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An efficient modelling strategy for temperature control of residential buildings

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Abstract

In this paper we propose a method for modelling the temperature of the rooms of a (retrofitted) residential building for control purposes. By assuming that only the temperature in the rooms and the solar radiation can be measured, the approach consists in writing the first-principle thermodynamics equations of the system and then, based on them, in developing a (black-box) ARMAX model. The model parameters are estimated by initially employing a standard method with a pseudo random set-point signal and then they are adapted by using a self-calibration method when the performance deteriorates. The selection of the user defined parameters in the overall procedure is thoroughly discussed by means of simulation results and experimental results are also shown in order to highlight the practical effectiveness of the methodology.

Keywords:temperature control, modelling, self-calibration, computational efficiency.

1 Introduction

The building sector consumes more than 40% of the world energy use and the resulting carbon emission is actually greater than that of the transportation sector [11]. Actually, a large part of the energy consumption in buildings is due to Heating, Ventilation and Air-Conditioning (HVAC) systems, for which energy efficient control solutions have to be sought in order to help the fulfilment of the global climate change targets. In fact, engineers and architects have influenced the energy use of the new buildings through the design of the envelope, the HVAC systems selection and the operation sequences specification. However, when the building has been finished or it already exists and can be just retrofitted, the consumption of energy is decided mainly by its control, maintenance and by its occupants. Dynamic models are of crucial importance in implementing an effective control strategy. As the industrial experience has shown, the modelling and identification is often the most demanding and costly part of the overall automation process. Thus, modelling the temperature of a building has been an active research field for decades, and many software packages have been developed such as EnergyPlus, Modelica, TRNSYS, etc. [30, 44]. They use validated thermodynamic models to describe as accurately as possible the thermal processes in buildings. However, in general these models are too complex to be used for the purpose of designing model-based controllers and therefore they are mainly employed for simulation purposes. In fact, a model of the transient thermal dynamics of a multi zone building can be constructed from energy mass balance equations. An attempt to model all the relevant physical phenomena will result in a set of coupled partial differential equations. These models based on first principles are accurate but they require the complete knowledge of the thermodynamical characteristics of the building [34, 36], otherwise identifying all the parameters is generally a complex procedure that requires many different tests [1, 5, 6, 7]. Further, many different sensors are necessary to obtain the required measurements. In any case, this approach results in general in too computationally heavy models to be effectively used for optimal control purposes.

For this reason, simplified models have been proposed in order to provide a good approximation of the thermal behaviour of buildings. In particular, physically based models, for instance RC-networks, describe the thermal system as a network of resistors and capacitors connected in series and in parallel form. The main problem in this case is to find a methodology to estimate the values of all the parameters. In fact, many different solutions have been proposed in literature. A possible approach is to derive the values of the parameters again from the thermodynamical characteristics of the building [13, 15, 17, 28, 29, 35]. However, it has to be recognized that a priori knowledge of all the characteristics of the building is not always possible or convenient and therefore this represents a serious drawback of the method. Thus, other techniques consists in evaluating all the unknown parameters by using different algorithms based on data acquisition and reasonable initial guesses, for which typical values are described in ASHRAE book [10, 20, 22, 38]. It has however to be noted that also in this case some knowledge of the building characteristics has to be assumed.

In order to overcome this issue, other papers propose a complete data driven identification methodology, where all the parameters are evaluated based on collected training data and on the application of different estimation methodologies. For example, the use of genetic algorithms has been proposed [39, 40, 42]. This is a well-known identification methodology but it is quite computationally heavy and in any case good initial guesses and bounds need to be used to avoid local minima. Another methodology applied to solve this problem is recursive least squares. For example, this procedure is used in [10, 16] to identify a second-order model for a specific room. However, the method needs good bounds or reasonable initial guesses. In these cases the total resistance of the wall need to be known a priori.

Other suitable methods can be based on the Extended Kalman Filter or on the Unscented Kalman Filter [14, 23, 24, 25], which require the nonlinear description of the model. Indeed, some characteristics for the filter needs to be selected and each sampling time the estimation of parameters is updated, which can be quite computationally demanding.

Another different way to solve the problem is to apply a maximum likelihood function to the linearized model. This has been proposed for example in [3, 37] where prior physical knowledge of the building is exploited.

In order to provide methods where there is no need to know any physical characteristic of the building and therefore in order to greatly reduce the complexity of the modelling task (this is especially the case of retrofitted residential buildings) black-box models, such as AutoRegressive Moving Average with eXogenous inputs model [27, 30, 31, 41] or Neural Network Autoregressive with eXogenous inputs (NNARX) models [8, 12, 43] have been used to build a relation between inputs and outputs of the system by using measurements only. In this case, however, in order to determine the order of the autoregressive part or the number of hidden layers of neurons, only trial and error methodologies have been implemented.

It is also worth highlighting that there are significant differences among the models considered in the literature, disregarding the employed parameters estimation methodologies. Actually, in order to decrease the complexity of the equations and consequently the order of the system, some models do not take into account the heat coming from adjacent rooms [18, 19], others suppose that all the thermal characteristics of the envelope are the same [4].

It appears therefore that, despite the significant research effort, there is still the need of a simple temperature modelling method that does not require any prior information on the structure of the building (and therefore it is particularly suitable for residential buildings) and where the model is not oversimplified and, at the same time, there is no need for an iterative procedure for the selection of the model order, which has to be consistent with the physical characteristics of the system.

In this paper a methodology for the determination of a simple and computationally efficient model of the temperature in the rooms of a building is proposed. In particular, we consider those buildings (like small houses with flats in the different floors) where only a few simple sensors can be installed and used and where the habits of the occupants do not change significantly along the time. At the same time, it is likely that the temperature set-point values change during the day so that the energy consumption of the heating/cooling system can be significantly reduced by means of a suitably designed control system. The purpose is therefore to obtain a simple temperature model, that can be easily employed for the design of model-based control algorithms (for example, model predictive control strategies). The methodology is intuitive, since it does not need any prior information on the building structure and it is suitable for real-time systems. In particular, firstly, the appropriate structure of an ARMAX model is determined by considering the first principle thermodynamics equations that describe the thermal behaviour of each single room in a house (in this context, the thermal mass of the interior walls is considered for generality). It is then assumed that only the measurements of the temperature of each room and of the external temperature and solar radiation are available, as they can be quite easily installed in existing buildings without excessive extra costs. Secondly, the procedure for the estimation of the parameters consists of performing an ad hoc experiment in the first day and then by applying a self-calibration technique when the modelling error increases above a given threshold. Practical issues related to the implementation of the overall procedure, and in particular related to the choice of user-defined parameters, are discussed in order to provide useful guidelines for an effective use of the method.

The paper is organized as follows. A description of the temperature model is developed in Section 2. The parameter estimation methodology is presented in Section 3. The model of a house, implemented in TRNSYS, which is then used to illustrate the proposed technique, is described in Section 4. Practical issues in the design of the procedure are discussed, by means of simulation results, in Section 5. The validation test on real data acquisition is described in Section 6. Finally, conclusions are drawn in Section 7.

2 Model description

The thermodynamic fundamental equation that describes the variation of the temperature of a single mass is:

$$Q = c_s m \frac{dT}{dt} \tag{1}$$

where Q [W] is the total amount of heat exchanged, c_s $\left[\frac{J}{kgK}\right]$ is the specific heat, m [kg] is the mass, T [K] is the temperature and t [s] is the time. By applying this equation to a room it is possible to describe the variation of its temperature by taking into account simplifying assumptions, such as: (i) the air inside the room is supposed to be at the same temperature; (ii) the room is considered a closed system with no mass exchange with the external environment; (iii) no occupancy driven loads have been taken into account.

