

José A. Gazquez*[1], Manuel Fernandez-Ros[1], Blas Torrecillas[2], José Carmona[2] and Nuria Novas[1]

[1]Department of Engineering, ceiA3, University of Almeria, 04120 Almeria, Spain; jgazquez@ual.es (J.A.G.); mfernandez@ual.es, (M.F-R.); nnovas@ual.es (N.N.)

[2]Department of Mathematics, University of Almeria, 04120 Almeria, Spain; btorreci@ual.es (B.T.); jcarmona@ual.es (J.C.)

***** Correspondence: jgazquez@ual.es

Received: 2020; Accepted: date ; Published: date

Abstract: The problem of finding an analytical solution for the circuit formed by a diode with series resistance (D-R circuit) has been investigated for a long time and several authors have proposed approximate solutions. Most solutions use the Lambert function and numerical methods. However, an analytical solution independent of experimental parameters is still missing to replace the D-R circuit with an analytical expression, in which only the diode and circuit parameters appear. In this paper, using a Taylor series expansion of the equation $V_D = f\ (V_S, R, I_D)$, we propose a new analytical solution that fits the exact solution with more accuracy than all known approximations. The average of the absolute error vector is the best, with an order of magnitude of difference compared to the other 3 approximations studied. This new function can be expressed by voltage analysis, which allows analytical solutions. Our solution may have many applications, such as a simple and accurate way, in circuit simulation programs or numerical calculation programs. In particular, it can be used as well

in circuits with D-R branches, exponential amplifiers, logarithmic amplifiers and waveform shapers. On another level, a comparison with other state of the art approximations is presented.

Highlights

► A new approximation is proposed for the diode - resistance series circuit.

► Our approximation depends exclusively on the parameters of the diode, Is, V_T and η.

► Other solutions need to use experimental coefficients and determination of parameters.

► Our approximation is the fastest in execution time, in a comparative study by MATLAB.

► The average of the absolute error is the best of the other 3 approximations studied.

Keywords: diode model; generalized diode equation; Lambert W-Function; nonlinear circuit.

1. Introduction

The solution of the current in the circuit formed by a diode with series resistance (D-R circuit) (Figure 1), generates a nonlinear equation (1).

$$I_D = \frac{1}{R}[V_S - V_D f(I_D)] \quad (1)$$

where I_D (2) is the diode current, V_D is the voltage of the diode and Vs is the supply voltage.

$$I_D = I_S\left(e^{\frac{V_D}{\eta V_T}} - 1\right) \quad (2)$$

In equation (2) Is is the reverse saturation current, η is the ideality factor, and V_T is the thermal voltage $V_T = KT/q$ (K is the Boltzmann constant, T is absolute temperature in Kelvin and q electron charge).

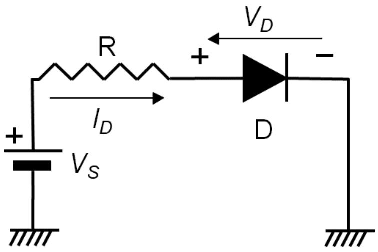

Figure 1. Diode and resistance series circuit.

This problem has been studied for a long time and several authors have given different solutions. The equation (1) does not have a solution in terms of elementary functions. In [1], [2], [3] and [4], approximate solutions, based on a function with adjustable coefficients, are proposed. More recently, in [5] and [6], exact solutions of the equation using the Lambert's function, which is solved by iterations, are obtained. The Lambert function, introduced in 1758, is classical and is defined as the inverse function for $y = xe^x$ (see [7] and their references for relevant mathematical information), it has had a great impact within the I-V modeling of solar cells [8]. In our solution, the use of this function is avoided due to its complexity. Another approximation that appeared in 2006 [9] is based on the method presented [2] and proposes an equation in several stages. The most recent known approximation [10] is also based in [2]. Both solutions have a high degree of accuracy but with a complex algorithmic formulation, which does not facilitate the theoretical analysis of circuits, restricting their use to numerical execution.

The fact of having a simple equation, capable of defining the behavior of a diode in series with a resistance as a branch of a circuit, is of interest. This would allow simple simulation of circuits with diodes with different applications: waveform shaper and clipping [11], would be the most interesting one. Through this method, it is possible to use any numerical calculation program to perform the simulations.

We propose an approximation that gets a new approximate analytical solution of the D-R circuit equation. The method consists of considering a two variable function obtained from the equation we wish to solve. Next, we approximate this function by the Taylor expansion series up to order three, because for higher orders the complexity increases, and the error is not reduced appreciably. Then, we obtain the inverse function from this approximation. Moreover, our solution is compared with the preceding approximations.

In addition to theoretical applications, this type of solution has direct application in the study and development of circuits that incorporate D-R branches. Furthermore, this type of solution depends solely on the diode parameters without the need of precalculated or experimental coefficients as other published solutions. The circuits that benefit from these solutions are limiters, signal clipping and waveform conformers, among others. The equation I-V solution can be applied in specific algorithms for each circuit, which can be solved with mathematical calculation programs, such as MATLAB, MathCad, Mathematica or Maple and in some cases with algorithms created ah-doc.

