

Optimal displacement of photovoltaic array's rows using a novel shading model

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Abstract

Photovoltaic energy has experienced tremendous growth in terms of implementation at facilities for power supply in rural areas and for energy dispatch to grid. The relative position of the fixed panels can present the problem of varying amounts of shadowing among them, which can reduce the overall energy produced from the array of photovoltaic panels on specific dates and times, in addition to the problems in each of the panels themselves. The existing methods calculate the distances between the rows of PV panels using a fixed height of the sun, such that the rays always strike perpendicular to the panels, thereby limiting the duration of solar gain to 4 hours. This paper proposes a method that optimises the minimisation of the distance between the rows of fixed photovoltaic panels. The proposed method is based on the exact calculation of the shadows of the panels for the different positions of the sun, which depends on the latitude of the facility, throughout the course of the day and for all of the planned hours of solar gain. To illustrate the proposed method, it has been applied to a case study for which the solutions obtained using the traditional methods are compared, indicating that the distance can be reduced by up to 40% when the tilt angle of the panel is 60°. In conclusion, the proposed general method for optimally minimising the distance between the PV panels in solar arrays, which is of particular interest for standalone photovoltaic (PV) systems in remote areas that act as isolated small power producing units for the supply of electricity.

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Nomenclature list

A	Azimuth of the Sun
A^*	$360-A$
d	Separation between the modules
d_i	Distance plan of the shadow of the point i
D	Length of the shadow cast by a module
h	Angle of the sun with respect to the horizon
H	Hour angle of the sun
H^*	$360-H$
H_i	Hour angle of the point i
k	Dimensionless factor depending on the latitude of the location
l	High PV panel
PV	Photovoltaic
PV_1	First row of PV panels
PV_2	Second row of PV panels
PV_{2h}	Second row of PV panels depending on the sun angle h
w	PV panel width
α	Azimuth of the panel
β	Angle relative to the horizontal PV panel
δ	Declination of the Earth
θ_{zs}	Zenith angle
Φ	Latitude
γ_s	Solar altitude angle

1 Introduction

Energy is a vital input for social and economic development (Baños et al., 2011). The demand for energy is expected to increase at a faster rate in upcoming years (Tolón-Becerra et al., 2011), partly due to the exponential growth of the world's population (Manzano-Agugliaro et al., 2013). The World Bank and International Energy Agency (IEA) estimated that the world will require a doubling in installed energy capacity over the next 40 years to meet the anticipated demands of developing countries (Türkyay and Telli, 2013). The realisation that the fossil fuel resources required for the generation of energy are becoming scarce and that climate change is related to carbon emissions to the atmosphere has increased interest in energy savings and environmental protection by reducing the use of fossil fuels (Vine, 2008). One strategy to achieve the goal of reducing the dependence on fossil fuel resources involves the use of renewable energy sources, not only for large-scale energy production but also for stand-alone systems (Zhou et al., 2010).

In response to concerns of environmental pollution, solar energy is playing a leading role in reducing the environmentally hazardous gasses produced during conventional electricity generation (Akikur et al., 2013). Photovoltaic solar systems are projected to prevent 100 Gt (Gigatons) of CO₂ emissions during the period from 2008–2050 (Zhang et al., 2012). Photovoltaic (PV) technology is one of the first among several renewable energy technologies that was adopted globally for meeting the basic electricity needs of rural areas that are not connected to the grid (Chaurey and Kandpal, 2010).

The installation of PV systems have played an important role worldwide because they are clean, environmentally friendly and secure energy sources (Khatib et al., 2013).

During the last few years, photovoltaic solar systems have become one of the most popular renewable energy sources in Europe (Cruz-Peragón et al., 2011), particularly in Spain (Montoya et al., 2014). PV systems represented 16% of all new power capacity installations in Europe in 2009 (Lüdeke-Freund and Loock, 2011).

Generally, the works related to the sizing of the PV panels are focused on the following:

1) the dimensions of the surface of each of the panels to meet an energy requirement (Kaldellis et al., 2009) or to optimise the costs of the PV system, which includes the photovoltaic panels, a battery bank, a battery charger controller and an inverter (Fathy et al., 2013), or 2) optimizing the Life Cycle Cost, LCC, which must be minimised (Kaldellis et al., 2009). When the photovoltaic cells were selected and the number of solar panels installed at a certain angle were determined, the PV array spacing becomes the most critical issue (Sun, 2011).

In photovoltaic (PV) systems, the solar cells are often connected in series, one completely shaded solar cell will act as loads, so draining power from fully illuminated cells and reducing the output of the whole string. In addition, it reduces array efficiency and causes hot-spot heating, a faulty mode of operation which can damage cell encapsulation materials, permanently reduce array power output and even put arrays out of action (Bishop, 1988). Although bypass diodes can be used to mitigate this effect by allowing current to flow in a different path, they increase both assembly time and material cost which lead to increased costs. This area has been and is still attracting immense interest from PV research communities as well as industrial players because it is the most economical way to improve the overall PV system efficiency (Ishaque and Salam, 2013).

Scientific communities have studied mainly the calculation of partially shaded conditions (PSCs) that often occur in large photovoltaic generation systems (PGSs). PSCs cause losses in system output power, hot spot effects, and system safety and reliability problems (Liu et al., 2014; Liu et al., 2015). Deline et al., (2013) focused their study on the performance impact of partial shading on a large PV system. Goss et al. (2014) studied the algorithms to calculate shading losses within an overall PV system energy yield model for two sub-models: irradiance incident on the cells and current and voltage (I-V) for each PV device.

To completely avoid shading, the methods found to resolve this issue in the literature, which will be described later (section 2), are limited by the number of hours of solar gain or are based on either a fixed angle of the sun or on empirical equations that are not useful worldwide. This paper proposes a method of determining the distance between fixed PV panels without limiting the useful hours of energy production, for any angle of the sun and for any latitude, thereby enabling the method to be applied anywhere. To validate the proposed method, a comparison with traditional methods will be made for a case study that represents the major energy production situations of a photovoltaic system.