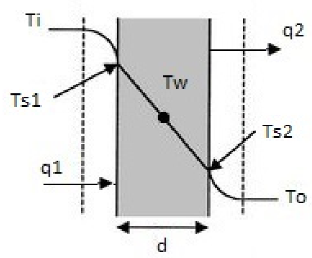

Figure 1: Temperature gradient and heat flow through a wall between two different rooms

The thermal flow can be transmitted from a room to another one passing through a wall as shown in Figure 1. This phenomenon occurs when the heat is exchanged from a fluid, such as air, to another one passing through a solid body, such as a window, a door or a wall. The wall is characterized by an own thermal capacity and a temperature that changes according to the quantity of heat that flows through it. Thus, by using (1), we obtain:

$$q_1 - q_2 = \Delta Q = C_w \frac{dT_w}{dt} \tag{2}$$

where q_1 [J] is the heat coming from the warmer room, q_2 [J] is the heat lost toward the colder room, T_w [K] is the wall temperature and C_w $\left[\frac{J}{K}\right]$ is the thermal capacity of the wall. Considering the heat balance on the surface between the wall and the room it is possible to determine q_1 and q_2 as:

$$\begin{aligned} q_1 &= h_i A(T_i - T_{s1}) = \frac{KA}{\frac{d}{2}}(T_{s1} - T_w) \\ q_2 &= h_o A(T_{s2} - T_o) = \frac{KA}{\frac{d}{2}}(T_w - T_{s2}) \end{aligned} \tag{3}$$

where T_i [K] and T_o [K] are the temperatures of the two rooms, T_{s1} [K] and T_{s2} [K] are the temperatures of the two wall surfaces, h_i $\left[\frac{J}{m^2K}\right]$ and h_o $\left[\frac{J}{m^2K}\right]$ are the convective coefficients, K $\left[\frac{J}{mK}\right]$ is the thermal conductance of the wall, A [m^2] is the area of the surface and d [m] is the thickness of the wall. By combining these two equations and substituting them in (2), we obtain:

$$C_w \frac{dT_w}{dt} = \frac{1}{R_1}(T_i - T_w) - \frac{1}{R_2}(T_w - T_o) \tag{4}$$

where R_1 and R_2 are coefficients that represent the combination of convection and conduction resistances.

A single room is usually bordering with more different areas, for example with the ground through the floor, with the external environment or with other rooms. In general, all the different heat flows to estimate the temperature evolution of a room, including exogenous sources like HVAC systems, the direct radiation from the sun, electrical devices, occupants, etc. that can heat or cool the room can be taken into account.

The whole system can therefore be written as follows:

$$\begin{cases} C_i \dot{T}_i = \sum_{j=1}^{n} q_j + \sum_{j=n+1}^{m} q_j = \sum_{j=1}^{n} \frac{1}{R_j}(T_i - T_{wj}) + \sum_{j=n+1}^{m} q_j \\ C_{w1}\dot{T}_{w1} = \frac{1}{R_1}(T_i - T_{w1}) - \frac{1}{R_{o1}}(T_{w1} - T_{o1}) \\ \vdots \\ C_{wn}\dot{T}_{wn} = \frac{1}{R_n}(T_i - T_{wn}) - \frac{1}{R_{on}}(T_{wn} - T_{on}) \end{cases} \tag{5}$$

where T_{on} is the temperature of the nth adjacent room, $q_1 \dots q_n$ are the heat flows gained and lost to the external rooms and $q_{n+1} \dots q_m$ are the exogenous heat sources, whose just the most significant components will be considered later in the proposed modelling strategy applied to residential buildings.

Note that this approach is similar to that proposed in [32] but here the thermal capacitances of the walls are explicitly considered.

The aim is to write a thermal model of the room based on:

- the temperature of the adjacent areas;
- the measured heat flow, or an estimation of it, of the exogenous inputs.

By applying the Laplace transform to (5) the system can be rewritten as:

$$\begin{cases} sT_i(s)C_i = \sum_{j=1}^{n} \frac{1}{R_j}(T_i(s) - T_{wj}(s)) + \sum_{j=n+1}^{m} q_j(s) \\ sT_{w1}(s)C_{w1} = \frac{1}{R_1}(T_i(s) - T_{w1}(s)) - \frac{1}{R_{o1}}(T_{w1}(s) - T_{o1}(s)) \\ \vdots \\ sT_{wn}(s)C_{wn} = \frac{1}{R_n}(T_i(s) - T_{wn}(s)) - \frac{1}{R_{on}}(T_{wn}(s) - T_{on}(s)) \end{cases} \tag{6}$$

As it is shown in (6), the temperature of the adjacent rooms appears only in the wall thermal balance equations, therefore the temperatures of the walls can be collected from all but the first equations and substituted into the first one. By assuming that the room has a parallelepipedal shape ($n = 6$) and each wall border has a different area (four lateral walls, floor and ceiling), the transfer function between the temperature of a room and the temperature of the other rooms becomes a seventh order transfer function. Note that T_i depends on several inputs, like temperatures of the border rooms and exogenous inputs in general, and unknown parameters:

$$T_i(s) = f(T_{o1}(s), \ldots, T_{o6}(s), q_7(s), \ldots, q_m(s); R_1, \ldots, R_6, R_{o1}, \ldots, R_{o6}, C_i, C_{w1}, \ldots, C_{w6}) \tag{7}$$

The second step is to transform this continuous transfer function in a discrete time transfer function by applying the Tustin bilinear transform [2] (for the sake of clarity we assume now that all the exogenous inputs are equal to zero):

$$s = \frac{2(z-1)}{\tau(z+1)} \tag{8}$$

and therefore, by substituting the s value into (7), the discrete time transfer function of the room is obtained:

$$T_i(z) = \frac{(a_{10}z^7 + \cdots + a_{16}z + a_{17})T_{o1}(z) + \cdots + (a_{60}z^7 + \cdots + a_{66}z + a_{67})T_{o6}(z)}{b_{10}z^7 + \cdots + b_{16}z + b_{17}} \tag{9}$$

By collecting the term z^7, we obtain:

$$T_i(z) = \frac{(a_{10} + \cdots + a_{17}z^{-7})T_{o1}(z) + \cdots + (a_{60} + \cdots + a_{67}z^{-7})T_{o6}(z)}{b_{10} + \cdots + b_{17}z^{-7}} \tag{10}$$

and therefore,

$$b_{10}T_i(z) = (a_{10}+\cdots+a_{17}z^{-7})T_{o1}(z)+\cdots+(a_{60}+\cdots+a_{67}z^{-7})T_{o6}(z)-(b_{11}z^{-1}+\cdots+b_{17}z^{-7})T_i(z) \quad (11)$$

which can be easily rewritten as follows:

$$T_i(k) = \mathbf{g_1}^T\mathbf{T}(k) + \cdots + \mathbf{g_8}^T\mathbf{T}(k-7) \quad (12)$$

where,

$$\mathbf{g_1} = \begin{bmatrix} \frac{a_{10}}{b_{10}} \\ \vdots \\ \frac{a_{60}}{b_{10}} \\ 0 \end{bmatrix} \quad \cdots \quad \mathbf{g_8} = \begin{bmatrix} \frac{a_{17}}{b_{10}} \\ \vdots \\ \frac{a_{67}}{b_{10}} \\ \frac{b_{17}}{b_{10}} \end{bmatrix} \quad \text{and} \quad \mathbf{T} = \begin{bmatrix} T_{o1} \\ \vdots \\ T_{o6} \\ T_i \end{bmatrix} \quad (13)$$