This paper has been structured in four sections. The first section dedicated to the introduction raises the problem of the series diode-resistance circuit and performs a review of the state of the art of this matter. In the second section, the main solutions developed by other authors are described and an analytical analysis of our proposed solution is carried out. In the third section, an improved equation of the Approximation of Gazquez, is given. This section also elaborates an application example in order to show the operation of the proposed equation. Finally, in the fourth section, the main conclusions are provided.

2. Analytical Analysis of Diode Equation

The equation that relates current and voltage in a diode is showed by the expression (2). In forward biasing ($V_D = +$), the equation (2) is simplified by the equation (3).

$$I_D = I_S e^{\frac{V_D}{\eta V_T}} \qquad (3)$$

The ideality factor η can take values between 1 and 2 in silicon diodes and, to do comparisons, here it is considered as $\eta = 1$.

Analyzing the circuit of Figure 1, we obtain the equation (4).

$$V_S = V_D + RI_S e^{\frac{V_D}{V_T}} \qquad (4)$$

Performing the change:

$a = \frac{1}{V_T},\ b = RI_S, x = V_D, y = V_S$ leads to (5),

$$y = f(x) = x + be^{ax} \qquad (5)$$

and the analytical approximation for V_D relies on an analytical approximation for the inverse of the function $f(x)$ in (5).

2.1. The preceding solutions

In [2], Taylor series expansion was used to approximate the inverse function directly from equation (5). However, this approximation must be performed around an approximated current, which increases the error function and requires a trial current function. This first approximation or trial function is proposed using the main properties of exponential and logarithmic functions. Although this method can be more precise through successive iterations, this would require a large computational cost, so the study of this solution is discarded and could not be used in direct models. The solutions in [5], [1] and [3] are studied for comparison with our solution.

a. *Approximation of Banwell* [5]. His solution is shown in the equation (6):

$$I_D = \frac{V_S - V_{do}}{R}\left(1 + \frac{V_T}{V_T + V_S - V_{do}} ln\left(\frac{I_{do}R}{V_S - V_{do}}\right)\right) \qquad (6)$$

I_{do} and V_{do} are a pair of data that verify the solution and were obtained experimentally. The values proposed in [2], are I_{do} = 100 µA, V_{do} = 0.504 V, V_T = 0.026 V, η = 1 and R = 100 Ω. With these values the saturation current equation (7) for the diode is obtained. We apply these values to all approximations to homogenize the results.

$$I_S = I_{do} e^{-\frac{V_{do}}{V_T}} \quad \rightarrow \; I_S = \; 3.81319 \cdot 10^{-13} A \quad (7)$$

b. Approximation of Abuelma'Atti. This author in [1], normalizes previously the voltage *VD* as *u:*

$$u = \frac{V_S + I_S R}{V_T + \ln\left(\frac{I_S R}{V_T}\right)} \qquad (8)$$

replacing and denormalized, you get the current in the diode, with:

$$I_D = \frac{V_T}{R I_S} \frac{\alpha + u^2}{\beta + u + \gamma e^{-\delta u}} \qquad (9)$$

The solution of equation (9) requires the previous calculation of four parameters. In [1], the next parameters are given $\alpha = 6.0$; $\beta = 3.35$; $\gamma = 7.17$ and $\delta = 1.29$.

c. *Approximation of Ortiz-Conde*. In [3], the author uses the same method of normalization that [1], I_D is given by

$$I_D = \frac{V_T}{R I_S} \ln\left[1 + \frac{1 + \frac{e^u}{2}}{1 + \alpha u^2 e^{-u}}\right] \qquad (10)$$

The solution of equation (10) requires the previous calculation of α parameter. In [3], the parameter $\alpha = 0.029$.

2.2. Our proposed: Approximation of Gazquez

In order to obtain an approximating inverse function for the equations (4)-(5), we consider the function $u(x,y)$ given by $u(x,y) = ln\frac{y-x}{b} - ax, \forall y > x$. Since the set $A = \{(x,y) \in \mathbb{R}^2 : y > x\}$ is open and convex and u is smooth on A we can use Taylor series expansion to assure, for every $y \in \mathbb{R}^+$and $x < y$, that

$$u(x,y) = u(0,y) + x\frac{\partial u}{\partial x}(0,y) + \frac{1}{2}x^2\frac{\partial^2 u}{\partial x^2}(0,y) + \frac{1}{6}x^3\frac{\partial^3 u}{\partial x^3}(\sigma x, y) \quad (11)$$

or some $0 \leq \sigma \leq 1$. Using the equation (11) for $0 \leq x, y$ related by equation (5) we have:

$$0 = u(x,y) = ln\frac{y}{b} - \left(a + \frac{1}{y}\right)x - \frac{x^2}{2y^2} - \frac{1}{3}\frac{x^3}{(y-\sigma x)^3} \quad (12)$$

Thus, we propose as approximation of the inverse of the function *f*, the solution in *x*, given by the equation (13):

$$0 = ln\frac{y}{b} - \left(a + \frac{1}{y}\right)x - \frac{x^2}{2y^2} \quad (13)$$

that is:

$$x = -y(1+ay) + y\sqrt{(1+ay)^2 + 2\,ln\frac{y}{b}} \quad (14)$$

Replacing in (14) in accordance with (4) and clearing I_D, it is obtained:

$$I_D = \frac{V_S}{R}\left(2 + \frac{V_S}{\eta V_T} - \sqrt{\left(\frac{V_S}{\eta V_T} + 1\right)^2 - 2\,ln\frac{RI_S}{V_S}}\right) \quad (15)$$