2 Installation of Photovoltaic panels: a brief review and mathematical modeling

The designs of photovoltaic panels systems are optimised to obtain the maximum energy efficiency. In the traditional design process, the surface of the solar panels is assumed to be perpendicular to the path of the incident sunlight. Due the terrestrial declination, the relative position of terrestrial hemispheres constantly varies relative

to the sun throughout the year. Therefore, to achieve the above perpendicularity of the solar rays to each PV panel, the inclination of the panels relative to the horizon must vary over the year. If the installation is fixed, a solution to maximize the energy production is to have the solar panels placed in the position as perpendicular as possible to the sun during the winter solstice.

The true twelve hours of the solar day (time of the zenith passage of the sun or the meridian of the place), establish the relationship between the declination (δ), latitude (Φ) angle and height of the sun on the horizon (h), which is given by the equation (Kalogirou, 2009):

$$\gamma_s = (\pi/2) - \Phi - |\delta| \quad (1)$$

$$\theta_{zs} = (\pi/2) - \gamma_s = \Phi + |\delta| \quad (2)$$

where γ_s is the solar altitude angle (the angle between the sun's rays and a horizontal plane), δ is the declination of the Earth at the winter solstice and θ_{zs} is the zenith.

Figure 1

As shown in Figure 1, the plane of the panel and the line t forms the angle β ; if r is perpendicular to t and s is perpendicular to the panel, then r and s also form the angle β . If we look at point P , noting that v is parallel to r , then we conclude

$$\beta = \theta_{zs} \quad (3)$$

This conclusion is valid only when the sun's rays strike perpendicular to the surface of the panel. Based on equations 1, 2 and 3, it is concluded that:

$$\beta = \Phi + |\delta| \quad (4)$$

From the above, it follows that there are two possible design solutions for the installation of photovoltaic panels:

1. Mobile Panels. Solar tracking systems adjust the position of mobile PV panels such that each day of the year, the inclination (β) and orientation (α) relative to the horizon of the place of the mobile panels are adapted according to the variation of the angle of elevation of the Sun at the time of zenith and azimuth angle of the hourly variation. There are basically two types of tracking systems, single-axis and dual-axis, and they usually either operate using an electric or thermal power mechanism. The advantages of using tracking with either one (Chang, 2009; Lubitz, 2011) or two axes (Ma et al., 2011) are well described.

2. Fixed panels. To enable the greatest energy capture possible, at the time of the peak solar irradiation (maximum input power all day), the surface of the plates or panels is facing the sun, and therefore the panel orientation should necessarily be inclined south (in the northern hemisphere). Note that in the execution of aligning the panels, true north does not always coincide with the magnetic north and thus the measurement using a compass can result in incorrect alignment, resulting in reduced energy acquisition. The calculation of the tilt angle and orientation is based on equation 4 in the following cases according to the β inclination, with several possibilities:

a) Certain systems may require that the input power during winter be as high as possible, i.e., ensure that the minimum energy production throughout the year is as high as possible to be able to meet a continuous demand, thus preventing the situation in which the production in December and January falls below the allowed low values. The use of standalone photovoltaic (PV) systems as the power source for pumping water is one of the most promising areas in photovoltaic applications (Bouzidi, 2013),

especially in regions far from power lines (Jakhrani et al., 2012). For this purpose, the solar rays should fall perpendicular to the surface of the panels at the winter solstice.

b) In other circumstances, it is necessary to supply the maximum power in summer and spring, for example, the use of photovoltaic panels as the power source for pumping water during the summer months (Bouzidi, 2013). In this case, $\beta = \Phi - |\delta|$.

c) If the goal is to generate the maximum amount of energy per year, i.e., maximise the annual production, for example, to deliver energy to the network, regardless of whether the energy is maximised at the solstices, then the sun's rays should impinge perpendicular to the panel at the equinoxes, as determined using the following equation, which is derived from applying the cosine theorem of spherical trigonometry (Appelbaum and Bany, 1979):

$$\cos \theta_{ZS} = \sin \delta \sin \Phi + \cos \delta \cos \Phi \cos w \quad (5)$$

Setting $w = 0$ because it is the time of the solar zenith and $\delta = 0$ because it is the time of the equinox, it follows that: $\beta = \theta_{ZS} = \Phi$.

d) If the goal is to produce an intermediate solution or compromise between the maximum annual cumulative production and the maximum production in winter, the average of the previous two solutions can be used.

The applicable formulas in each of the cases presented for fixed panels are summarised in Table 1.

Table 1

Once the requirements are determined, the process of sizing the photovoltaic system starts with the input data for the calculation of the daily global radiation for the horizontal surface at zero degrees south facing in the desired geographical location.

The sizing process aims to find the values for different inclinations and to compare the corresponding different model results. The process of implementing the theoretical model is well known (IDEA, 2011).

The sizing of the PV system depends on the amount of solar radiation collected by the PV panels and this in turn depends on the installation of the PV panels. For example, we have calculated the annual global incident radiation depending on the angle of inclination of the plate, with angular increments by degrees. The results are shown in Figure 2.

Figure 2

Once the value of the annual minimum of daily radiation is determined (kW h/m^2 day), the irradiance is calculated in kW h/m^2 . However, the energy losses in the charge and discharge of the batteries (accumulators performance: 6 % and regulators: 10 %) must be considered, in addition to an allocation of part of the energy captured for possible battery charging (14 % power reserve for possible situations where a recharge is required). All told, the power consumption must be increased by 30 % to meet the above needs.

Next, the radiation is calculated for the dimensioning of a PV system for a radiation range between the minimum and the average annual (kWh/m^2) the irradiance is calculated. Currently there are many programs that provide the value of the irradiance based on the meteorological data recorded at the installation site, such as PVGIS (Huld et al., 2012).