Therefore by applying the same consideration to the exogenous inputs, the complete black-box model is defined as follows:

$$T_i(k) = \sum_{j=1}^{8} \left\{\mathbf{g_j}^T\mathbf{T}(k-j+1) + \mathbf{p_j}^T\mathbf{q}(k-j+1)\right\} \quad (14)$$

where

$$\mathbf{p_1} = \begin{bmatrix} l_{11} \\ \vdots \\ l_{1m} \end{bmatrix}, \quad \cdots \quad \mathbf{p_8} = \begin{bmatrix} l_{81} \\ \vdots \\ l_{8m} \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} q_1 \\ \vdots \\ q_m \end{bmatrix} \quad (15)$$

Furthermore the system is also characterised by measurement noise, thus, the black-box model becomes an ARMAX model:

$$T_i(k) = \sum_{j=1}^{8} \left\{\mathbf{g_j}^T(\mathbf{T}(k-j+1) + \boldsymbol{\varepsilon}(k-j+1)) + \mathbf{p_j}^T\mathbf{q}(k-j+1)\right\} \quad (16)$$

where $\boldsymbol{\varepsilon}$ denotes the measurement noise.

It appears that the resultant ARMAX model depends also on the inputs at the time instant k. However, the system is affected by the thermal inertia which slows down the thermal transmission of the energy. Hence, the contribution of the heat sources at time instant k is actually very small and therefore, for the sake of simplicity, it is possible to approximate expression (16) by setting $\mathbf{g_1}$ and $\mathbf{p_1}$ to zero:

$$T_i(k) = \sum_{j=2}^{8} \left\{\mathbf{g_j}^T(\mathbf{T}(k-j+1) + \boldsymbol{\varepsilon}(k-j+1)) + \mathbf{p_j}^T\mathbf{q}(k-j+1)\right\} \quad (17)$$

The parameters estimation technique that is used to evaluate all the unknown parameters can be done with the application of the Matlab [26] function called `armax`; this function minimizes a robustified quadratic prediction error criterion by using an iterative Gauss-Newton algorithm [21]. This method is applied on (12) to determine all the $\mathbf{g_j}$ and $\mathbf{p_j}$ unknown parameters. Even though this equation is a direct consequence of the thermodynamical equations, during all the passages the parameters have lost their physical meaning, in fact such ARMAX models are defined as black-box models.

At this point, in order to keep the system as simple as possible, we consider that, for the kind of (residential) buildings we are considering (houses, flats), the number of occupants and electrical devices is small and more or less constant and that the people living in the building have the same habits (for example, windows are kept generally close unless the heating/cooling system is switched off and therefore the model is useless). Thus, the only exogenous inputs we consider as relevant are the solar radiation and the output of the heating/cooling device, which can be represented as the temperature controller output, as it is explained in the next section.

3 Parameters identification methodology

For each room we consider the temperature control system shown in Figure 2. It is a standard cascade control scheme based on proportional-integral (PI) controllers, where the inner loop represents the heating/cooling system, which is assumed it cannot be modified by the user. The outer loop controls the room temperature by acting on the set-point of the inner loop so that the output of the primary PI controller can be considered as a measure of the heat/cool provided to the room. Thus, there is no need for an additional sensor for this exogenous input to the system (17) as the variable u_1 can be used for this purpose.

In order for the parameters estimation methodology to be effective, it is essential to take into account that suitable exciting signals are necessary but, most of all, that all the nonlinearities (for example due to the solar irradiance effect) present in the true systems [9] have been implicitly linearized. For this reason, a self-calibrating methodology is proposed in order to automatically update the estimation when the operating conditions change.

It has to be taken into account that, as we are considering a residential building, we assume that the set-points $\bar{T}_i$ selected by the user for the different rooms change during the day according the the occupancy of the rooms themselves. For example, it is reasonable that the set-point of the living room of a flat is higher during the day and lower during the night (viceversa for the bedrooms).

The identification procedure consists therefore in performing an experiment during the first 24 hours of usage of the system (which can be any day of the year, when the fan-coil units are employed) and in estimating the ARMAX parameters based on the collected data. Note that by considering a reasonable sampling period of 4 min, the dataset is composed by 360 values for each measured variable, that is, a quite large number of values with respect to the order of the ARMAX model.

In this context, the choice of the set-point signals $\bar{T}_i$ for the different rooms represents a crucial issue (see Section 5.1). Then, each x_1 hours, the estimation temperature error for the ith room, defined as

$$e_i = \frac{\sum_k |T_m(k) - T_i(k)|}{N} \tag{18}$$

is evaluated, where T_m is the actual measured temperature and N is the number of samples in the considered interval which is determined by taking into account the preceding x_2 hours. If the estimation error (18) is greater than a given threshold x_3, then the identification procedure is applied again by using the data collected in the preceding x_4 hours. In this context a maximum variation of $\pm 5\%$ for each parameter is allowed in order to account for typically slow weather changes from one season to another.

Note that x_i, $i = 1, \ldots, 4$ are design parameters that have to be selected properly in order for the overall procedure to be effective. The choice of these parameters, as well of the other factors which might influence the results is discussed in Section 5.

It is in any case worth highlighting that, thanks to the simplicity of the black-box model description, the overall methodology can be implemented both in a centralized and a decentralized controller.

4 Model of the test building for simulations

In order to discuss the practical issues involved in the method described in the previous section, a large number of simulations have been performed by using TRNSYS (using a sampling period of 4 min), which is a well-known software that implements non-linear equations to simulate, with a very good accuracy, the real thermal behaviour of the building. In this

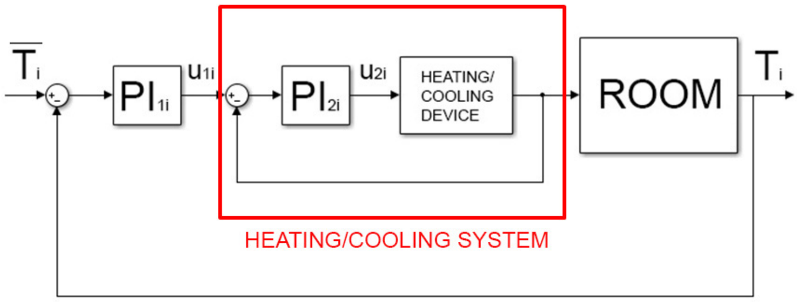

Figure 2: The control scheme for the temperature of the ith room.

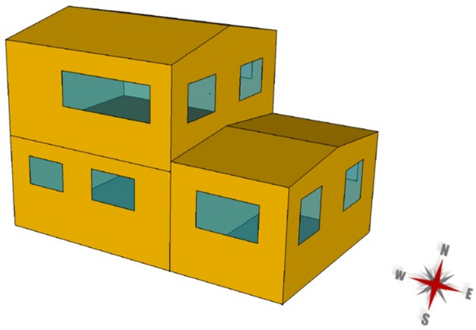

Figure 3: Model of test building implemented in SketchUp 2013.

paper we consider tests performed on the two floor building shown in Figure 3, which has been implemented in SketchUp 2013 [33]. The ground floor is composed by two rooms while there is just one room in the first floor. The characteristics of the three rooms are given in Table 1.