3. Results and Discussion

Using numerical analysis, we have estimated the I_D-Vs response of all approximations and the exact solution given by equation (4) in the interval [0, 10 V] for Vs. The absolute error committed between each approximation and the exact solution has also been calculated. By logarithmic scale, this error will be displayed to highlight the differences between all solutions. Note that this is the most interesting case because from that voltage there will be no significant change in the trend of the approximation

Figure 2 shows a comparison of the I_D-Vs response of the approximate solution proposed in this work (Gazquez), the approximations of Banwell [5], Abuelma'Atti [1] and Ortiz-Conde [3], and the exact solution, the equation (4). We refer to the exact solution, to the obtained by iteration with enough steps so that the error is less than numerical resolution. As you can see in detail (a) in figure 2, our solution provides excellent agreement with the exact numerical solution, better than the rest, beyond the driving threshold, where our approximation and the exact solution overlap. Banwell's solution also overlaps. Below the threshold voltage shown in detail (b), the Abuelma and Ortiz-Conde approximations adjust with great accuracy and the

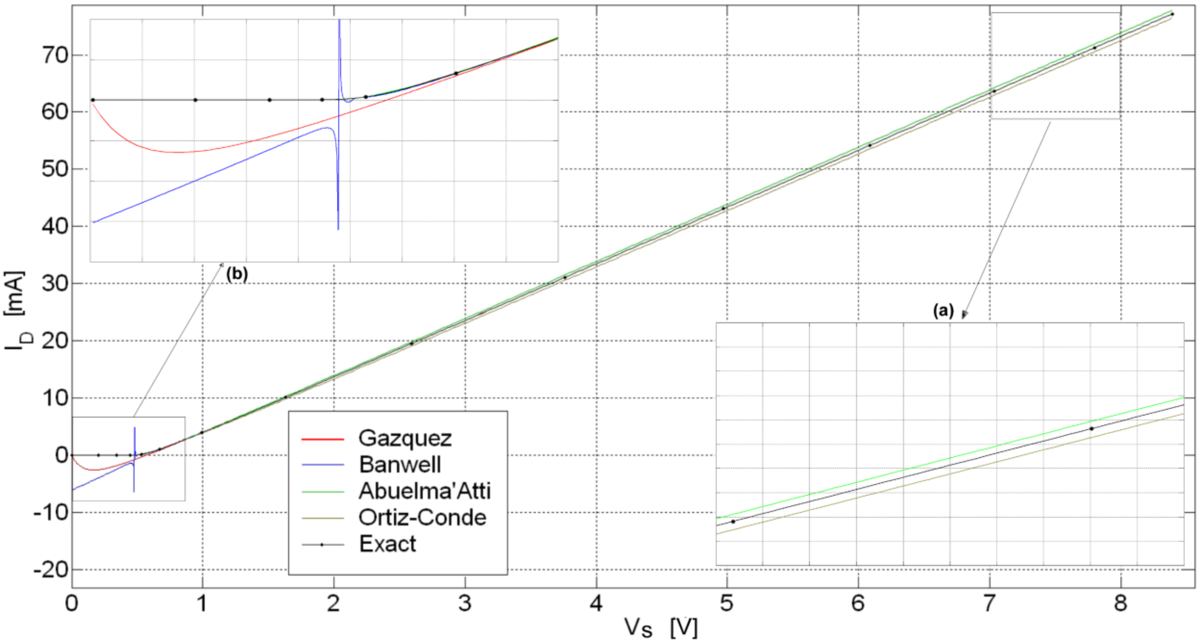

Figure 2. I_D-V_S response for the approximations, (a) final detail, (b) initial detail.

Banwell approximation has the greatest error, as well as a second species discontinuity.

For our approximation, a defect is also observed for values lower than the threshold voltage (detail (b)), so an improved approximation is proposed to correct this defect. Gazquez's first approximation presents a negative response current close to the threshold zone of the diode. To solve this defect, the properties of the conjugate are used to convert the negative response to zero. This modification responds to the expression $I_D = \frac{I_D + \sqrt{I_D I_D^*}}{2}$ giving as a result a new model for approximation of Gazquez, which is included in the equation (16).

$$I_D = \frac{V_S}{2R}\left[\left(2+\frac{V_S}{\eta V_T}\right) - \sqrt{\left(1+\frac{V_S}{\eta V_T}\right)^2 + 2\ln\frac{V_S}{RI_S}} + \sqrt{\left[\left(2+\frac{V_S}{\eta V_T}\right) - \sqrt{\left(1+\frac{V_S}{\eta V_T}\right)^2 + 2\ln\frac{V_S}{RI_S}}\right]^2}\,\right] \qquad (16)$$

The + sign in front of the second root is used to obtain the positive part of it.

From here on, any mention to approximation of Gazquez refers to equation (16).