The overall battery capacity also depends on the chosen value of the irradiance because the choice of this item determines the energy deficit to be covered. The

problem lies in choosing the combination of panels and batteries that minimises the overall cost of the installation for the irradiance levels available. Once selected, we know the number of panels that are necessary. Then, the problem becomes physically locating the photovoltaic modules for solar gain in a manner that uses the minimum amount of space and minimises the losses by shadows; there are several traditional methods to addressing this problem geometrically. However, it is always recommended to calculate the orientation of the panels to the height of the worst-case for the sun, which is for the Winter Solstice. This calculation in turn requires knowledge of the curve of the PV panel shading that is projected during the entire year. In this way, the panels can be placed in the available space, with the use of the largest number of panels to ensuring that each panel will not cast a shadow on each other.

3 Classical methods for calculating the distance between panels in PV arrays

3.1 Classical method 1: trigonometric method.

This method is based on calculating the distance between panels (D) depending on the sun's height (h) (equation 5) for which the installation is designed. In figure 3, the relationship between the trigonometric magnitudes is shown, and equation 8 is deduced:

$$D = \ell \cdot \cos \beta + d \quad (6)$$

$$d = \ell \frac{\sin \beta}{\operatorname{tg} h} \quad (7)$$

$$D = \ell \left(\cos \beta + \frac{\sin \beta}{\operatorname{tg} h} \right) \quad (8)$$

where ℓ is the height of the panel and β is the tilt angle of the panel.

Figure 3 shows the shadow cast by the first row of panels (PV_1) over the second row of panels (PV_{2h}). Under these conditions, the shadow reaches a horizontal distance D from the first row of panels (for $h = 30^\circ$), and therefore the second row should be positioned at PV_{2h} , whereas if the sun has a height of $h = 10^\circ$, the shadow reaches a horizontal distance D' and the second row should be placed at PV_{2h}' . For example, for a panel with $\ell = 1.665$ m and a tilt $\beta = 60^\circ$, the obtained distances are: $D = 3.33$ m and $D' = 9.01$ m.

Figure 3

3.2 Classical method 2: empirical method.

This method is designed to ensure 4 hours of sun around noon at the winter solstice for Spain, instead of calculating the positioning for a specific height of the sun; the given latitude is the input data for this method. The distance (d) measured between the horizontal rows of the panels of height ℓ is given in eq. 9 (see Figure 4), (IDEA, 2011).

$$d \leq \ell \cdot k \quad (9)$$

k is a dimensionless factor that depends on the latitude of the location

$$k = \frac{1}{\operatorname{tg}(61^\circ - \Phi)} \quad (10)$$

where Φ is the latitude ($^\circ$).

The distance obtained (d) must be added to the horizontal projection of the panel of length l at an angle of inclination (β), $l \cdot \cos \beta$, as described in equation 11.

$$D = l \cos \beta + d = l \cos \beta + \frac{l}{\tan (61^\circ - \Phi)} = l \cdot \left(\cos \beta + \frac{\sin \beta}{\tan (61^\circ - \Phi)} \right) \quad (11)$$

Figure 4

4 Proposed method to avoid shading: exhaustive method

This method allows the calculation of the exact shadow of PV panels for each solar hour. In this way, we can determine the optimum use of space in terms of the energy requirements of PV facilities located at a given latitude (Φ) defined by number of hours of solar gain and panel tilt (α , β).

The shadow curve projected on the ground for each PV panel (or group of them), are calculated through 3 directions: West, North and East. This requires knowledge of the azimuth of the sun at the solstices of winter and summer, for both the sunrise and sunset (depending on the selected energy configuration, described by four cases, table 1).

With the geometry and inclination angle of the panel (for the selected energy design situation) and using the trigonometric relationships data, the shadow curves for both corners of each panel is calculated, and then the envelope of the shadow is determined. This envelope will be the outer contour of the shadow, as shown in figure 6. So we can determine the minimum distance between the rows of the panels to avoid the effect of shadows.

For a given latitude, the solar height is a function of every hour, which are calculated based on the principles of celestial mechanics, where the Earth is at the centre of the celestial sphere (Fig. 5). The Equatorial plane of the Earth is also the Equatorial plane of the celestial sphere (NS), where the azimuth is positive (clockwise) from the north (A), $A^* = 360^\circ - A$.

The solar height (h) is the angle between the astronomical meridian with the equatorial plane, the Earth declination angle is (δ), and the latitude is (Φ). Then, to calculate the solar height (h) at a specific hour (H) to a point on the Earth (with latitude Φ), spherical trigonometry equations are used for a spherical triangle, as shown in Figure 5.

Figure 5

In a spherical triangle, point Z is usually taken as the coordinate origin. The sides are the measured angles (in radians), with the triangle sides in our case being ($90^\circ - \delta$), ($90^\circ - \Phi$) and ($90^\circ - h$), and their respective angles are H (hour angle), A^* (supplement of azimuth) and h (solar height).

To relate these variables into a single system of equations, we use the law of sines, in which we replace the first two sides and their angles (Kalogirou, 2009):

$$\frac{(90^\circ - \delta)}{\sin(A^*)} = \frac{(90^\circ - h)}{\sin H} \quad (12)$$

$$\frac{\sin(90^\circ - \delta)}{\sin(360^\circ - A)} = \frac{\sin(90^\circ - h)}{\sin H} \Rightarrow \frac{\cos \delta}{-\sin A} = \frac{\cos h}{\sin H} \quad (13)$$

For the last two fractions, the sines of the angles of 90° and 360° are 1 and 0, respectively; furthermore, $\sin(90^\circ - v) = \cos v$, $\sin(360^\circ - r) = -\sin r$, resulting in the following equality:

$$\cos \delta \cdot \sin H = -\sin A \cdot \cos h \quad (14)$$

Applying the first law of cosines of spherical trigonometry to the side $(90^\circ - \delta)$, the following equation is obtained:

$$\sin \delta = \sin \Phi \sin h + \cos \Phi \cos h \cos A \quad (15)$$

Next, applying the first law of cosines of spherical trigonometry to the side $(90^\circ - h)$, the following equation is obtained:

$$\sin h = \sin \Phi \sin \delta + \cos \Phi \cos \delta \cos H \quad (16)$$

At this point, we have the value of h in terms of the three desired variables (δ, Φ, H) .