The building is fictitiously placed in Milan Linate (Italy) and it is not surrounded by other houses. Thanks to TRNSYS 17 Weather Data, a file with the weather data of the whole year is available and therefore, by using the TRNSYS functionality to introduce it into the simulation algorithm, it is possible to take into account the real environmental conditions. The most important parameters that characterize the building are the thermal conductivities (u-values) of the walls and of the windows. These parameters describe the thermal insulation coefficients of the main parts of the building and therefore they are responsible of the heat exchange rate between the rooms. They have been set as written in Table 2. It is worth stressing that these values are assumed to be completely unknown in the modelling procedure. The employed HVAC system consists of one fan-coil unit (Type 600 block in the TRNSYS HVAC library) and one temperature sensor for each room (to keep the system as

Room	Dimensions	Windows
East side ground floor	3x4x2.5	4
West side ground floor	5x4x2.5	5
First floor	5x4x2.5	4

Table 1: Area and number of windows of each room in the test building.

Element	u-value $\left[\frac{W}{m^2K}\right]$
external walls	0.510
ground floor	0.039
external roof	0.316
internal walls	0.508
internal ceiling	4.153
external windows	2.890

Table 2: Thermal conductivities of the test building.

simple as possible). The temperature is controlled by means of PI low level controllers (see Section 3).

As it has been already mentioned, as the measurement of the temperature of the HVAC output air might not be available in general (or the sensors to be used for this purpose can be too expensive), the output value of the primary PI controller which controls the temperature is used as exogenous input, thus making the method effective to implement in practical case. Summarising, the sensors used for the implementation of the devised modelling strategy are those to measure the indoor temperatures of the single rooms, the external temperature, and the solar radiation.

5 Practical issues

As already mentioned in Section 3, for an effective implementation of the estimation methodology it is essential to appropriately choose the design parameters x_i, $i = 1, \ldots, 4$ as well as considering the set-point signals to be employed during the first 24 hours.

In order to simplify the analysis (and the design in practical cases), a reasonable selection can be done for the parameters x_1, x_2 and x_3. In particular, the value x_1 can be fixed equal to 8 hours in order to provide an accurate model without increasing too much the computational burden. By applying a similar reasoning, x_2 is selected equal to 12 hours and x_3 equal to 1 [K].

The other design choices are discussed in the following subsections (where simulation results are obtained with the previous values of x_1, x_2 and x_3).

5.1 Signals for the initial estimation

Since the first day estimation plays obviously a key role in the overall method, it is essential to accurately select the set-point signals to be employed in this context. From a practical point of view, it would be very nice to employ routine operating set-points, in order to avoid a possible uncomfortable situation for the inhabitants. However, the result provided in this case depend on the occupants' choice and they are in general not accurate, that is, the self-calibrating procedure will be applied many times in the subsequent days, as it will be shown hereafter. In order to solve this issue we propose to use a pseudo-random signal, which is built by following some restrictions related to the thermal comfort of the residents:

- the upper acceptable temperature is 22 [^{o}C];
- the lower acceptable temperature is 17 [^{o}C];
- the set-point of each room is changed each hour and it can increase or decrease only with a $\Delta T = 1$.

Obviously, the first two constraints are related to the case where the first 24 hours are in the cold season but the same rationale can be easily applied to other periods. Actually, a difference of five degrees is necessary to provide a good estimation and it has to be stressed that the values are in any case in a liveable temperature range.

The advantage of using a pseudo-random signal is illustrated in Figures 4 and 5 where the proposed method is compared with the use of routine set-points. These have been selected as shown in Tables 3 and 4. Two cases are considered where different pseudo-random signals and different routine set-point signals are chosen. Obviously, if the set-point signals are selected by the inhabitant, it should be taken into account that the choice is entirely subjective and this also represents a drawback with respect to the pseudo-random signal approach. All the simulations have been performed by using a value $x_4 = 48$ (see subsection 5.2). They are related to the first days of November (however, the considerations done hereafter can be done also for other periods of the year). From this illustrative examples it appears that a more accurate model is obtained by using the pseudo-random set-point signals than by using the routine set-point signals. Indeed, the model parameters need to be updated much less times during the following 19 days.

It has to be stressed that, since the direct radiation of the Sun and the external temperature are not controllable inputs, the parameters related to these inputs may not be identified correctly at the first attempt. In order to give a deeper insight on this issue, the first five days of Figure 4 (top) are considered in Figure 6 for a ground floor room, where the solar radiation and the external temperature are also plotted (note that the pseudo-random set-point signal strategy has started at 1 pm). It can be seen that the initial parameter estimation is obtained when the solar radiation is not very high. For this reason, an estimation error occurs during the fourth day because in that day the solar radiation is much higher than the previous days. For this reason the model is recalibrated and then it is capable to predict the thermal behaviour of the building in the subsequent days.

Room	Time intervals (hours of the day)		
	0-10	10-12	12-24
East side ground floor	17	17	18
West side ground floor	19	18	18
First floor	20	20	18

Table 3: Case 1 of the inhabitant-chosen set-point signals (in Celsius degrees) for the first 24 hours.

Room	Time intervals (hours of the day)			
	0-8	8-10	10-12	12-24
East side ground floor	17	17	17	21
West side ground floor	20	20	19	19
First floor	19	18	18	18

Table 4: Case 2 of the inhabitant-chosen set-point signals (in Celsius degrees) for the first 24 hours.

5.2 Size of recalibration dataset array

As already mentioned, the changes in the set-point signals defined by the user as well as the changes in the external temperature and in the solar radiation during the day make a recalibration strategy possible by considering previous data. In this context it is necessary to select the value of the parameter x_4, namely, of the size of the array of data to be used for a new estimation of the model parameters when the error exceeds the given threshold, the following reasoning can be done. From one point of view, in order to reduce the computational burden and to consider the change of climate along the year it is sensible to use a small amount of data, for example 24 hours. However, this choice can introduce errors because the less samples are used the less filtered is an unusual behaviour of the system. For instance, in winter season the model is calibrated to work with low environmental temperature and if a day hotter than usual occurs, the model error can exceed the $1[K]$ threshold and the algorithm will estimate again the parameters. However, if only the last 24 hours are taken into account, the parameters will be estimated based on a uncommon winter day and this may provoke another error the day after. On the other hand, if 48 hours data are used, the model will be recalibrated based on two days data and the winter day will filter the contribution of the unusual one, it is important to stress that the possible modification in all the parameters is bounded in $\pm 5\%$ of their previous value. For this reason, a suitable choice is to select $x_4 = 48$ hours. Illustrative examples with $x_4 = 24$ hours choice are shown in Figure 7 (two examples with a different pseudo-random initial set-point signals are considered) and it can be compared with the 48 hours choice employed in Figure 4. It appears that the 48 hours time interval provides a more accurate modelling with less recalibration procedures applied in the subsequent days.

Table 5 shows the mean absolute temperature error in all the performed tests. The error is defined as in (18) where N represents the number of instants of the whole test time interval.

5.3 Model evaluation during different periods of the year

Along the whole year weather conditions obviously change, for example the winter conditions are completely different from the summer ones. Thus, the HVAC system operates with different temperatures, the solar irradiance changes in direction and intensity, etc. It is therefore necessary to evaluate if the black-box model is capable to describe the system during the different periods of the year or if it has to be recalibrated completely when the

Test	West side ground floor	East side ground floor	First Floor
Fig. 4 Top	0.4062	0.5436	0.4382
Fig. 4 Bottom	0.2766	0.3793	0.3763
Fig. 5 Top	0.5715	0.6113	0.4306
Fig. 5 Bottom	0.3908	0.4969	0.3862
Fig. 7 Top	0.7476	0.5243	0.3999
Fig. 7 Bottom	0.4491	0.7151	0.3819

Table 5: Mean absolute temperature error in all the performed tests.

conditions are too different from the first identification day.
Many simulations have been performed to investigate this issue, by assuming that the same HVAC system is used to heat and to cool the room. In fact, it is obvious that different considerations have to be done if the actuation system changes, for example radiators are used for the heating task and air conditioners for the cooling one.
More specifically, two different tests have been performed on the model to investigate the behaviour of the simulated temperature in different conditions:

- firstly, the model identified during the month of November is applied directly in February (starting from the 3rd of February);
- secondly, a case study from the 1st of January to the 4th of September is performed.