In Figure 3, a new comparison of the I_D-Vs response is shown. It shows the good adjustment of our approximation for voltages below the threshold voltage, as can be seen in detail (b). On the other hand, the best fit between our approximation and the exact solution is

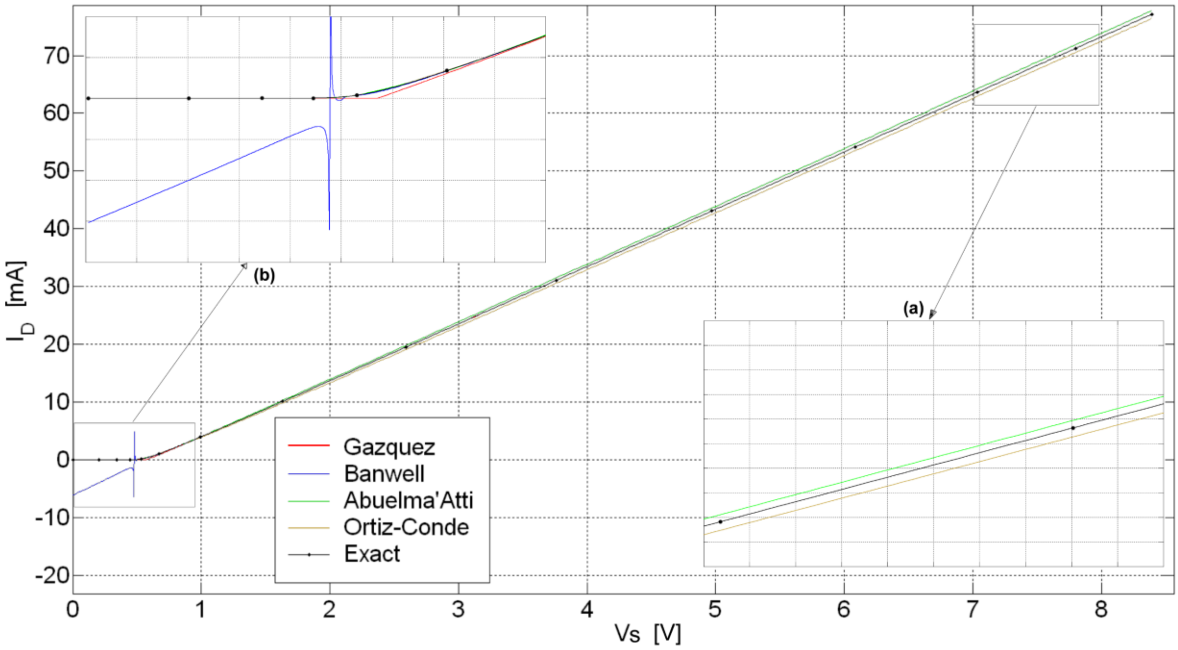

Figure 3. Modified I_D-V_S response for the approximations, (a) final detail, (b) initial detail.

maintained (detail (a)).

Following is the Gazquez function in MATLAB format:

```
a=2+vi/VTN - sqrt((1+vi/VTN).^2+2*log(vi/(R1*Is)));

id=vi/(2*R1).*(a+sqrt(a.^2));
```

In relation to the computational load, the Gazquez function only requires the calculation of one logarithm and two square roots. Regarding the execution times, using the MATLAB (version R20-19b) utility "tic - toc", the following results are obtained for 500 calculated points, including the solution with the Lambert equation [4]: Gazquez = 1.405 ms, Banwell = 2.129 ms, Abuelma = 3,760 ms, Ortiz-Conde = 2.603 ms and Lambert = 14.413 ms. Lambert's function is a Matlab script (Symbolic toolbox) and is implemented with an iterative algorithm. The execution time of Lambert function is undoubtedly much longer than any of the others. The calculations have been carried out on a laptop with an Intel i5 processor - 1.8 GHz.

In Figures 2 and 3, in some areas two or more curves overlap, in which case the exact solution has been highlighted.

Figure 4 shows the error of each approximation with respect to the exact solution. It is observed that for high voltages our approximation is more accurate than the others, and there are several orders of magnitude difference with Abuelma and Ortiz-Conde's approximations. Although Banwell's approximation is somewhat better than ours between values from the threshold voltage to average values, it is the one that makes the greatest error from the origin to the threshold voltage, presenting a discontinuity in that section as do Abuelma and Ortiz-Conde's approximations, which present several discontinuities. The absolute error is used as a