To calculate the shadow, the dimensions and inclinations (β, α) of the panels are also required. To determine the end points of the shadow, first consider that the solar hours are designed to capture the PV system. The solar hours consist of the central peak hour (H_0) and the hours that are distributed equidistantly, backward and forward from this central time. The central peak is considered to be $H_0 = 0^\circ$ for the prime meridian (Greenwich), with each hour corresponding to 15 degrees, with addition to the right by 15 degrees per hour of solar collection and subtraction to the left by 15° per hour (with 0 considered as 360° to avoid negative angle values). For example, for Spain, which is located in the meridian of Greenwich, for a setting of 8 hours of sunlight, there would be an H ranging from 300° (8:00 h) to 60° (16:00 h), as shown in figure 6.

Figure 6

With known values of δ, Φ, H , we can calculate h (eq. 16) for at least 5 points for each top corner of each panel. Every corner produces a shadow in the shape of a parabola; see the shadow points 1 to 5 for the right corner point P (figure 6) and points 1' to 5'

for left corner point P' (figure 7). The outer contour of both shadows will be the envelope, which is the desired complete shadow contour that is marked in red in figure 8.

Finally, we must calculate the distance (d) to the shadow for each point according to equation 17, relating the tilt angle of the panel, the length of the panel and the height of sun h.

$$d = \frac{\ell \cdot \sin \beta}{\text{tg}(h)} \quad (17)$$

So far, we have projected distances of shadows from the corners of the PV panel (d_i). To draw these distances on the ground, polar coordinates are used, where the angle is the hour angle of the point (H_i), as shown in figure 6. The most critical case is the shortest day of the year, where h is the lowest in the design of the PV system. When the shadow envelope of a panel is known, one may draw the shadow cast for each entire row of the PV panel assembly, so that no shadow may arise between rows.

The data to be calculated is the distance between the panel rows (D), as shown in figure 9. As shown in figure 7, we must solve the triangle whose vertices are: point 1 (shadow first hour), point 5 (shadow last hour), and P (projection of the corner of PV panel). The vertex angle P is the difference in the hour angles of the other two vertices, $(360 - (H_1 - H_5))$, and the sides from point P to point 1 and to point 5 are d_1 and d_5 , respectively, with the values of d_1 and d_5 calculated before for h_1 and h_5 , respectively. Now it is possible to calculate the area occupied for several assumptions (figure 10), where

$$x = d_5 \sin \left(\frac{360 - (H_1 - H_5)}{2} \right) = d_5 \sin H_5 \quad (18)$$

and

$$d = d_5 \cos\left(\frac{360 - (H_1 - H_5)}{2}\right) = d_5 \cos H_5 \quad (19)$$

Figure 7

5 Results and discussion

5.1 Case study: results

This methodology is applied to analyse an example involving isolated small power producing units for the supply of electricity for pumping water in agricultural use. The energy system, which is a photovoltaic facility with 30 panels, is designed to be optimised for the day of the winter solstice. The projected shadows by each row of panels were calculated, thus enabling the minimum distance from the next row of panels to be determined. The case study is located in the south of Spain (latitude 37° 5' 34.80" N and longitude 2° 38' 0" W). Equations 15, 16 and 17 and the data of declination $\delta = -23^\circ 27'$ and latitude $\Phi = 37,093^\circ$ are used.

The azimuthal angle A is based on the position of the observer-north, while the hour angle H corresponds to the observer-sun position. Next, we develop the calculation of shadows at sunrise, Ortho, $h = 0^\circ$, i.e., $\sin h = 0$; using equation 16, where in all the formulas, the angles are in radians, while the solutions are in degrees to make it more intuitive.

$$0 = \text{tg } \Phi \text{ tg } \delta + \cos H \quad (20)$$

The calculations result in $H_{\text{ORTHO}} = 289.09^\circ$ and $H_{\text{SUNSET}} = 70.91^\circ$.

Next, table 2 presents the results of the calculations for two points, 1 and 5, as shown in Figures 6 and 7. The known angles $H_1 = 300^\circ$; $H_5 = 60^\circ$ and $\beta = 60^\circ$, angles (h_1, h_5) and

distances d (d_1 y d_5) are calculated, considering the size of the panel $\ell = 1.665$ m for the distance calculation.

$$\sin h_5 = \sin 37.093^\circ \sin(-23.45^\circ) + \cos 37.093^\circ \cos(-23.45^\circ) \cos 60^\circ \quad (21)$$

The calculations result in $h_5 = 6.88^\circ$, and d_5 was calculated using the equation 17:

$$d_5 = \frac{1.665 \cdot \sin 60^\circ}{\text{tg } 6.88^\circ} = 11.48 \text{ m} \quad (22)$$

Table 2 presents the results of the calculations for each shadow of the 5 points. Note that when the inclination of the panel decreases, the distance of the shadow also decreases. If we take the distance (d) from the main panel tilt ($\beta = 60^\circ$) as a reference, with a decrease in tilt to $\beta = 45^\circ$, the distance is decreased to 82 %; if the tilt decreases to $\beta = 40^\circ$, then the distance decreased to 77 %; and finally, if $\beta = 15^\circ$, then the distance is decreased to 30 %. These results are valid for all the times at which the shadow distance is calculated.

Table 2

For the study of the total area of the facility, we have studied two possible distributions of 30 modules: three rows with ten panels (figure 10) or five rows with six panels.