The first test simulates the situation in which an inhabitant switches off the system for two months and then switch it on in a different period of the same season. In Figure 8 it can be seen that the model is simulating the temperature evolution with a good accuracy even if it was identified two months before.
The second test shows the normal evolution of the system during a whole year. In particular, the initial parameter estimation is performed (by applying a pseudo-random set-point signal) on the first of January and then the model is applied (by recalibrating it if necessary). The heating system is switched off on the 1st of May and then the cooling system is switched on on the 1st of July and applied for two months with different set-point values (ranging from 23 to 27 [^{o}C]). The results for the whole period are shown in Figure 9 and a zoom of the period starting from the beginning of July is shown in Figure 10. It can be seen that the number

of recalibration is small compared to the number of days, which means that the model is capable to well capture the dynamics of the system.

6 Model validation test

After having discussed the practical issues in a simulation environment, the modelling strategy has also been tested in a real environment. For this purpose, the CIESOL building located in the Campus of the University of Almería, in the southeast of Spain has been used. A detailed description of this building can be found in [9]. In order to calibrate and validate the model proposed in this paper, a typical room of this building has been selected. This room is situated on the top floor of the CIESOL building between two rooms with analogous characteristics. More specifically, it has North orientation, a total volume of 76.8 m^3 and a window of 4.49 m^2 located in the north wall. Even if the room is characterized by the presence of different sensors (air temperature, air velocity, plane radiant temperature, humidity, etc.) and actuators (a fan-coil system, a shading system and a window opening/closing system), the typical situation of a standard building has been recreated as much as possible. In particular:

- the temperatures of the walls of the room have been employed instead of the temperatures of the adjacent rooms, because these are the available sensors;
- the temperature of the output air of the fan-coil system has been used as the only exogenous input;
- a temperature sensor placed in the middle of the room measures the inside temperature.

The experiment in the room has been performed between May 11 and May 17, 2015. The weather conditions in that period were quite stable as it is shown in Figure 11, where the external temperature and the direct solar radiation are shown. It can be noted that the external temperature requires that the fan-coil system is used in cooling mode to possibly decrease the temperature during the day time.

Figure 12 shows the results of the application of the proposed methodology. It can be observed that, after the first calibration, which takes place at the end of the first day, the model does not need to be retuned during the whole period of the test. The maximum error is always less than 0.5 [K], that is, the behaviour of the real temperature is well approximated

by the model even though the room has been used by people during that period (note that, even if there are also sensors to count the number of people entering or exiting the room, see Figure 13), this information is not taken into account.
It is then worth stressing that, because of the low external temperature, in some time intervals (in the second half of the period), the fan-coil system, which acts in cooling mode, is off and it is not possible to attain the set-point value (see the bottom part of Figure 12).

7 Conclusions

In this paper we have proposed a methodology for the determination of an ARMAX model of the temperature of the rooms of a smart building. The technique is based on a first initial estimation of the model parameters (during the first 24 hours, by applying pseudo-random set-point signals) and then on the application of a self-calibration procedure. Practical issues related to the main design choices have been discussed and it has been shown that the proposed solution are capable to provide the required balancing between the need to follow the system changes along the year (mainly due to the weather change) and to avoid too many recalibration procedures.
Furthermore a real test has been performed to prove the effective performances of the model. Future work will include the comparison between the presented modelling approach with others proposed in the literature and the use of the obtained model for (optimal) control purposes.

Acknowledgments

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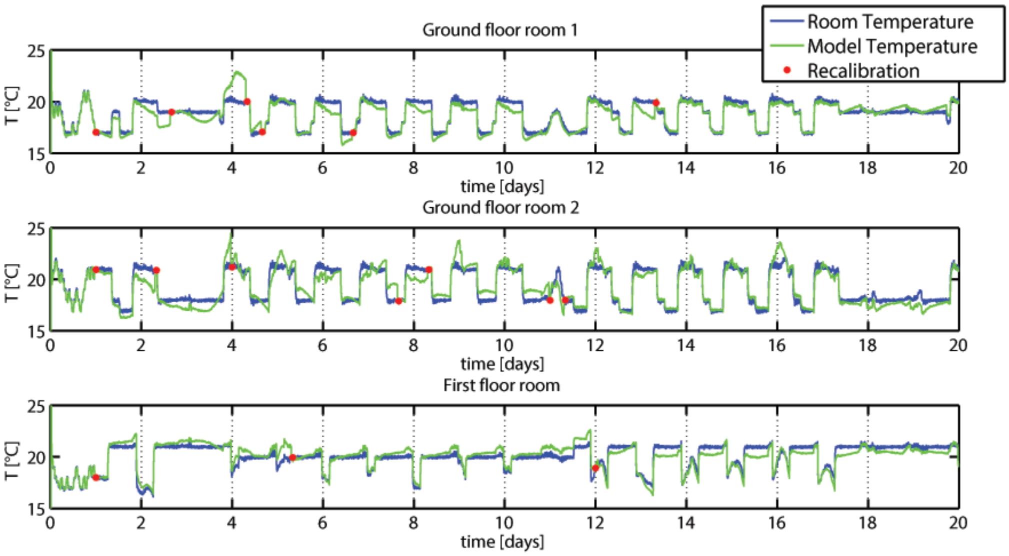

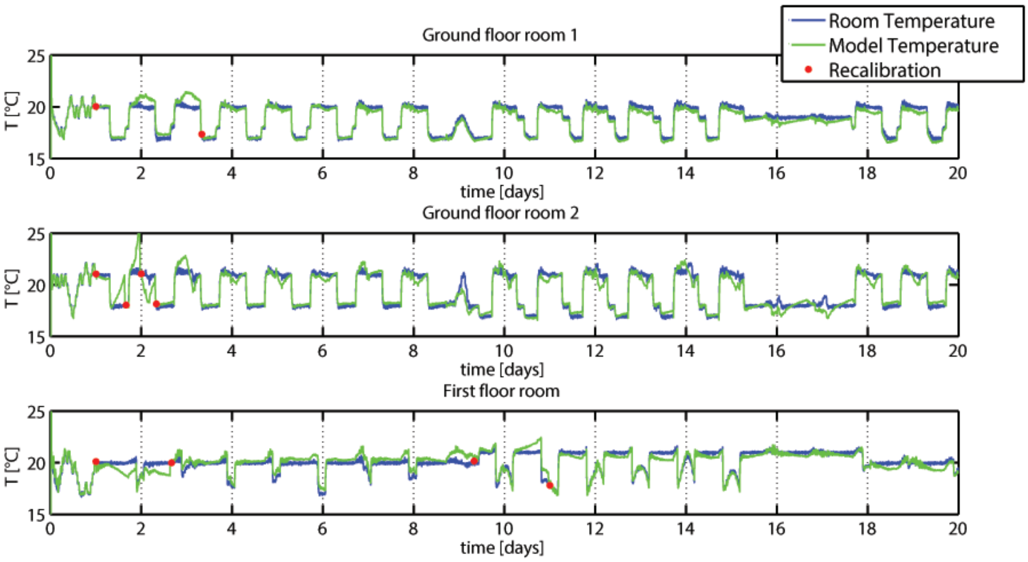

Figure 4: Results related to the use of pseudo-random set-point signals. Top: case 1. Bottom: case 2. Blue line: measured temperature. Green line: model temperature. The application of the recalibration strategy is indicated by '*'.