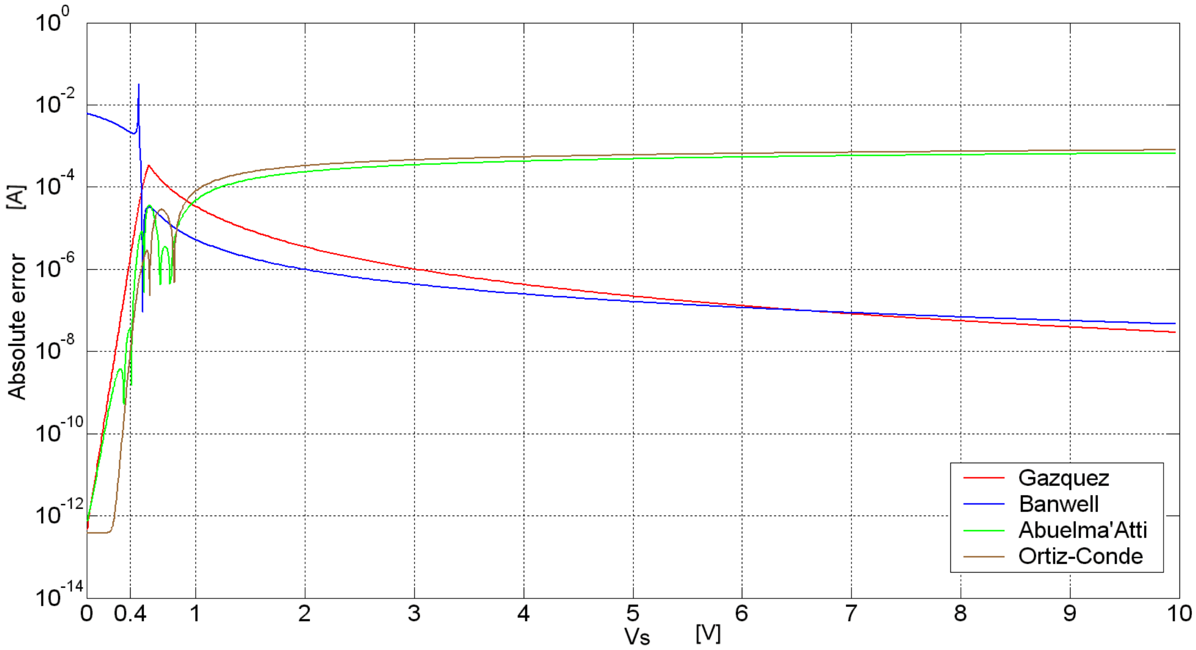

Figure 4. Error of the approximations with respect to the exact solution.

comparison factor because the relative error in the proximity of zero is not representative.

Table 1 presents a comparative value of the absolute error vs different voltages of the source. It should be noted that for values equal to or greater than 7 V the most accurate is the

Gazquez equation. Gazquez's approach is only the worst between 0.5 and 0.8 V and without much difference with the others and the average (Avg) of the absolute error vector with 500 points, is the best with one order of magnitude of difference.

The Banwell approximation has been used in half-wave rectifiers to suppress a 10 Gbps on and off signal [12] or for the calculation of protection diodes in power interfaces of aeronautical equipment [13] or in the I-V characterization of organic photocells [14], among other applications. However, the Approximation of Abuelma'Atti has been applied exclusively to the field of solar cell characterization [15-16]. The Approximation of Ortiz-Conde was applied to the model of organic solar cell [17] and it has also found application in the mathematical modelling of high-performance, low-cost optoelectronic devices such as the heterojunction-based hybrid UV light sensor [18], or in the application of modelling to develop high-performance semiconductor devices [19].

Table 1. Absolute Error vs Voltage Supply comparison.

	Absolute Error [A]			
Vs[V]	**Gazquez**	**Banwell**	**Abuelma'Atti**	**Ortiz-Conde**
0.2	$7.26 \cdot 10^{-10}$	$5.50 \cdot 10^{-3}$	$3.06 \cdot 10^{-10}$	$3.10 \cdot 10^{-13}$
0.4	$1.60 \cdot 10^{-6}$	$2.16 \cdot 10^{-3}$	$3.53 \cdot 10^{-8}$	$4.78 \cdot 10^{-9}$
1.0	$3.39 \cdot 10^{-5}$	$5.30 \cdot 10^{-6}$	$4.85 \cdot 10^{-5}$	$8.17 \cdot 10^{-5}$
2.0	$3.62 \cdot 10^{-6}$	$1.00 \cdot 10^{-6}$	$2.33 \cdot 10^{-4}$	$3.29 \cdot 10^{-4}$
4.0	$3.35 \cdot 10^{-7}$	$2.50 \cdot 10^{-6}$	$4.24 \cdot 10^{-4}$	$5.42 \cdot 10^{-4}$
7.0	$8.22 \cdot 10^{-8}$	$8.85 \cdot 10^{-8}$	$5.74 \cdot 10^{-4}$	$6.99 \cdot 10^{-4}$
10.0	$2.91 \cdot 10^{-8}$	$4.43 \cdot 10^{-8}$	$6.70 \cdot 10^{-4}$	$7.97 \cdot 10^{-4}$
Avg.	$2.80 \cdot 10^{-5}$	$9.51 \cdot 10^{-4}$	$2.72 \cdot 10^{-4}$	$3.38 \cdot 10^{-4}$

The Banwell approximation has been used in half-wave rectifiers to suppress a 10 Gbps on and off signal [12] or for the calculation of protection diodes in power interfaces of

aeronautical equipment [13] or in the I-V characterization of organic photocells [14], among other applications. However, the Approximation of Abuelma'Atti has been applied exclusively to the field of solar cell characterization [15-16]. The Approximation of Ortiz-Conde was applied to the model of organic solar cell [17] and it has also found application in the mathematical modelling of high-performance, low-cost optoelectronic devices such as the heterojunction-based hybrid UV light sensor [18], or in the application of modelling to develop high-performance semiconductor devices [19].