Calculation of the row spacing (D) is performed using equation 6, where d is the height of the triangle formed between the points: P, 1 and 5 (see figure 9). The calculation of the occupied area is performed using equation 23.

$$\text{Area} = (w \cdot n_p) \cdot (D \cdot (n_r - 1)) \quad (23)$$

where w is the width of the panel (in our case $w = 1$ m), n_p is the number of panels per row y , and n_r is the number of rows.

Table 3 shows the results of the calculation of the distance between the rows of panels

(B) for each of the four tilt angles of the panels described in Table 1 and for the two arrangements of the panels. The results of Tables 3 and 4 allow us to deduce the importance of choosing an appropriate distribution of rows of panels, where the panels can concentrate on the smallest number of rows that requires a minimum distance between rows due to the shadows. In all the methods studied, the choice of 10 panels in 3 rows reduces by 16.6 % the occupied space with respect to the choice of 6 panels in 5 rows. In addition, it is also observed that the higher tilt panel ($> \beta$) has the largest area of occupation, as expected.

Table 3

Table 4

In all PV systems, it is very important to select the correct location and to avoid undesirable shadows that reduce its energetic production. The existing technology of solar cells used in the PV panels does not operate well under partial shading conditions (Zhou et al., 2013). Shading of a PV array, either complete or partial, can have a significant impact on its power output and energy yield (Paraskevadaki and Papathanassiou, 2011). Some cells in a PV module that are partially shaded become reverse biased, acting as loads instead of generators (Liu et al., 2008). A 10 % shade on a PV panel can cause a decrease of up to 90% of its generation capacity. After the selection of the photovoltaic cells of the solar panels and the determination of the angle of installation of the panels, the PV array spacing becomes the most critical issue (Sun, 2011). This paper proposes a simple estimation method that takes into account all energy options (tilt angle of the panel, β), and then provides a comparison of the results with the classical methods.

Analysing the results of the proposed method (Table 3) and the results of the classical methods (Table 4) reveals that the proposed method reduces the area occupied by 23 % compared to the classical method 1, regardless of the arrangement of the panels in rows, for $\beta = 15^\circ$ and $h = 15^\circ$, and compared with the classical method 2 or the empirical approach, the area is reduced by 5.3 %. However, with increasing tilt panel (β) while maintaining an $h = 15^\circ$, these percentages are increasing; and for $\beta = 40^\circ$, the proposed method reduces the area by 35 % compared to the area occupied method 1 and by 9 % compared to Method 2. For $\beta = 45^\circ$, the proposed method reduces the area used by 36 % compared with method 1 and by 10 % compared to Method 2. Finally, for $\beta = 60^\circ$, the proposed method reduces the area required by 40 % compared to method 1 and by 11 % compared to Method 2. Therefore, the proposed method is advantageous for all possible situations.

Additionally, because the proposed method has no limitation on the number of hours of solar gain, this makes it particularly useful for areas where many hours of sun per day exist, such as southern Spain and North Africa, where up to 10 hours of solar gain can occur. Therefore, the proposed method can be applied to standalone photovoltaic (PV) systems used as the power source for pumping water (Bouzidi, 2013), especially in areas far from power lines (Jakhrani et al., 2012).

6 Conclusions

This paper proposed a simple estimation method that minimises the distance between rows of fixed PV panels while avoiding the shadows between them. Furthermore, the proposed method has no limitations on the height of the sun or the estimated times for

solar collection, nor is it limited to a specific geographical area, as is the case for empirical methods. The results of the proposed method was compared with the results of the classical methods, with the comparison results indicating that the proposed method can reduce the area by up to 40 % if the tilt angle of the panel is 60° compared with any of the methods considered. In conclusion, the proposed method is a global method that will allow the minimisation of the distance between PV panels of PV arrays while avoiding shadowing, which is of special interest for areas with many hours of sun for maximum energy uptake, especially for standalone photovoltaic (PV) systems in remote areas used as isolated small power producing units for the supply of electricity.

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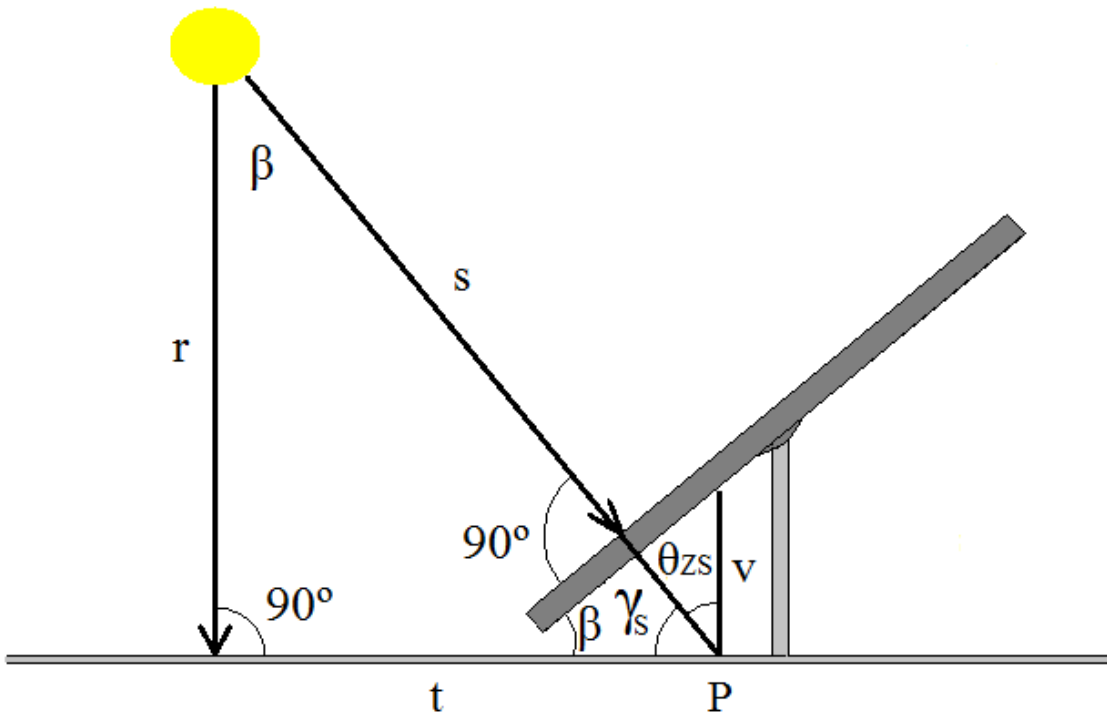


Figure 1. Incidence of the sun's rays on a photovoltaic panel (frontal view).