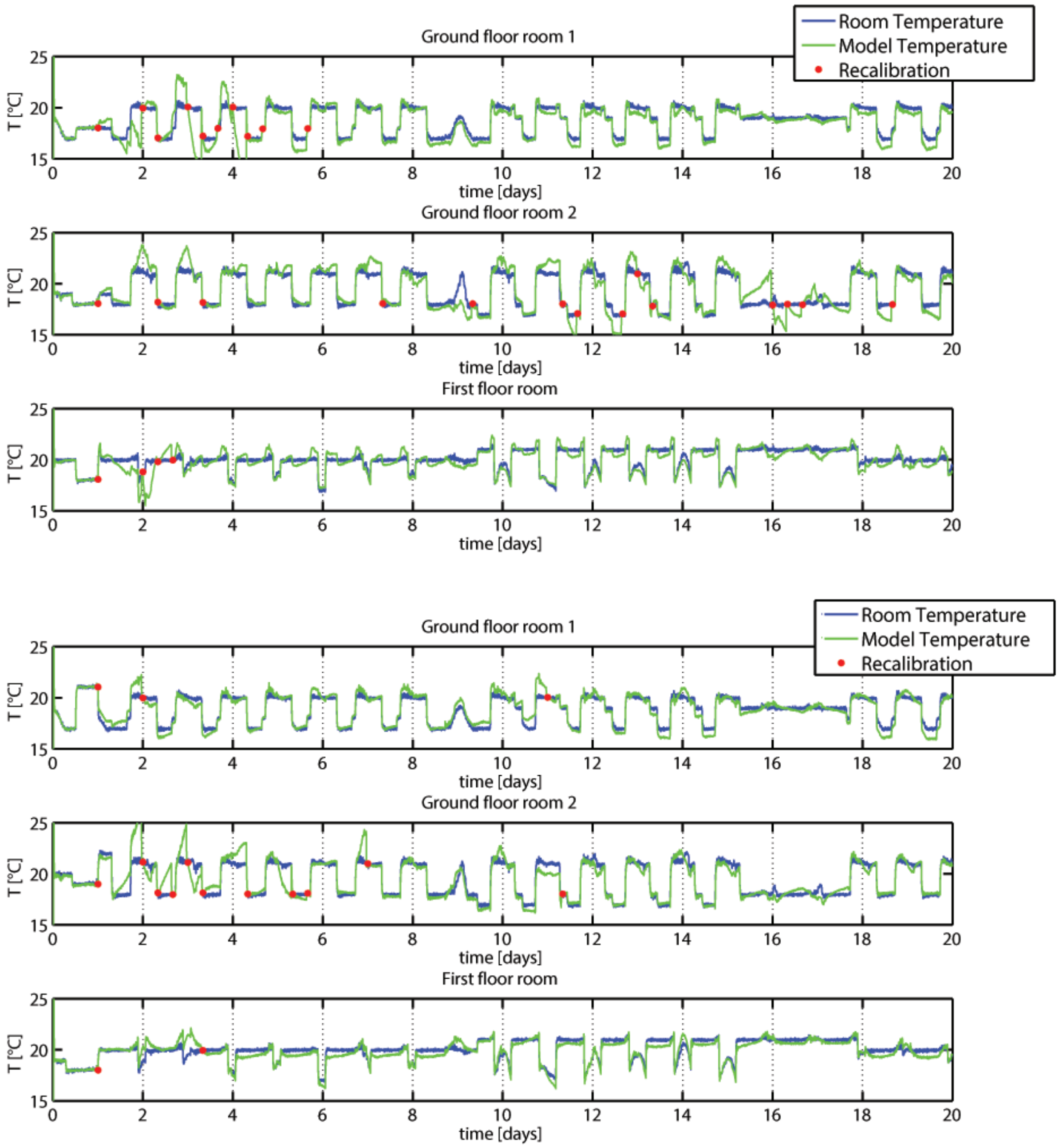

Figure 5: Results related to the use of inhabitant-chosen set-point signals. Top: case 1 (Table 3). Bottom: case 2 (Table 4). Blue line: measured temperature. Green line: model temperature. The application of the recalibration strategy is indicated by '*'.

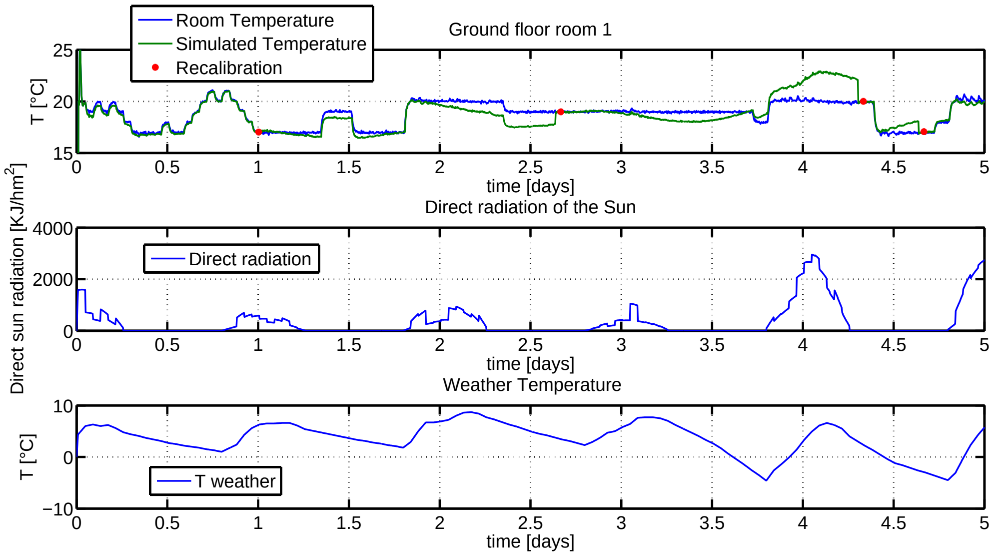

Figure 6: Zoom of the results for the first days of the ground floor room 1 shown in Figure 4 (top) with the solar radiation and the external temperature. Blue line: measured temperature. Green line: model temperature. The application of the recalibration strategy is indicated by '*'.