As an example of application (Figure 5), the comparative result of the simulation in the time domain of an inverter amplifier detector circuit is shown, using the Gazquez equation implemented in MATLAB versus the PSpice circuit simulator. Figure 5 shows the circuit; the

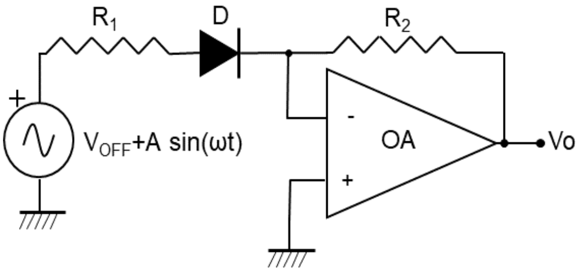

Figure 5. Simulated circuit.

simulation data, obtained from the PSpice model is as follows:

- Diode type 1N4148, Is = 2.68 10^{-9} A, V_T = 0.025 V, η = 1.8, O.A.= ideal (MATLAB) and LM324 (PSpice).
- Circuit: R_1 = 200Ω, R_2 = 500 Ω; generator V_{OFF} = 1 V, A = 1 V, ω = 2π▪1KHz.

Figure 6 shows the result of the simulation of this circuit with all approximations including the exact function. Figure 7 shows the result of the simulation using the PSpice program, there is no difference in the signals in the time domain of the Gazquez equation.

This equation is suitable for the theoretical study of circuits with series D-R branches in forward biasing, with the help of numerical calculation programs. It is also used in embedded applications with microcontrollers with low computing capacity.

There are a wide variety of applications where the diode equation is very useful for modelling the I-V behaviour of a diode, in [20] use half-wave rectifier as a demo application of two generalizations of the Lambert *W* function, the Lambert–Tsallis *Wq* and the Lambert–Kaniadakis *Wκ* functions. Computer-aided design (CAD) tools need efficient and realistic modeling for the design of new challenges in microelectronics, the model of the diode in series with a resistor is the basis of a wide variety of applications [21].

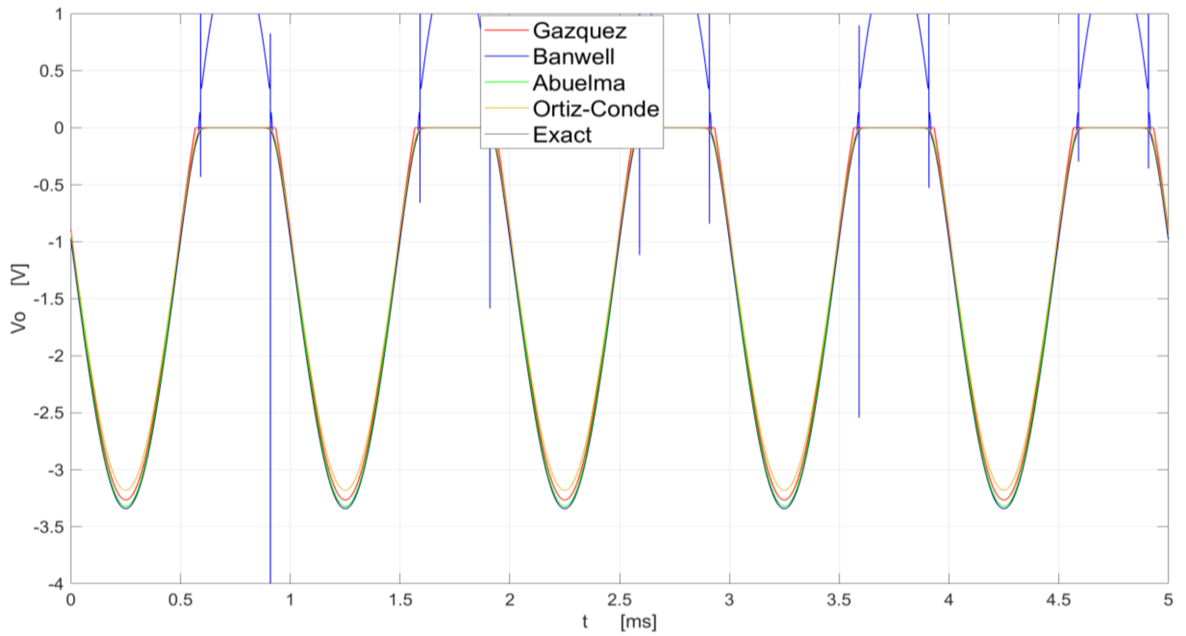

Figure 6. Results of the simulation with MATLAB of the equations of Gazquez, Banwell, Abuelma'Atti, Ortiz-Conde and exact function.