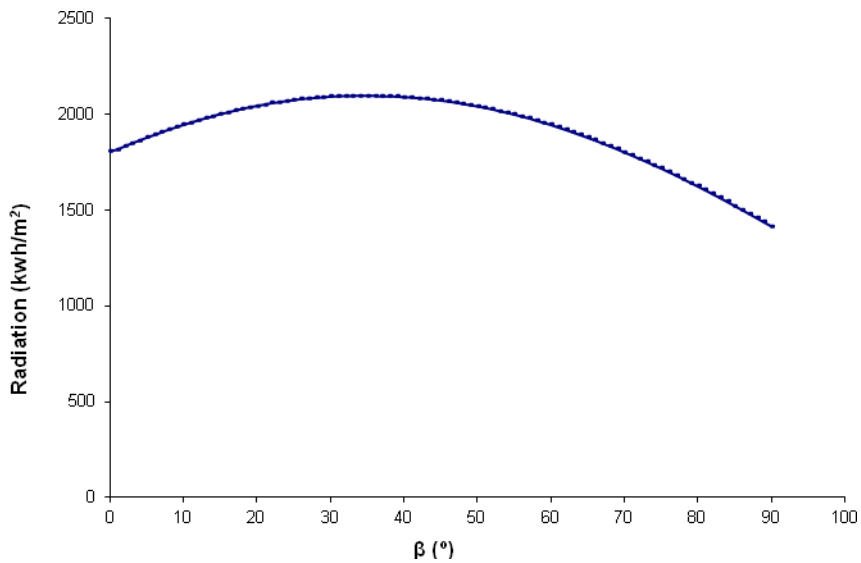


Figure 2. Intercepted radiation (kWh/m²) depending on the tilt angle of the PV panel for Southern Spain.

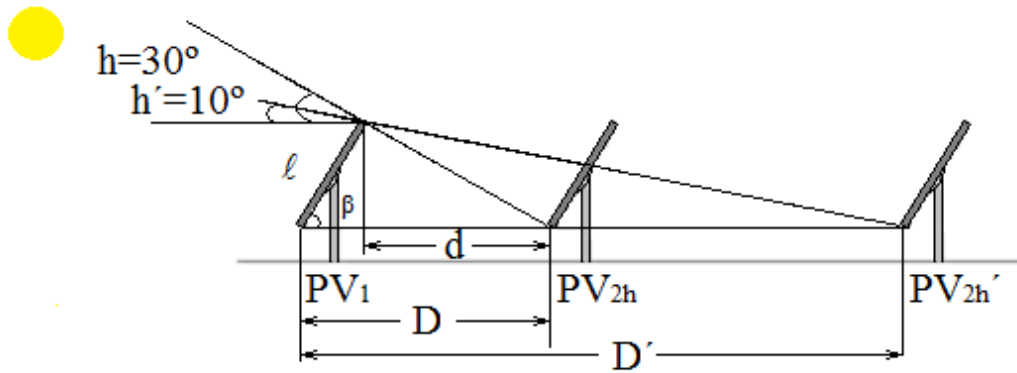


Figure 3. Row alignment minimum PV panels as a function of the solar altitude (Classical Method 1).

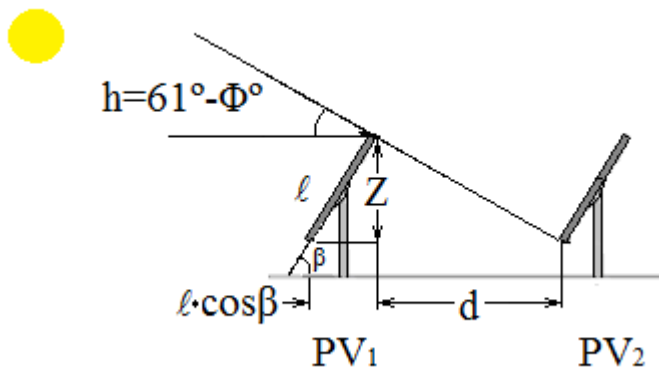


Figure 4. Row alignment minimum PV panels as a function of the solar altitude (Classical Method 2).