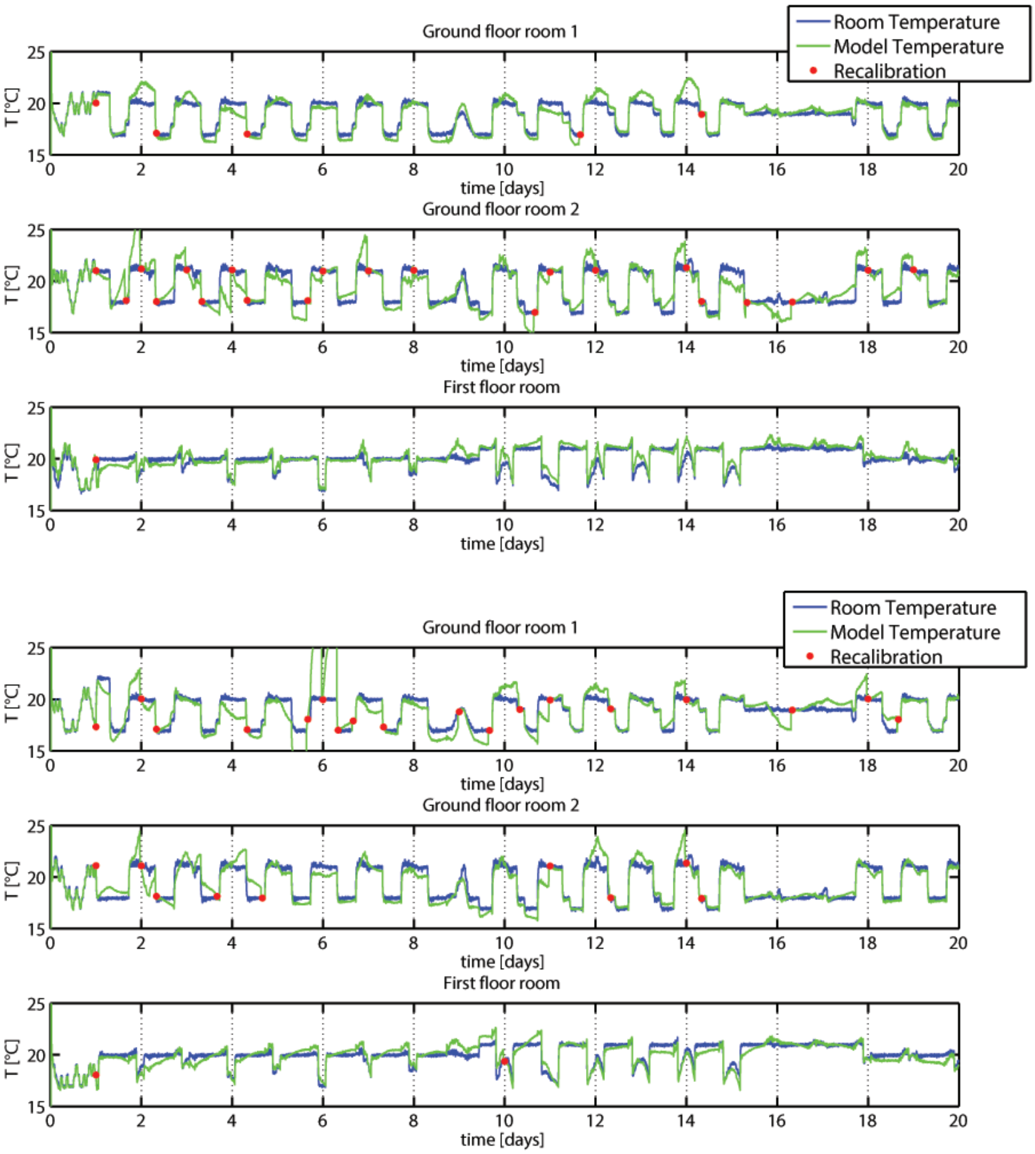

Figure 7: Results related to the use of $x_4 = 24$ hours and different pseudo-random set-point signals (top and bottom). Blue line: measured temperature. Green line: model temperature. The application of the recalibration strategy is indicated by '*'.

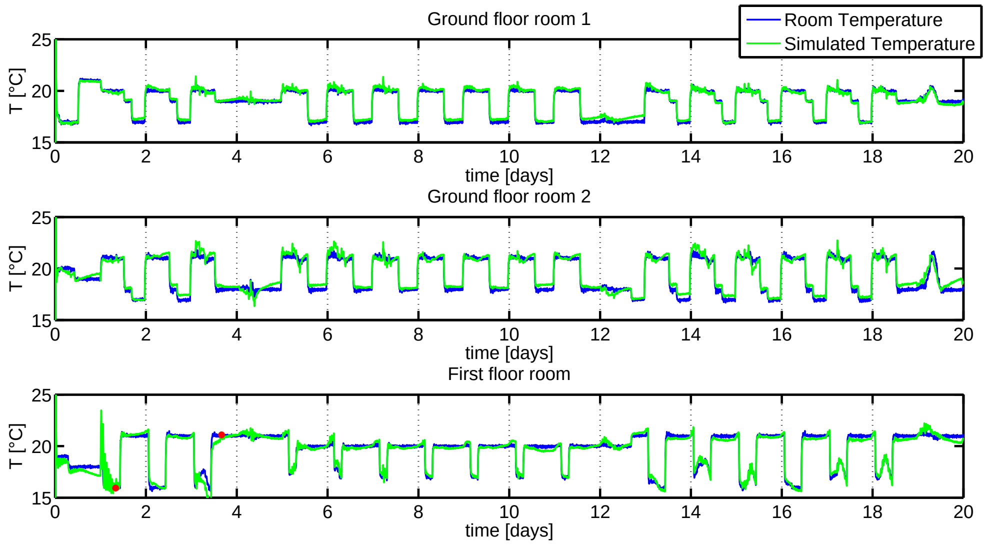

Figure 8: Results related to the use of a model tuned in November during the month of February. Blue line: measured temperature. Green line: model temperature. The application of the recalibration strategy is indicated by '*'.

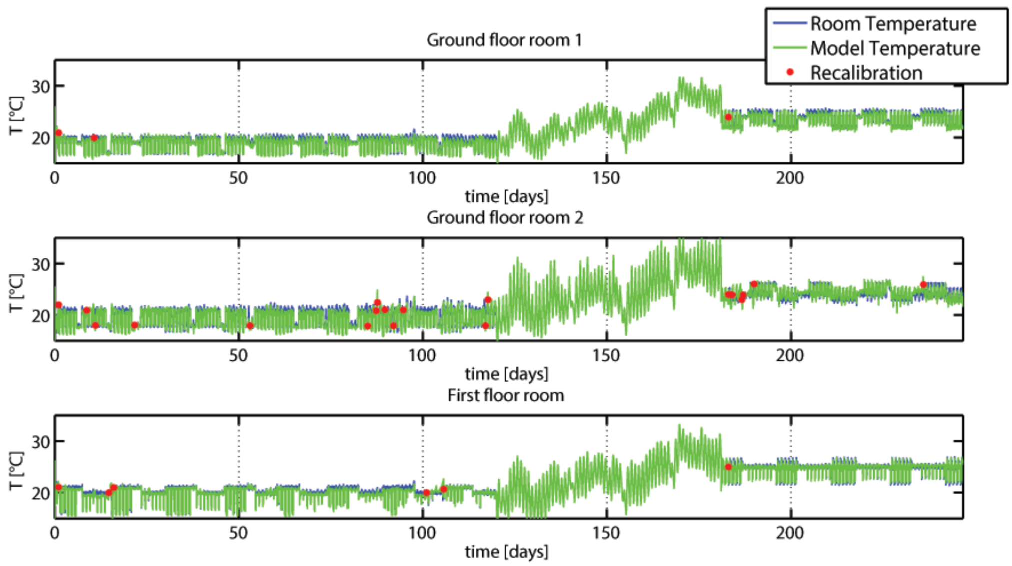

Figure 9: Results related to the use of a model tuned in January during eight months (the system is switched off on May and June). Blue line: measured temperature. Green line: model temperature. The application of the recalibration strategy is indicated by '*'.