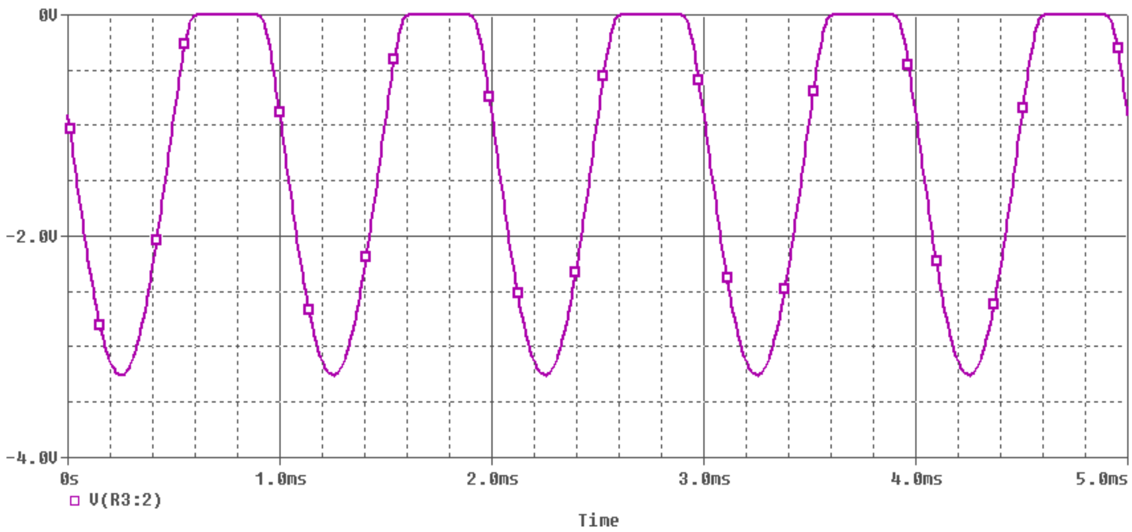

Figure 7. Simulation result using PSpice.

Other applications of interest at present are focused on the field of simulation or emulation of photovoltaic panels [16], [22]. In this type of works where the fact of being able to transform the implicit expression of the I-V characteristic into an explicit analytical expression supposes an important help. The modelling of photovoltaic cells is always carried out with different levels of precision. The model is based on an equivalent circuit and using concentrated parameters and variables. The models are mainly based on the use of one, two or three diodes in parallel with a resistance that build a mathematical model. The accuracy of the model makes it possible to simulate the performance of solar cells under various operating conditions, which is why many researchers use complex algorithms to obtain the parameters [23].

4. Conclusions

A new approximation is proposed for the equation of the current in the circuit formed by a diode with series resistance. This solution is more accurate than existing ones past the conduction threshold and is more direct without the need to use experimental coefficients, parameter determination and without normalization/denormalization processes. Our approximation depends exclusively on the physical parameters of the diode, I_S, V_T and η and the other circuit elements. The average of the absolute error is $2.80 \cdot 10^{-5}$ A. That error is the best with an order of magnitude of difference versus the other 3 models' approximations. This does not only make it more accurate, but also easy to use in circuit simulation programs and in mathematical models of complex circuits. Our approximation may be of interest in the analysis of circuits with D-R branches, exponential amplifiers, logarithmic amplifiers and waveform shapers. This equation is suitable for the theoretical study of this kind of circuits (when the diode is forward biased) with the help of numerical calculation programs and in the implementation of simulators or embedded systems with low computer resources, such as embedded or cloud execution, since it can be executed with few mathematical operations. Our approximation is the fastest in execution time, in a comparative study by MATLAB.

Author Contributions: Conceptualization, Jose A. Gazquez; Formal analysis, Blas Torrecillas and Jose Carmona-Tapia; Investigation, Jose A. Gazquez; Methodology, Manuel Fernandez-Ros; Visualization, Nuria Novas; Writing – original draft, Jose A. Gazquez and Manuel Fernandez-Ros; Writing – review & editing, Nuria Novas.

Funding: Under I+D+I Project UAL18-TIC-A025-A, University of Almeria, the Ministry of Economy, Knowledge, Business and University and the European Regional Development Fund (FEDER).

Acknowledgments: Under I+D+I Project UAL18-TIC-A025-A, University of Almeria, the Ministry of Economy, Knowledge, Business and University and the European Regional Development Fund (FEDER) and The Ministry of Economic and Competitiveness of Spain financed this work, under Project TEC2014-60132-P, in part by Innovation, Science and Enterprise. Andalusian Regional Government through the Electronics, Communications and Telemedicine TIC019 Research Group of the University of Almeria, Spain and in part by the European Union FEDER Program and CIAMBITAL Group.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Abuelma'Atti, M.T. Improved Approximate Analytical Solution For Generalised Diode Equation. *Electron. Lett.* **1992**, Volume 28, no. 6, pp. 594–595.
2. Fjeldly, T.A.; Moon, B.-J.; Shur, M. Approximate Analytical Solution of Generalized Diode Equation. *IEEE Trans. Electron Devices* **1991**, Volume 38, no. 8, pp. 1976–1977.
3. Ortiz-Conde, A.; Garcia Sanchez, F.J. Approximate Analytical Expression For Equation of Diode with Series and shunt resistences. *Electron. Lett.* **1992**, Volume 28, no. 21, pp.

1964–1965.