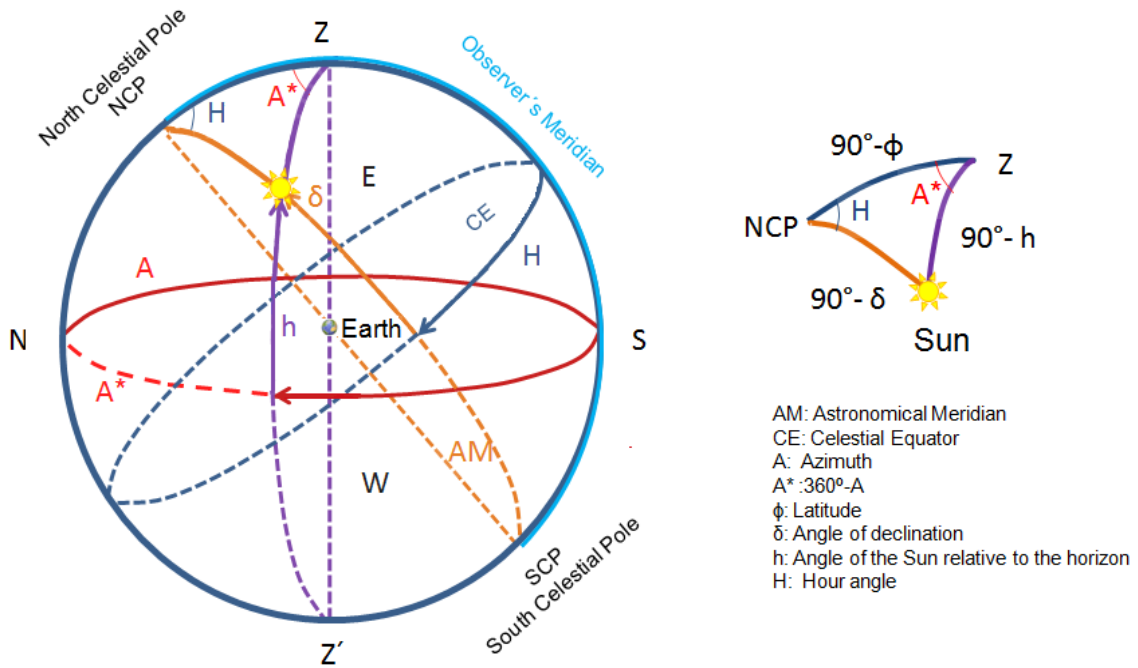


Figure 5. Spherical triangle for measuring the projected shadow by the sun on the panel.

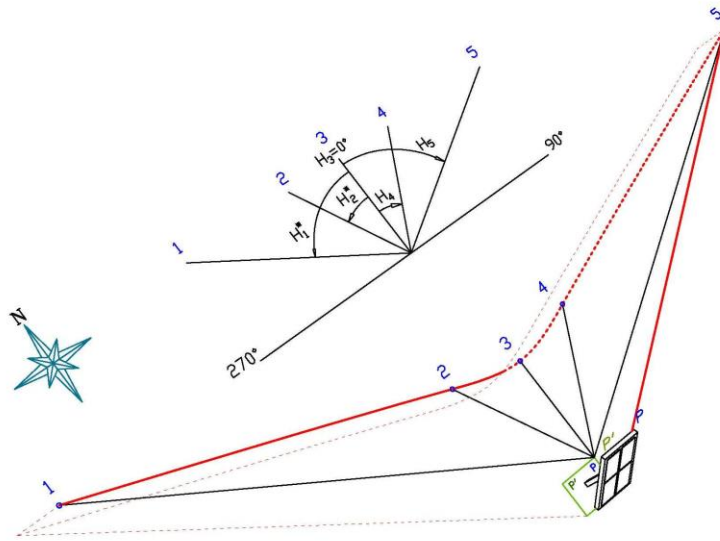


Figure 6. Proposed method: isometric view of the calculated shadow produced by the right corner of PV panel (P).

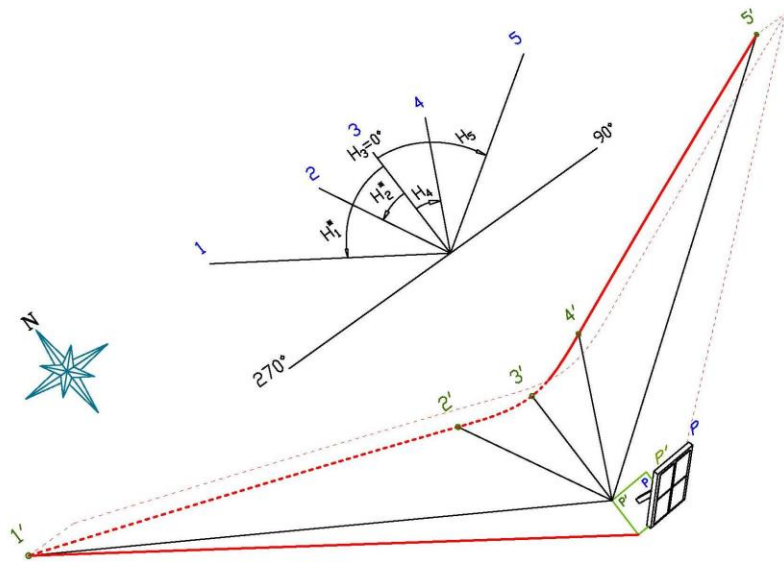


Figure 7. Proposed method: isometric view of the calculated shadow produced by the left corner of PV panel (P').

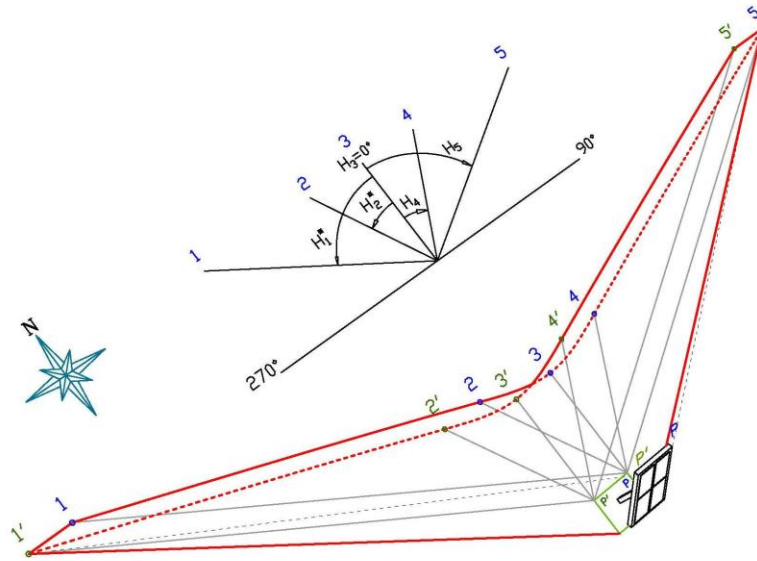


Figure 8. Proposed method: isometric view of the entire shadow calculated for a PV panel.