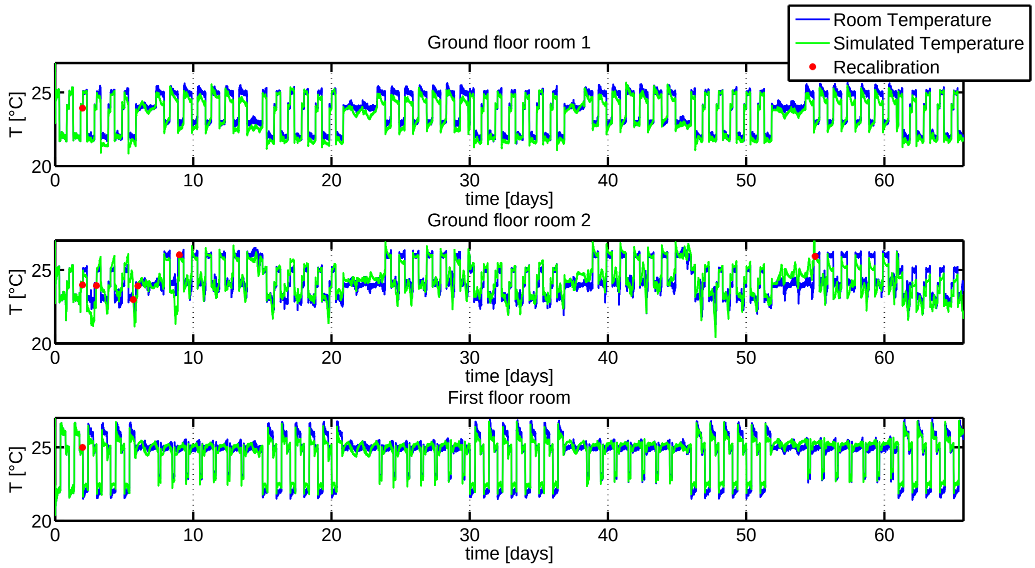

Figure 10: Zoom of the results of Figure 9 related to the months of July and August.

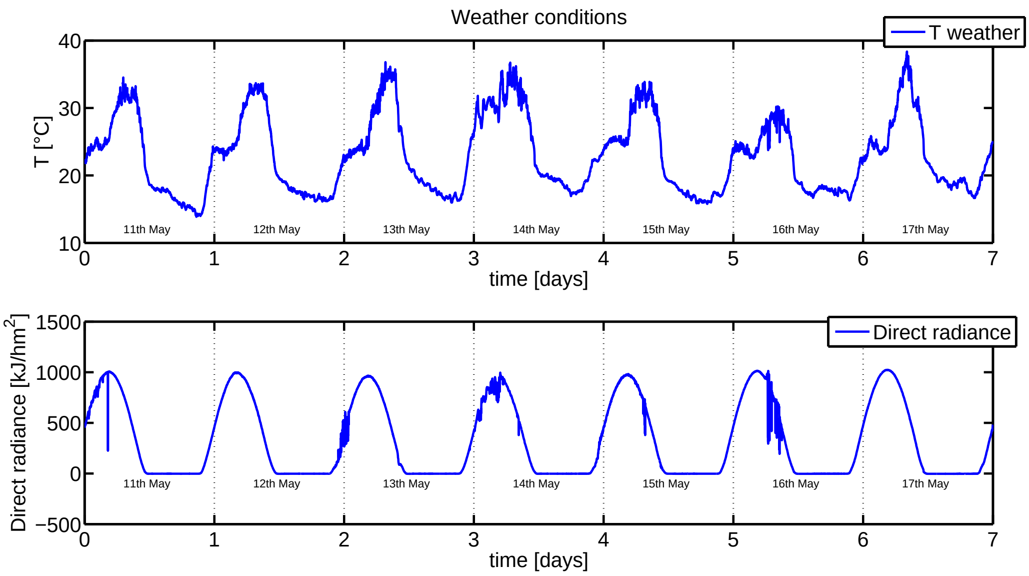

Figure 11: Temperature and direct solar radiation in Almería during the experiments.

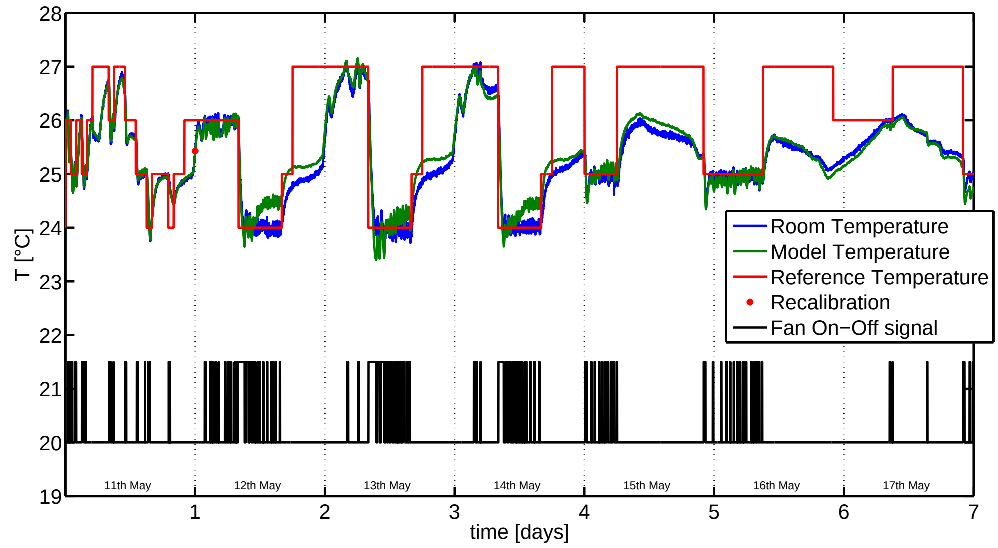

Figure 12: Results related to the real acquisition dataset during the month of May in CIESOL building. Blue line: measured temperature. Green line: model temperature. Red line: set point value. Black line: On-Off signal of the fan-coil system. The application of the recalibration strategy is indicated by '*'.

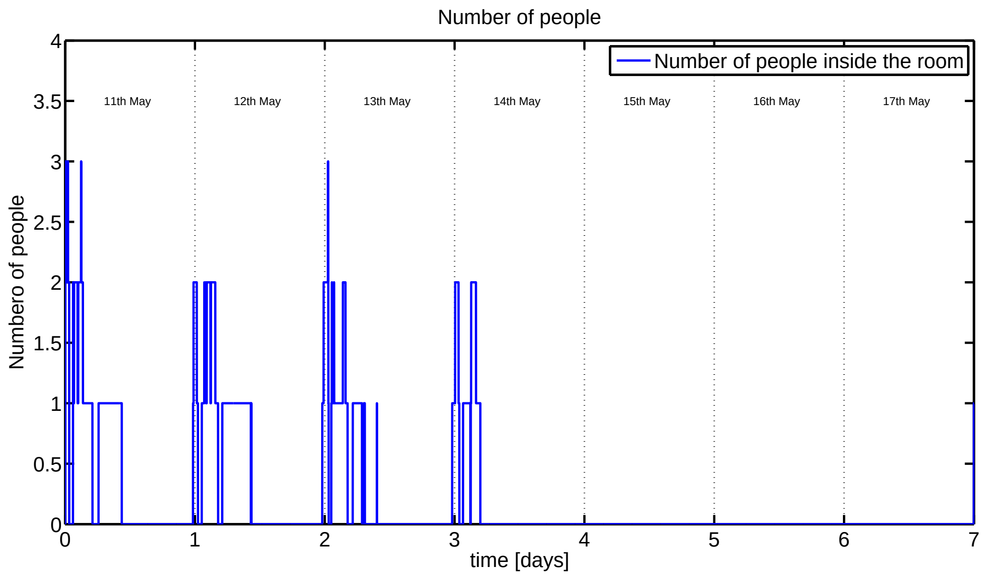

Figure 13: Results related to the real acquisition dataset during the month of May in CIESOL building. The number of people is changing during all test long.