4. Bihan, J.L. Analytical approximation of normalised diode equation with highly improved accuracy. In *The 10th IEEE International Symposium on Electron Devices for Microwave and Optoelectronic Applications*, **2002**, pp. 259–264.
5. Banwell, T.C. Bipolar transistor circuit analysis using the Lambert W-function. *IEEE Trans. Circuits Syst. I Fundam. Theory Appl.* **2000**, Volume 47, no. 11, pp. 1621–1633.
6. Banwell, T.C.; Jayakumar, A. Exact analytical solution for current flow through diode with series resistance. *Electron. Lett.* **2000**, Volume 36, no. 4, pp. 291–292.
7. Fritsch, F.N.; Shafer, R.E.; Crowley, W.P. Solution of the Transcen- dental Equation wew=x [C5]. *Commun. AC* **1973**, Volume 16, no. 2, pp. 123–124.
8. Batzelis, E.I.; Anagnostou, G.; Chakraborty, C.; Pal, B.C. Computation of the Lambert W Function in Photovoltaic Modeling. *Lect. Notes Electr. Eng.* **2020**, Volume 604, no. May, pp. 583–595.
9. He, J.; Fang, M.; Li, B.; Cao, Y. A new analytic approximation to general diode equation. *Solid. State. Electron.* **2006**, Volume 50, no. 7–8, pp. 1371–1374.
10. Fang, J.; Li, M.; Chen, B.; He, Y. A new analytic approximation to general diode equation. *Guti Dianzixue Yanjiu Yu Jinzhan/Research Prog. Solid State Electron.* **2009**, Volume 29, no. 4, pp. 547–550.
11. Hren, A.; Korelic, J.; Milanovic, M. RC-RCD Clamp Circuit for Ringing Losses Reduction in a Flyback Converter. *IEEE Trans. Circuits Syst. Express Briefs* **2006**, Volume 53, no. 5, pp. 369–373.

12. Kaneko, Y.; Higashino, T.; Okada, M. An interference suppression for transporting radio frequency signals with 10 Gbps optical on-off keying. *IEICE Transactions on Electronics*, E101C (4) 2018, pp. 285-291. DOI: 10.1587/transele.E101.C.285
13. Meier, F. Calculation of the protective effect of protection diodes in power interfaces of aeronautical equipment through the Lambert W function [Berechnung der schutzwirkung von schutzdioden in leistungs-schnittstellen von luftfahrtgeräten durch die lambert-w-funktion]. *EMV 2016 - Internationale Fachmesse und Kongress fur Elektromagnetische Vertraglichkeit*, *2016-February*, pp. 513-519.
14. Tada, K. Parameter extraction from S-shaped current-voltage characteristics in organic photocell with opposed two-diode model: Effects of ideality factors and series resistance. *Physica Status Solidi (A) Applications and Materials Science* **2015**, Volume 212, no 8, pp. 1731-1734. DOI: 10.1002/pssa.201431877
15. Fairooz, F.; Choudhury, S.A.; Ashraf, N.I.; Mahdi, W.; Chowdhury, M.H. Use of Bimetallic Plasmonic Nanoparticle Complexes for Enhancing Thin-Film Solar Cell Efficiency. *2nd International Conference on Electrical, Computer and Communication Engineering*, ECCE 2019, art. no. 8679450. DOI: 10.1109/ECACE.2019.8679450
16. Lun, S.; Du, C.; Xu, C. An explicit i - V model of solar cells based on padé approximants. *Proceedings of the 28th Chinese Control and Decision Conference,* CCDC 2016, art. no. 7532106, pp. 6169-6172. DOI: 10.1109/CCDC.2016.7532106
17. Jin, J.W.; Jung, S.; Bonnassieux, Y.; Horowitz, G.; Stamateri, A.; Kapnopoulos, C.; ... Logothetidis, S. Universal compact model for organic solar cell. *IEEE Transactions on*

Electron Devices **2016**, Volume 63, no 10, pp. 4053-4059.

18. Bazine, A.; Jemmeli, D.; Belhaj, M.; Dridi, C. New modeling method for UV sensor photoelectrical parameters extraction. *Optik* **2019**, Volume 181, pp. 906-913.
19. Elangovan, H.; Kesavan, A.V.; Chattopadhyay, K.; Ramamurthy, P.C. 2D layering of silicon nanocrystals at TiO2/CuI heterojunction for enhanced charge transport. *Journal of Applied Physics* **2019**, Volume 125, no 24, pp. 245302.
20. Ramos, R.V. The Rq,Q function and the q-Diode. *Phys. A Stat. Mech. its Appl.* **2020**, Volume 556.
21. ElGabry, O., Hussien, F., Mohieldin, A.N. Integrated circuit technology characterization and evaluation using automated CAD tool. AEU - International Journal of Electronics and Communications **2018**, 97, pp. 68-78. DOI: 10.1016/j.aeue.2018.09.035
22. Merenda, M.; Iero, D.; Carotenuto, R.; Corte, F.G.D. Simple and low-cost photovoltaic module emulator. *Electron.* **2019**, Volume 8, no. 12.
23. R. Abbassi, A. Abbassi, A. A. Heidari, and S. Mirjalili, “An efficient salp swarm-inspired algorithm for parameters identification of photovoltaic cell models,” *Energy Convers. Manag.* **2019,** Volume 179, pp. 362–372.