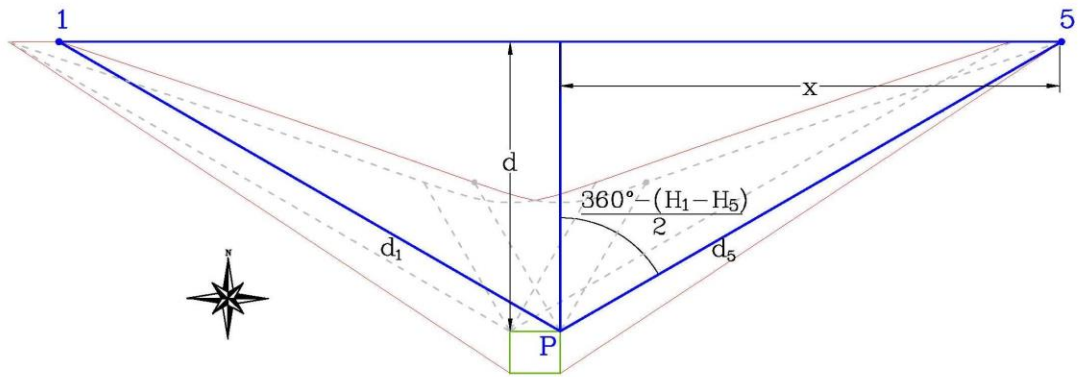


Figure 9. Proposed method: diagram of the combined panel-shadow and the triangles obtained to calculate the minimum distances between the panels.

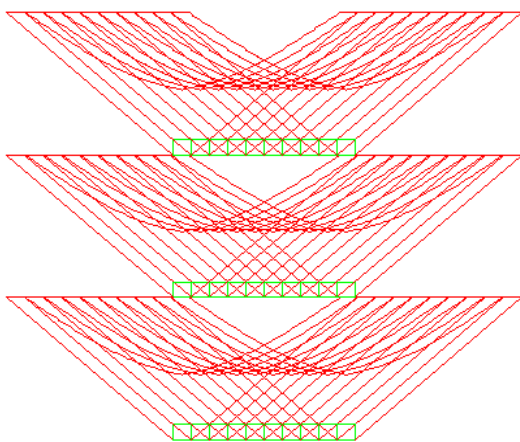


Figure 10. Proposed method: shadowing of 30 PV panels (3 rows of 10 panels).

Table 1. Results of the feedback systems.

Objective	β (°)
(a) Maximum power at the winter solstice.	$\Phi + \delta $
(b) Maximum power at the summer solstice tips	$\Phi - \delta $
(c) Maximum annual energy production	Φ
(d) Intermediate solution between the case (a) and (c)	$\Phi + (\delta /2)$

Table 2. Proposed method: shadow distance (d) calculation for different PV panel tilt values (β) from 8:00 to 16:00 hour, (eq. 17).

			Point					
	unit		1	2	3	4	5	
Solar hour		Ortho	8:00	10:00	12:00	14:00	16:00	Sunset
H	(°)	289.09	300.00	330.00	0.00	30.00	60.00	70.91
h	(°)	0.00	6.88	23.16	29.50	23.16	6.88	0.00
d ($\beta= 60^\circ$)	m	∞	11.48	3.41	2.55	3.41	11.48	∞
d ($\beta= 45^\circ$)	m	∞	9.38	2.78	2.08	2.78	9.38	∞
d ($\beta= 40^\circ$)	m	∞	8.52	2.53	1.89	2.53	8.52	∞
d ($\beta= 15^\circ$)	m	∞	3.43	1.02	0.76	1.02	3.43	∞

Table 3. Proposed method: distance and area occupied of a solar field with 30 panels considering different panel tilt values (β , $\alpha = 0^\circ$) and for different solar gain hours.

Tilt panel (β)	Gain solar hours	D (m) (eq. 6)	Area (m ²) (eq. 23)	
			$n_p = 10$ $n_r = 3$	$n_p = 6$ $n_r = 5$
15°	4	2.48	49.62	59.54
	6	2.67	53.36	64.03
	8	3.32	66.50	79.80
40°	4	3.44	68.87	82.64
	6	3.91	78.14	93.77
	8	5.71	114.30	137.16
45°	4	3.56	71.24	85.49
	6	4.07	81.45	97.74
	8	5.87	117.44	140.93
60°	4	3.75	75.08	90.09
	6	4.38	87.58	105.09
	8	6.58	131.57	157.89

Table 4. Classical methods: summary of the occupied area for a solar field of 30 panels and for 4 hours of solar catchments.

Tilt panel (β)	Classical Method	h	D (m) (eq. 6)	Area (m ²) (eq. 23)	
				$n_p = 10$ $n_r = 3$	$n_p = 6$ $n_r = 5$
15°	Method 1	10°	4.05	81.05	97.26
		15°	3.22	64.33	77.20
		30°	2.35	47.10	56.52
	Method 2	$61-\Phi^\circ$	2.62	52.40	62.88
40°	Method 1	10°	7.35	146.92	176.31
		15°	5.27	105.41	126.49
		30°	3.13	62.60	75.12
	Method 2	$61-\Phi^\circ$	3.79	75.78	90.94
45°	Method 1	10°	7.85	157.11	188.53
		15°	5.57	111.45	133.73
		30°	3.22	64.35	77.22
	Method 2	$61-\Phi^\circ$	3.94	78.85	94.62
60°	Method 1	10°	9.01	180.25	216.30
		15°	6.22	124.32	149.18
		30°	3.33	66.63	79.96
	Method 2	$61-\Phi^\circ$	4.22	84.39	101.27